Improving speed with orthogonal beamforming

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Berlin Beamforming Conference 2010

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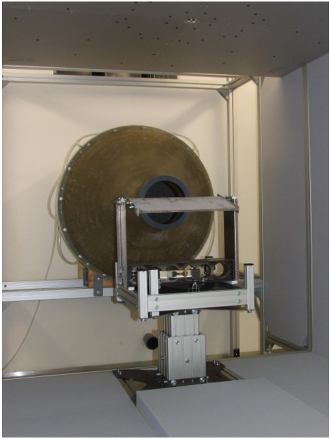
Airfoil trailing edge noise

Setup in open jet aeroacoustic wind tunnel



Porous airfoils

- experimental survey using 56ch phased array
- 17+ different airfoils
- U: 25...50 m/s
- α: -16...24°
- $\approx$ 3500 measurements
- fast method for absolute level determination needed !

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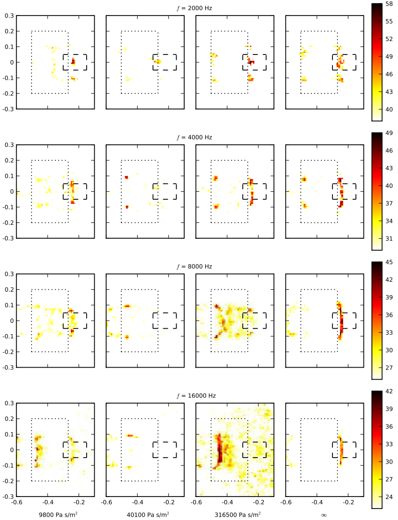
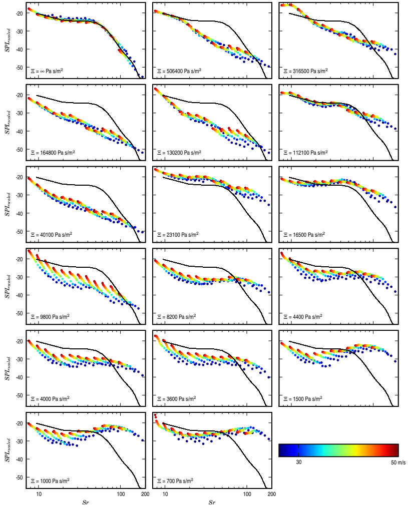
Airfoil trailing edge noise

Porous airfoils: some results





*T. Geyer et.al. Experiments in Fluids, Volume 48(2), 2010, 291 - 308

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How does it work?

Phased array beamforming (frequency domain)

- uses information from N microphone signals
- $N \times N$ cross spectral matrix (CSM)
- M sources (wanted+unwanted, $N > M$)

$\longrightarrow$ CSM has M non-zero eigenvalues

Eigendecomposition

CSM eigenvalues and eigenvectors

- contain all information about the sources
- M eigenvalues / eigenvectors $\longrightarrow$ M sources
- practical complication: "noise" in the signals

$\longrightarrow$ CSM has full rank ($N > M$ eigenvalues)

- two groups:
  - "large" eigenvalues (eigenvectors span signal subspace)
  - "small" eigenvalues (eigenvectors span noise subspace)

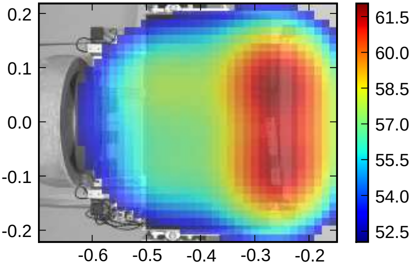
Hypothesis

- signal subspace eigenvalues map to sources

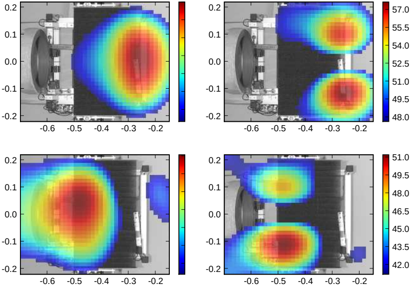
Example: Airfoil

2 kHz octave band

delay & sum



first four eigenvalues



- conventional delay & sum beamforming
- CSM resynthesised from eigenvalue / eigenvector pairs
- seems to work, but mapping is only *approximate*
- spatial resolution not improved

