GeoFlow: Numerical Simulations for the Non-Rotating Case

K. Bergemann[1], F. Feudel[1], L.S. Tuckerman[2]

[1] University of Potsdam, Germany
[2] PMMH-ESPCI, Paris, France

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Outline

- Experimental setup built by Astrium GmbH (Friedrichshafen)

Equations

Governing equations

$$Pr^{-1}\left[\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\right] = -\nabla p + \nabla^2 \boldsymbol{u} + \frac{Ra\,T}{\beta^2 r^5}\boldsymbol{e}_r - \sqrt{Ta}\,\boldsymbol{e}_z \times \boldsymbol{u}$$

$$+\widehat{Ra}\,T\,r\,\sin\theta\,\boldsymbol{e}_{\mathbf{eq}}$$

$$\nabla\cdot\boldsymbol{u} = 0\ , \qquad \frac{\partial T}{\partial t} + \boldsymbol{u}\cdot\nabla T = \nabla^2 T$$

with no-slip b.c. for $\boldsymbol{u}$ and $T(\eta) = 1, T(1) = 0$

Parameters:

$$\eta = \frac{R_1}{R_2} = 0.5,\ Pr = \frac{\nu}{\kappa} = 64.64,\ Ta = \left(\frac{2\Omega R_2^2}{\nu}\right)^2,\ Ra = \frac{2\epsilon_0\epsilon_r\gamma}{\rho\nu\kappa}V_{\mathrm{rms}}^2\,\Delta T$$

($\widehat{Ra} = \frac{\alpha\Delta T}{4}\,Ta\,Pr$ is not independent)

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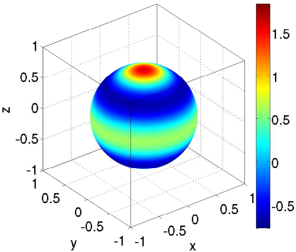
Numerical methods

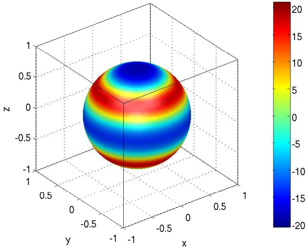
- Pseudo-spectral time-stepping code using decomposition of the primary variables into poloidal and toroidal potentials
  (R. Hollerbach, 2000)
- Steady-state solving via Newtons method used to perform path-following of stationary branches
- quadratic extrapolation in order to trace solutions around turning points
- Computation of the leading eigenvalues of the Jacobian matrix via the Arnoldi method
  (C.K. Mamun, L.S. Tuckerman, 1995)

