GeoFlow: 3D numerical simulation of supercritical thermal convective states

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Outline

Research on convection in spherical shells, geophysically motivated

- central symmetry of buoyancy force field to simulate gravity fields
- parameter values very high (turbulence)

  e.g. Ra_{core} up to 10^{30}
- rich variety of influences

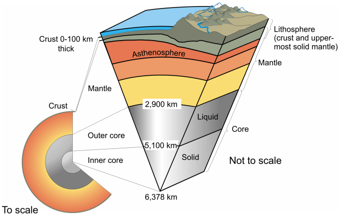
  e.g. Coriolis force due to rotation, centrifugal force due to rapid rotation

  e.g. differential rotation vs. rigid body rotation
- physical properties of fluids

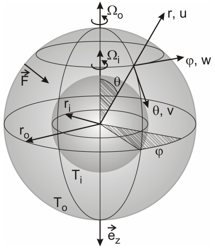
  e.g. low, moderate or high viscosity
- and the magnetic field?

GeoFlow experiment: spherical Rayleigh-Bénard conv.

- central force field by dielectrophoretic effect
- non-rotating case (Busse, 1982, J. Fluid Mech.)
- rotational effects: transition to stabilizing effects by rapid rotation (Busse, 2002, Phys. Fluids)
- without magnetic effects



Earth's interior, Kious (1996)



Rotating spherical model, inner sphere heated, with central force field

Equations for convection in rotating spherical shells with dielectric force field

$$\nabla \cdot \mathbf{U} = 0$$

$$Pr^{-1}\left[\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U}\cdot\nabla)\mathbf{U}\right] = -\nabla p + \nabla^2\mathbf{U} + \frac{Ra_{centr}\ T}{\beta^2 r^5}\,\hat{\mathbf{e}}_r$$
$$-\sqrt{Ta}\,\hat{\mathbf{e}}_z \times \mathbf{U} + \widetilde{Ra}\ T\ r\sin\theta\hat{\mathbf{e}}_{eq}$$

$$\frac{\partial T}{\partial t} + \mathbf{U}\cdot\nabla T = \nabla^2 T$$

no-slip boundary conditions for velocity $\mathbf{U}$,
temperature fixed $T(\eta) = 1$, $T(1) = 0$

Parameters

geometry	radius ratio	$\eta = \frac{r_i}{r_o}$
physical prop. of fluid	Prandtl number	$Pr = \frac{\nu}{\kappa}$
buoyancy (central force)	central Rayleigh number	$Ra_{centr} = \frac{2\epsilon_0\epsilon_r\gamma}{\rho\nu\kappa}\ V_{\mathrm{rms}}^2\,\Delta T$
Coriolis force	Taylor number	$Ta = \left(\frac{2\Omega r_o^2}{\nu}\right)^2$
centrifugal force	additional Rayleigh number	$\widetilde{Ra} = \frac{\alpha\Delta T}{4}\ Ta\,Pr$

Spectral method

- decomposition of primary variables into poloidal and toroidal parts
- decomposition of these into Chebyshev polynomials and spherical harmonics
- solving equations for spectral coefficients

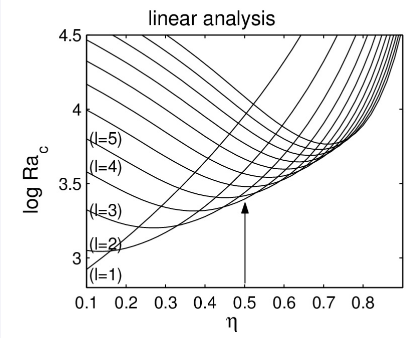
$$e(r,\theta,\varphi,t)=\sum_{m=0}^{M}\sum_{\ell=m'}^{L}\sum_{k=1}^{K}e_{k\ell m}T_{k-1}(x)P_{\ell}^{|m|}(\cos\theta)e^{im\phi}$$

- truncation with (K,L,M)=(30,60,20)
- R. Hollerbach: *A spectral solution of the magneto-convection equations in spherical geometry*, *Int. J. Numer. Meth. Fluids*, 2000, **32**, 773-797