Improving resolution

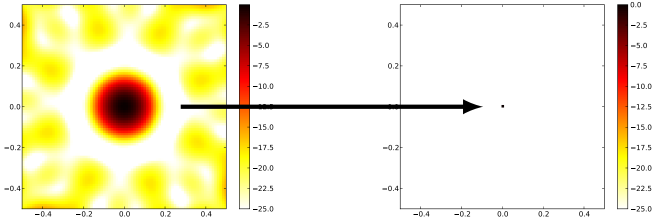
Beamformer as spatial filter

A source signal passes the filter without attenuation

B all other signals are attenuated as much as possible

Source location

- single source: maximum in map = source location



- multiple sources: ??? (maxima do not need to be sources)
- eigendecomposition may help !

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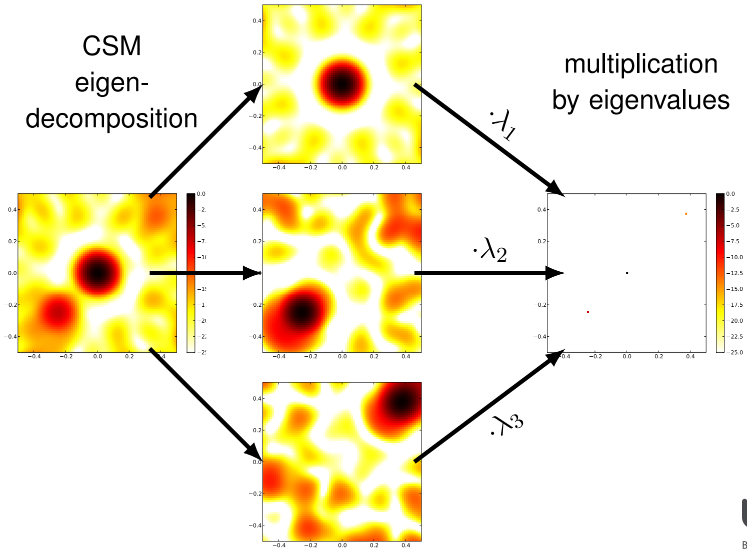
Orthogonal beamforming

Location and strength from eigendecomposition

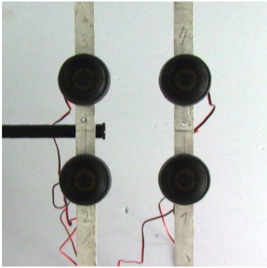


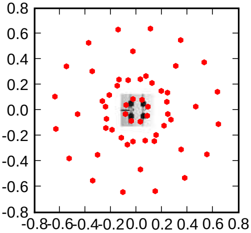
Algorithm(simplified)

- for each frequency:
  - compute cross spectral matrix (CSM, $N \times N$)
  - compute eigendecomposition $(\lambda_i, \mathbf{v}_i)$
  - estimate number of sources M
  - for each i in (1,...,M):
    - compute beamforming map from resynthesised CSM
    - store location of map maximum
    - store eigenvalue λ_i as source strength
  - new map: accumulate all strengths at stored locations

Generic test case

Four loudspeakers



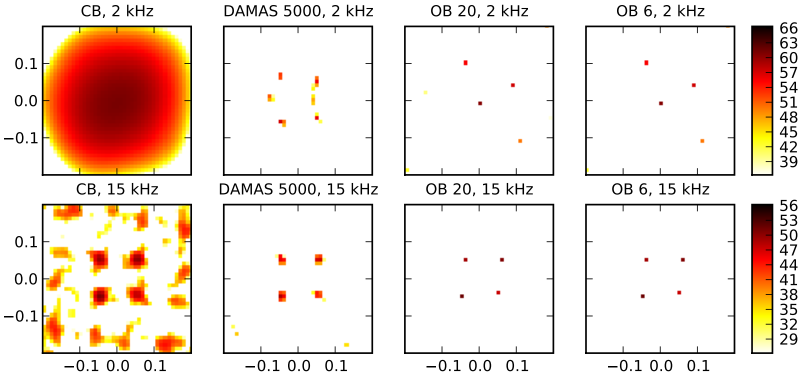


- four "identical" tweeters
- narrow spacing (10 cm)
- 56ch array, aperture (150 cm)
- distance 72 cm
- uncorrelated noise signals:

case I: "identical" amplitude
case II: 0, -6, -12, -18 dB

Case I: identical amplitudes

Maps for 2 kHz and 15 kHz frequency line



- conventional delay & sum (CB)
- DAMAS (5000 iterations)
- orthogonal beamforming (OB) with M=20 and with $M = 6$

