Covariance Matrix Fitting for Aeroacoustic Application

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Introduction

Microphone arrays have become a useful and common tool for acoustic source localisation. In addition to the classical delay-and-sum (DAS) beamforming technique more sophisticated deconvolution algorithms like DAMAS [2] or CLEAN-SC [6] are widely used. These algorithms try to reconstruct a detailed source distribution map based on the rather blurred result obtained by the DAS algorithm.

The recent covariance matrix fitting (CMF) method pursues a different approach. It solves a convex optimisation problem in order to fit the unknown source powers to the measured cross spectral matrix through the sound propagation model.

In this contribution, the CMF method is applied to measurement data obtained for a reference setup in an aeroacoustic wind tunnel. The results are then compared with those acquired using beamforming and deconvolution algorithms. The accuracy of the method is discussed and some remarks on the computational cost are made.

Theory

Sound propagation model

The propagation of sound from a source position $\boldsymbol{x}$ to a receiver at position $\boldsymbol{y}$ can be calculated via a Green's function. Assuming the source can be described by a monopole the complex sound pressure amplitude at the receiver position for a discrete frequency ω is defined by

$$p(r,\omega) = q_0 \frac{1}{r} \mathrm{e}^{-\mathrm{j}\omega\frac{r}{c_0}} \;, \tag{1}$$

with q_0 being the source strength, $r = \|\boldsymbol{y} - \boldsymbol{x}\|$ denoting the distance between source and receiver, and the speed of sound c_0 .
The signals from any given source are evaluated at a reference point y_0:

$$p_0(r_0,\omega) = q_0 \frac{1}{r_0} \mathrm{e}^{-\mathrm{j}\omega\frac{r_0}{c_0}} \;. \tag{2}$$

Inserting (2) into (1) such that q_0 is eliminated yields the transfer function

$$p(r,\omega) = a \cdot p_0(r_0,\omega) \tag{3}$$

with

$$a = \frac{r_0}{r} \mathrm{e}^{\mathrm{j}\omega\frac{r_0 - r}{c_0}} \;. \tag{4}$$

Equation (3) calculates the sound pressure amplitude at a receiver position depending on the sound pressure induced at the reference position by the source.

To evaluate the sound pressure at M receiver positions equation (3) can be expanded:

$$\boldsymbol{p} = \begin{pmatrix} p(\boldsymbol{y}_1,\omega) \\ p(\boldsymbol{y}_2,\omega) \\ \vdots \\ p(\boldsymbol{y}_M,\omega) \end{pmatrix} = \begin{pmatrix} \frac{r_0}{r_1}\mathrm{e}^{\mathrm{j}\omega\frac{r_0-r_1}{c_0}} \\ \frac{r_0}{r_2}\mathrm{e}^{\mathrm{j}\omega\frac{r_0-r_2}{c_0}} \\ \vdots \\ \frac{r_0}{r_M}\mathrm{e}^{\mathrm{j}\omega\frac{r_0-r_M}{c_0}} \end{pmatrix} \cdot p_0(r_0,\omega) \;. \tag{5}$$

To account for more than one source the source region can be discretized by a grid of N source positions. Defining the transfer function from source to receiver by

$$a_{m,n} = \frac{r_{0,n}}{r_{m,n}} \mathrm{e}^{\mathrm{j}\omega\frac{r_{0,n}-r_{m,n}}{c_0}} \;, \tag{6}$$

$$m = 1 \ldots M, \quad n = 1 \ldots N, \tag{7}$$

the system now is described by

$$\boldsymbol{C} = \boldsymbol{A}\boldsymbol{D}\boldsymbol{A}^{\mathrm{H}} \tag{8}$$

with $\boldsymbol{C} = \boldsymbol{p}\boldsymbol{p}^{\mathrm{H}}$ being the cross spectral matrix (CSM) or covariance matrix with the dimension $M \times M$, the $N \times N$ matrix $\boldsymbol{D} = \boldsymbol{p}_0\boldsymbol{p}_0^{\mathrm{H}}$, and the transfer matrix

$$\boldsymbol{A} = \begin{pmatrix} a_{1,1} & \cdots & a_{1,N} \\ \vdots & \ddots & \vdots \\ a_{M,1} & \cdots & a_{M,N} \end{pmatrix} \;. \tag{9}$$

The $^{\mathrm{H}}$ denotes the Hermitian transpose, i.e. the transpose with complex conjugated entries.