States after onset of convection at Ra $= 2491$

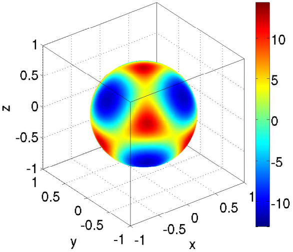
axisymmetric state at Ra$= 2500$



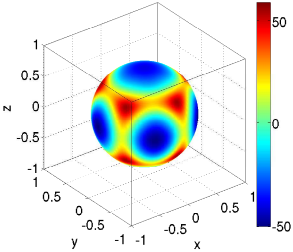
different axisym. state at Ra$= 4000$



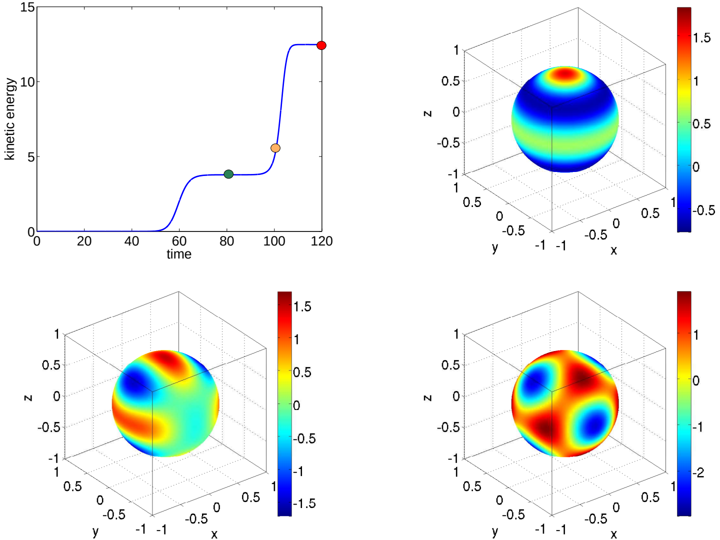
cubic symmetric state at Ra$= 3000$



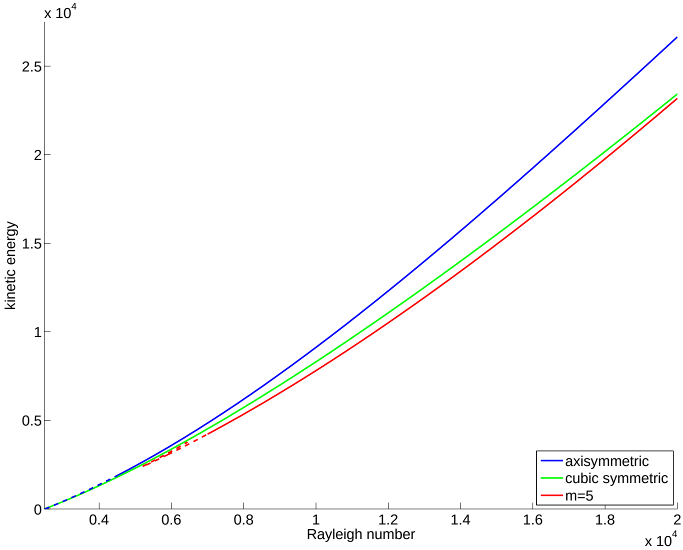
$m = 5$ state at Ra$= 9000$



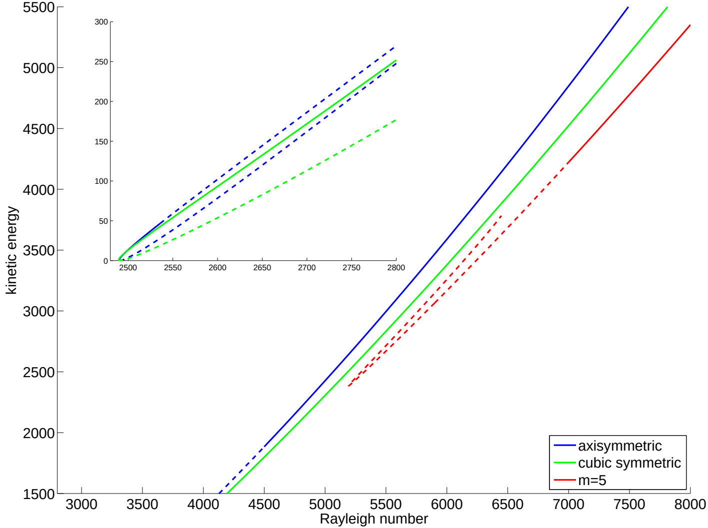
Time evolution of the system at Ra $= 2500$



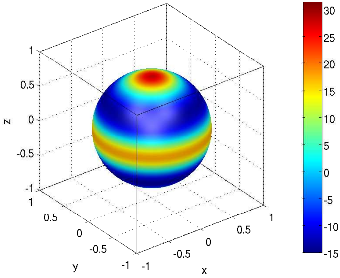
Bifurcation diagram



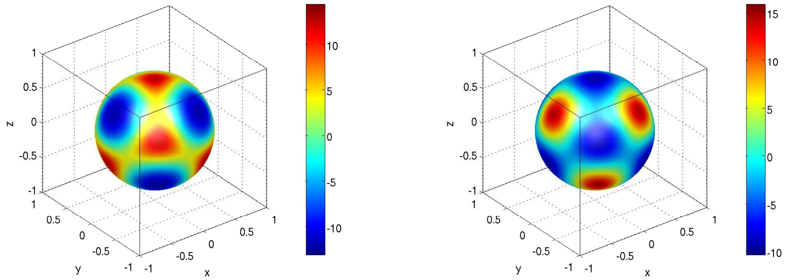
Bifurcation diagram (zoom)



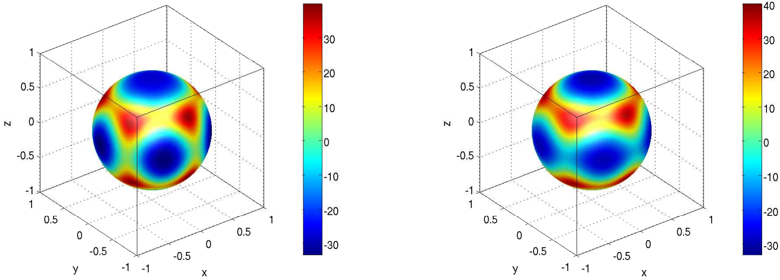
Axisymmetric steady-states at Ra $= 4000$