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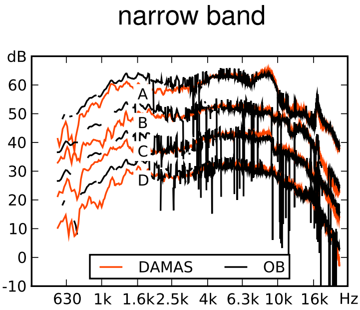
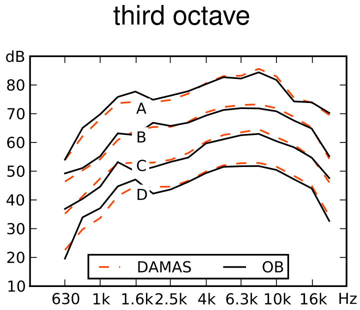
Case I: identical amplitudes

Spectrum





- integration over loudspeaker sectors A, B, C, D
- B, C, D shifted by -10 dB, -20 dB, -30 dB respectively

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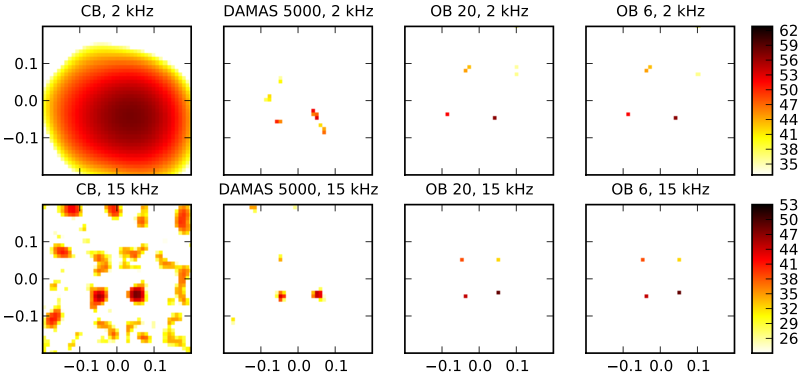
Case II: different amplitudes

Maps for 2 kHz and 15 kHz frequency line



- conventional delay & sum (CB)
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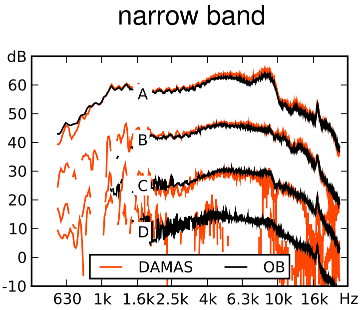
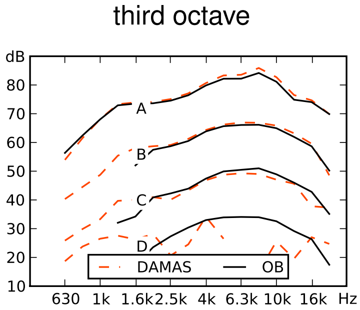
Case II: different amplitudes

Spectrum





- integration over loudspeaker sectors A, B, C, D
- B, C, D shifted by -10 dB, -20 dB, -30 dB respectively

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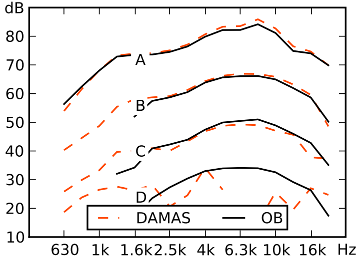
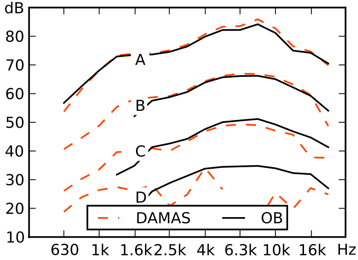
Case II: additional noise

Spectrum

no noise



+ noise (SNR 3 dB)



- additional noise in each microphone channel (-3 dB)