Experimental constraints

gap width	$r_o - r_i[mm]$	13.5	$\rightarrow$	η	0.5
viscosity	$\nu[m/s^2]$	$5 \cdot 10^{-6}$	$\rightarrow$	Pr	64.64
high voltage	$V_{rms}[V]$	10	$\}$	Ra_{centr}	$\leq 1.4 \cdot 10^5$
temperature grad.	$\Delta T[K]$	≤ 10			
rotation rate	$n[Hz]$	≤ 2	$\rightarrow$	Ta	$\leq 1.3 \cdot 10^7$
				$\widetilde{Ra}$	$\leq 9.37 \cdot 10^6$

Linear analysis for $Ta = 0$

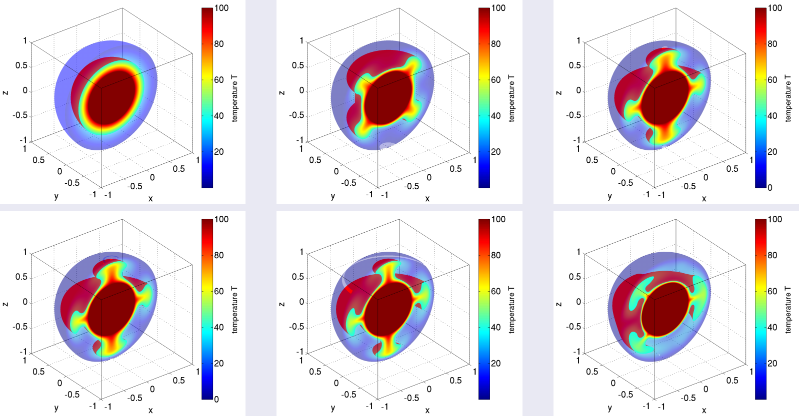


- critical $Ra_{central}$ for onset of convection independent on Pr
- larger $\eta \rightarrow$ larger critical mode l

Source: Travnikov, V., Egbers, C., Hollerbach, R.: The GEOFLOW-experiment on ISS. II: Numerical simulation, Adv. Space Res. 32 (2003), pp. 181–189
Travnikov V.: Thermische Konvektion im Kugelspalt unter radialem Kraftfeld, Dissertation, Cuvillier Verlag Göttingen, 2004

$\eta = 0.5$, $Pr = 64.64$, $Ta = 0$, Ra_{centr} increases: temperature field visualized in hemispherical shells, scaled to 100 %

$2 \cdot 10^3$ | $5 \cdot 10^3$ | $8 \cdot 10^3$



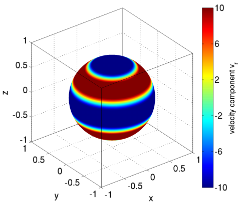
$1 \cdot 10^4$ | $2 \cdot 10^4$ | $5 \cdot 10^4$

$Ra_{central} = 2 \cdot 10^3$ - conductive state, $Ra_{central} \leq 2 \cdot 10^4$ - steady axisymmetric,
$Ra_{central} = 5 \cdot 10^4$ - time-dependent (irregular)

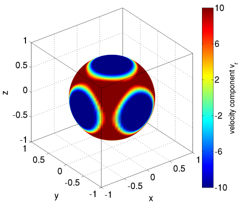
What is the stable state?

$\eta = 0.4, \ Pr = 64.64, \ Ta = 0$
$Ra_{central} = 5 \cdot 10^3$

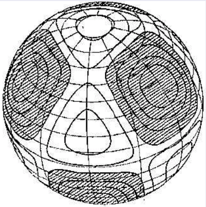
Busse (1975), J. Fluid Mech.



axisymmetric



cubic symmetry

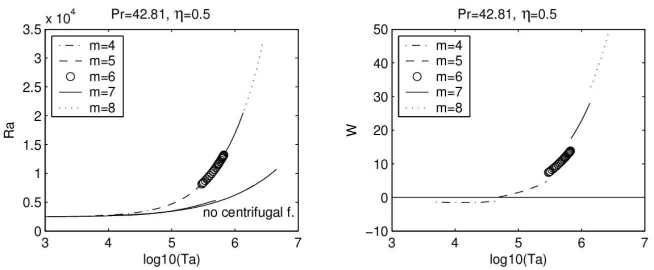


cubic sym.