The CMF method

To obtain an approximation $\hat{\boldsymbol{C}}$ of the CSM when performing acoustic measurements, the time signal recorded by each microphone is windowed and FFT-transformed. The cross spectra for each frequency band are then averaged over all calculated windows.
To account for modelling and measurement errors, a quantity σ^2 for uncorrelated additional noise is introduced into the model (8):

$$\hat{\boldsymbol{C}} = \boldsymbol{A}\boldsymbol{D}\boldsymbol{A}^{\mathrm{H}} + \sigma^2 \mathbf{I} \;. \tag{10}$$

While $\hat{\boldsymbol{C}}$ is known from the measurements and the transfer matrix $\boldsymbol{A}$ from modelling the sound propagation with monopoles the matrix $\boldsymbol{D}$, which contains the cross spectrum of the sound pressure amplitudes, is not known à priori.
Yardibi et al. [8] proposed the so-called "Covariance Matrix Fitting" by assuming that all sources are uncorrelated and $\boldsymbol{D}$ therefore only has entries d_n on its diagonal and solving

$$\operatorname{minimize}_{\{d_n\}_{n=1}^{N},\sigma^2} \|\hat{\boldsymbol{C}} - \boldsymbol{A}\boldsymbol{D}\boldsymbol{A}^{\mathrm{H}} \;\; - \sigma^2\boldsymbol{I}\|_{\mathrm{F}}^2 \tag{11}$$

$$\text{subject to} \;\; d_n \geq 0 \;, \quad \sigma^2 \geq 0 \tag{12}$$

with an additional sparsity constraint on the sum of the entries d_n. The subscript $_\mathrm{F}$ denotes the Frobenius norm (i.e. the sum of all squared entries of a matrix).
Earlier on, a similar approach called "Spectral Estimation Method" was proposed by Blacodon & Elias [1], who reasoned that the system can be solved by minimizing the error σ^2 between model and measurement and thus simply by solving

$$\mathrm{minimize}_{\{d_n\}_{n=1}^{N}} \|\hat{\boldsymbol{C}} - \boldsymbol{A}\boldsymbol{D}\boldsymbol{A}^{\mathrm{H}}\|_{\mathrm{F}}^2 \,. \qquad (13)$$

In the presented work the system is reduced even further in leaving out the main diagonal of $\hat{\boldsymbol{C}}$, since it does not contain any information about phase differences between the signals or any additional information about the amplitudes but comprises uncorrelated additional noise from the measurements. For solving, equation (13) with reduced $\hat{\boldsymbol{C}}$ is brought into a form that can be solved via the Least Angle Regression Lasso algorithm (LARS-Lasso [7]).

Measurements

The experiments were conducted in an aeroacoustic wind tunnel at BTU Cottbus. As can be seen in Fig. 1, the setup consists of an airfoil with chord length 235 mm mounted in the open jet in front of the nozzle. The flow speed at the nozzle exit is 50 $\frac{\mathrm{m}}{\mathrm{s}}$. For the acoustic measurements an array consisting of 56 microphones and positioned 68 cm above the airfoil is used. A more detailed description of the measurements and the setup is given in a previous paper [3].
The measured data was processed using Orthogonal Beamforming [5], DAMAS [2] with 100 iterations, CLEAN-SC [6], and the presented CMF method.

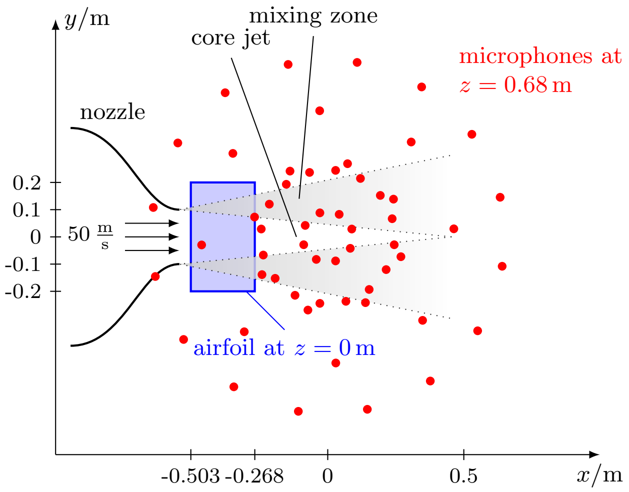

Figure 1: Setup used for the measurements [3].

Results

For a qualitative comparison of the four methods sound maps for the third-octave bands with centre frequencies 2000 Hz and 8000 Hz were generated. The dotted outline in the Figures 2 and 3 show the location of the airfoil in the map, the dashed line marks the trailing edge region used for integration of the calculated levels.

As can be seen in Figure 2, the main noise generating

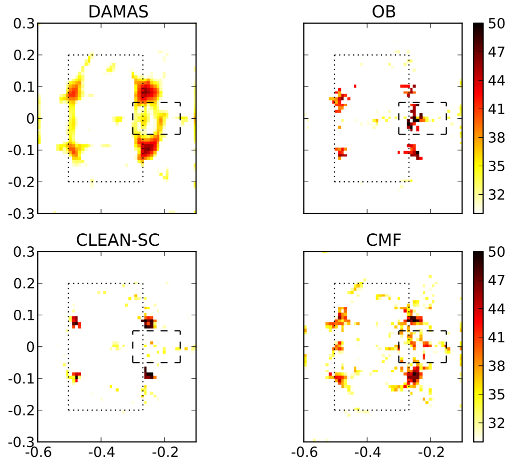

Figure 2: Resulting sound maps (scale in dB) at 2000 Hz third-octave band.

mechanisms at 2000 Hz are the interactions between the shear layer and the leading and trailing edge of the airfoil. This is confirmed by all methods. While CLEAN-SC and Orthogonal Beamforming show quite sharp peaks, the resulting maps of DAMAS and CMF seem more blurred but show a larger source region at the trailing edge, as can be expected from a broader shear layer. When looking at the framed trailing edge region, all methods differ in their reconstruction of the exact source positions, with OB yielding the highest level.
For higher frequencies trailing edge noise is more dominant.