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Errors

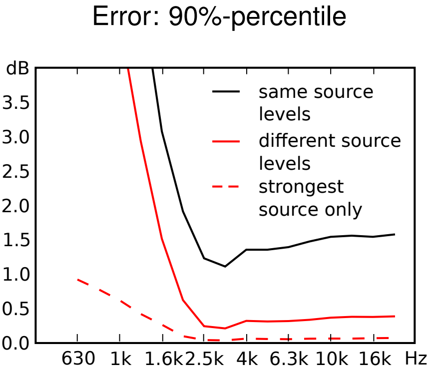
From test case:

- good performance for high frequencies and different source strengths
- low frequencies: imprecise localisation
- same source strengths: errors in source strength estimation

Theory

- mapping assumption (signal subspace eigenvalues - sources) is approximate
- theoretical error bounds depend on difference of source strengths (Gershgorin Circle Theorem 1931)
- practical error bounds from Monte Carlo simulation

Errors: Monte Carlo simulation



- 4 loudspeakers at random positions
- statistics from 10,000 runs
- small error in many relevant cases

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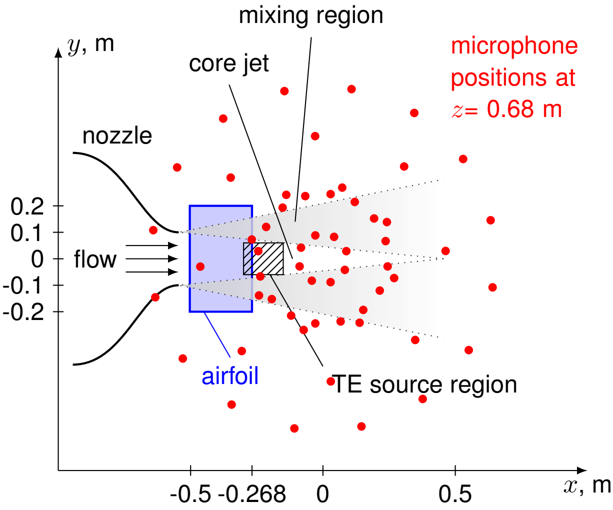
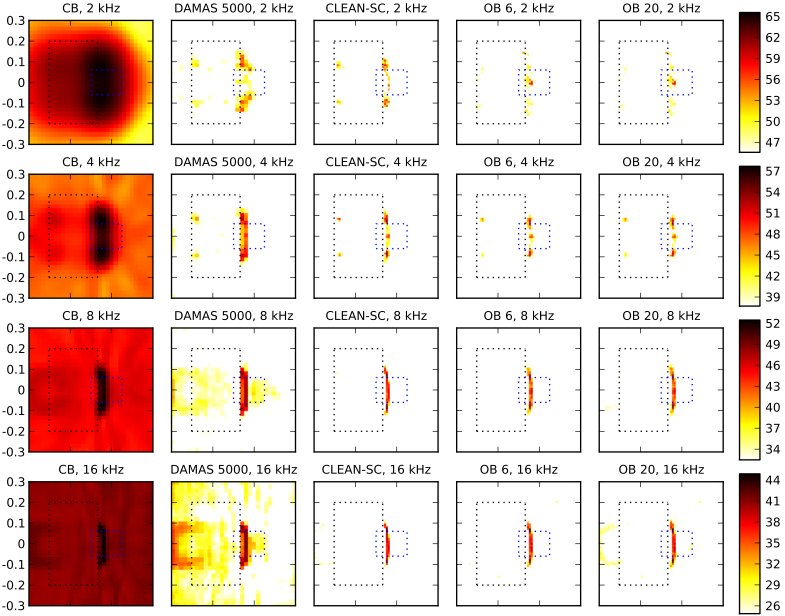
Practical test case

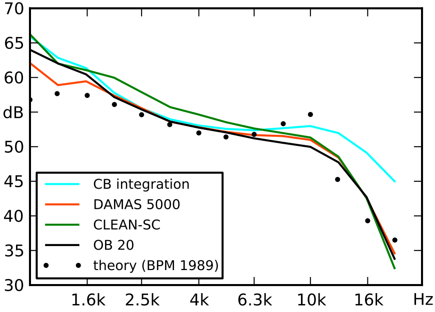
Airfoil trailing edge noise - setup




CB, 2 kHz
DAMAS 5000, 2 kHz
CLEAN-SC, 2 kHz
OB 6, 2 kHz
OB 20, 2 kHz
CB, 4 kHz
DAMAS 5000, 4 kHz
CLEAN-SC, 4 kHz
OB 6, 4 kHz
OB 20, 4 kHz
CB, 8 kHz
DAMAS 5000, 8 kHz
CLEAN-SC, 8 kHz
OB 6, 8 kHz
OB 20, 8 kHz
CB, 16 kHz
DAMAS 5000, 16 kHz
CLEAN-SC, 16 kHz
OB 6, 16 kHz
OB 20, 16 kHz
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
65 62 59 56 53 50 47
57 54 51 48 45 42 39
52 49 46 43 40 37 34
44 41 38 35 32 29 26