More details in following talk:

- K. Bergemann, F. Feudel, L. Tuckerman: *GeoFlow: Symmetry breaking bifurcations in a spherical shell convection*

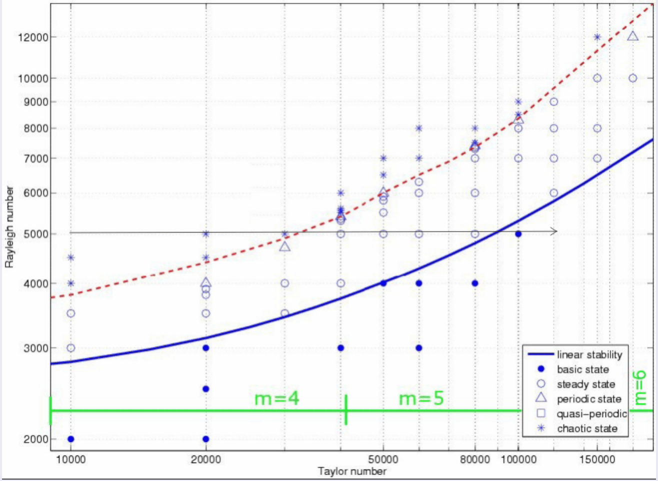
Linear analysis for $Ta \neq 0$



- shape of stability curves nearly independent on Pr
- instability due to Hopf bifurcation
- for large Ta: $Ra_{centr} \sim Ta^{2/3}$ (P. Roberts 1968, Philos. Trans. R. Soc. London)
- drift velocity W changes sign (slows down or fastens rotation)

Source: Travnikov et al. (2003), Travnikov (2004)

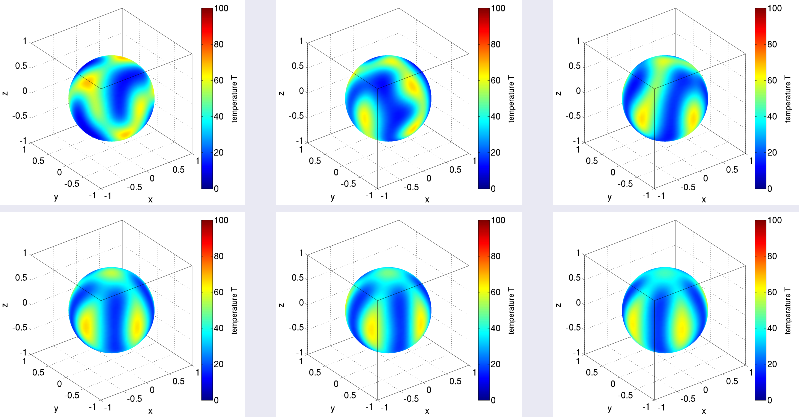
Stability diagram and flow states



Source: Gellert et al. 2005, J. Phys.: Conf. Ser.

$Ra_{centr} = 5 \cdot 10^3$, Ta increases: influence of initial conditions
temperature field visualized on spherical surface in the gap, scaled to 100 %

$1 \cdot 10^4$ $2 \cdot 10^4$ $4 \cdot 10^4$



$6 \cdot 10^4$ $8 \cdot 10^4$ $1 \cdot 10^5$

$Ra_{central} = 1 \cdot 10^4, 2 \cdot 10^4$ - irregular flow, $Ra_{central} = 4 \cdot 10^4$ - periodic flow,
$Ra_{central} \leq 1 \cdot 10^5$ - steady flow

Summary and Conclusion

- non-rotating case
  - stability analysis gives onset of convection
  - DNS shows transition from steady to irregular flow
  - request for analyses of stable states with path following methods $\rightarrow$ K. Bergemann et al.
  - resolve of transition to chaos
  - timeseries analysis with nonlinear methods
- rotating case
  - stability analysis shows influence of centrifugal forces
  - DNS shows complex pattern drift
  - request for path following methods for steady states
  - hydrodynamic instabilities: transition to chaos