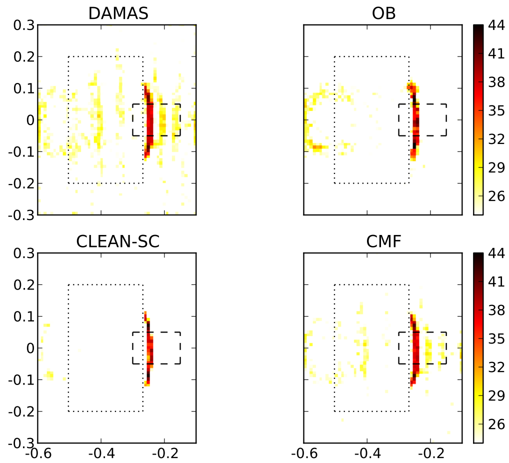

Figure 3: Resulting sound maps at 8000 Hz third-octave band.

At frequency 8000 Hz (Figure 3) all methods yield more or less the same result, DAMAS and CMF reconstruct secondary sources (10 dB below the highest level) up- and downstream from the main source at the trailing edge.
To compare the results of the localisation methods quantitatively, the reconstructed sound pressure levels at third-octave bands between 500 Hz and 20000 Hz were

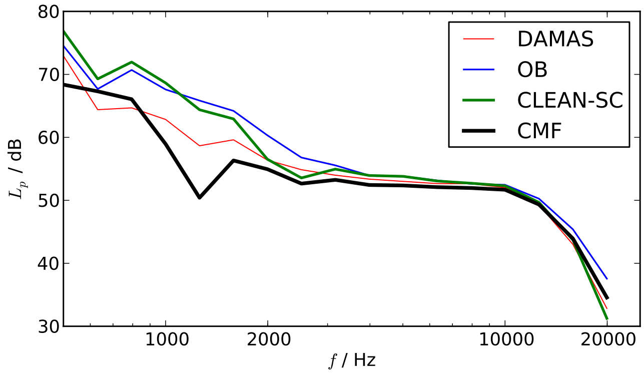

Figure 4: Third-octave band spectra integrated over the trailing edge core jet region.

method	time (seconds)
DAMAS	58
CLEAN-SC	1.1
OB	1.1
CMF	86

Table 1: Mean calculation time for one frequency.

integrated over a rectangular region that comprises the core jet at the trailing edge of the airfoil. The resulting spectra are shown in Figure 4.
For frequencies above 3000 Hz the sound pressure level at the trailing edge as reconstructed by the methods is nearly the same for all algorithms. At lower frequencies the levels differ more. The highest deviation between the localisations can be observed at 1250 Hz with a range of 15 dB between the reconstructed levels and CMF yielding the lowest value by 8 dB. The question which method is giving the correct level cannot be answered with certainty and is subject to further research.

Computational cost

All calculations were carried out on a 2.2 GHz CPU each. The used focus grid (i.e. the assumed source positions) has 3111 points. An array of 56 circular arranged microphones was used. The time needed by each method for the calculation of the map for one frequency is compared in Table 1.
Orthogonal Beamforming and CLEAN-SC are by far the fastest algorithms calculating only for roughly one second. DAMAS (100 iterations) is considerably slower, with one minute computing time, whereas the CMF method calculates even longer with one minute and a half.
However, these values highly depend on the implementation of the methods. In our case, all algorithms are implemented in Python except for the most CPU intensive parts which were implemented in C. The CMF algorithm is completely implemented in Python using the *scikit-learn* module [4].
Concerning the memory usage, the cost of CLEAN-SC and Orthogonal Beamforming is negligible as well. The memory needed for DAMAS increases quadratically with the number of grid points ($\propto N^2$) while the memory for CMF grows linearly with grid size and number of microphones ($\propto M \cdot N$). In the case studied here this amounts to roughly 75 MB per frequency using DAMAS and about 1.5 MB using CMF.

Conclusion

The inverse Covariance Matrix Fitting method has successfully been implemented and applied to measured data.

In contrast to established source localisation methods like DAMAS, CLEAN-SC and Orthogonal Beamforming, the presented algorithm is not based on beamforming but inversely solves an equation describing the sound propagation from sources to receiver positions. The comparison of CMF with sophisticated beamforming algorithms shows that CMF is on par with these techniques.

The test case used was flow noise generated by an airfoil in the aeroacoustic wind tunnel at BTU Cottbus. While the results calculated at lower frequencies have to be analysed further to be validated, at high frequencies (above 3000 Hz) the maps generated by the different methods as well as the integrated sound pressure levels are consistent. For the test scenario used in this paper, the calculation using the CMF implementation takes considerably more time than the other algorithms.

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