Practical test case

Airfoil trailing edge noise - results



- medium and high frequencies:
  - good agreement with theory
  - performance comparable to DAMAS/CLEAN-SC
  - better than integration of maps from delay and sum (CB)

Improving **speed** ... ?

Delay and Sum

- vector-matrix-vector multiplication ($\hat{\mathbf{G}}$ = CSM)

$$B(\mathbf{x}_t) = \mathbf{h}^H(\mathbf{x}_t)\hat{\mathbf{G}}\mathbf{h}(\mathbf{x}_t)$$

- $4(N^2 + N)$ flop per grid point

Orthogonal beamforming

- vector-vector multiplication ($\hat{\mathbf{G}}_i = \lambda_i \mathbf{v}_i \mathbf{v}_i^H$, resynthesised CSM)

$$B_i(\mathbf{x}_t) = \mathbf{h}^H(\mathbf{x}_t)\hat{\mathbf{G}}_i\mathbf{h}(\mathbf{x}_t) = \lambda_i |\mathbf{h}^H(\mathbf{x}_t)\mathbf{v}_i|^2, \qquad i = 1 \ldots M$$

- $(4N + 1)M$ flop per grid point (+ maximum finding)
- can be faster than delay and sum !
- easily parallelisable (one thread per eigenvalue)

Speed

Practical results

Loudspeaker example

- $N = 56$ (microphones)
- grid size 41×41=1681
- 2048 frequency bins
- time (incl. steering vector calculation)
  - CB: 190 s
  - OB ($M = 6$): 119 s
  - OB ($M = 20$): 185 s

Orthogonal beamforming - conclusions

Orthogonal beamforming

- signal subspace method
- suppresses noise effectively

Determination of absolute levels

- results comparable to deconvolution methods for medium and high frequencies
- works with minor sources (-20 dB)
- theoretical errors bounds established

Speed

- very fast, feasible for huge grids
- parallelisable

Reference

Ennes Sarradj: *"A fast signal subspace approach for the determination of absolute levels from phased microphone array measurements"*, Journal of Sound and Vibration 329 (2010), pp. 1553-1569, DOI: 10.1016/j.jsv.2009.11.009

- full mathematical details
- error bounds
- detailed example results

and now for something completely different ...

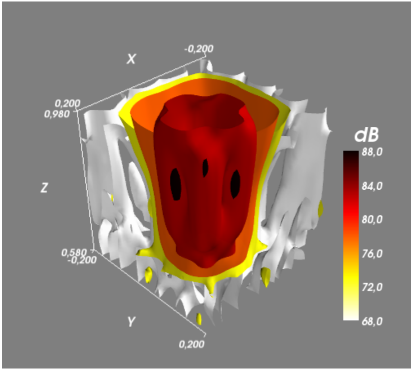
3D beamforming

- beamforming maps usually on planar grids (2D)
- reality is 3D ! (at least ;-)
- problem: 3D grids are huge (e.g. $50 \times 50 \times 50 = 125000$ points)
- solution: fast method
- problem: planar arrays – bad 3D resolution
- solution: deconvolution or similar method

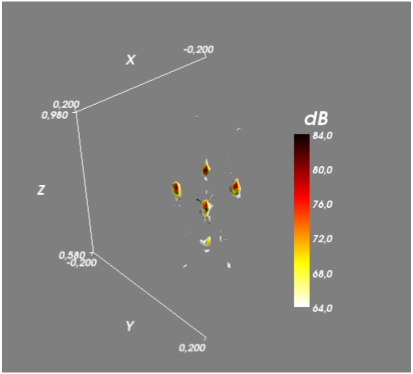
3D beamforming

Four loudspeakers - 4 kHz octave band

delay & sum



orthogonal beamforming



3D beamforming

Airfoil - 4 kHz octave band - orthogonal beamforming