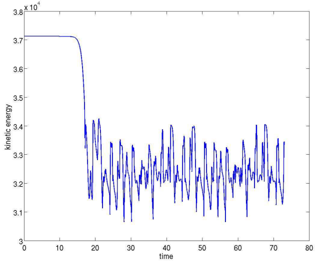
Cubic symmetric steady-states at Ra $= 3000$

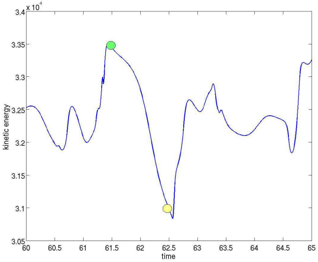


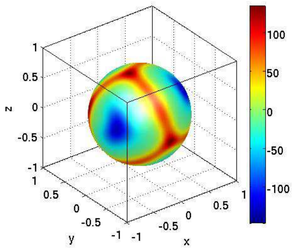
$m = 5$ states at Ra $= 6000$

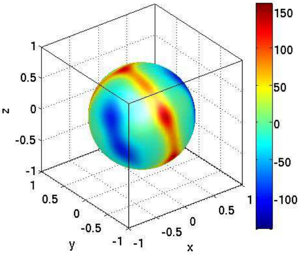


Onset of time-dependency found numerically for Ra $= 28000$



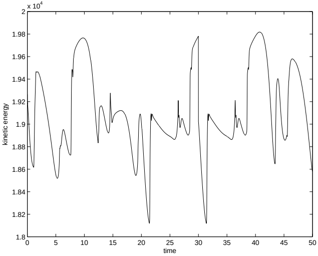


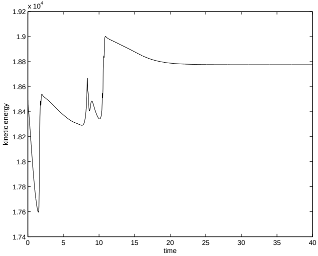


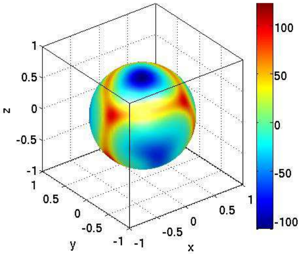


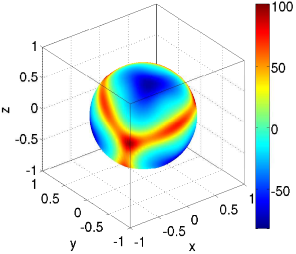
Remnant of a tetrahedral symmetry!

Hysteresis of chaos at Ra $= 19000$ (left side) frozen state is reached for Ra $= 18600$ (right side)









Conclusion & Outlook

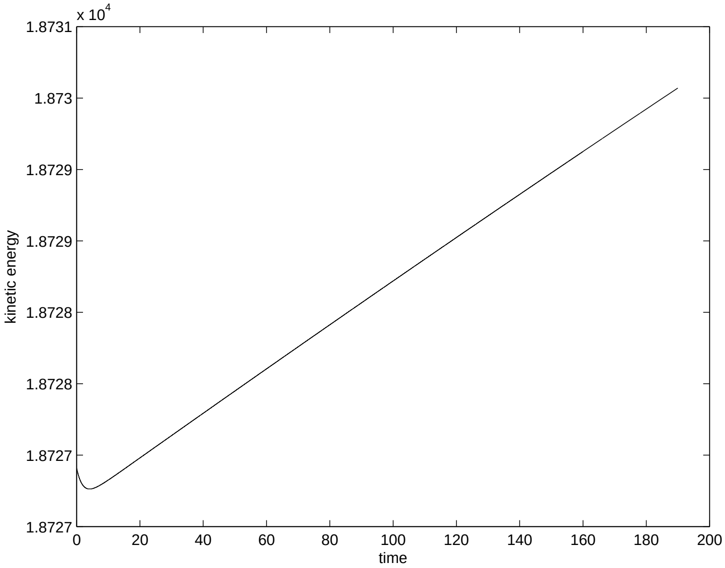
Conclusion

- Stationary solution branches have been computed by means of a path-following method
- Existence of multistability
- Sudden onset of chaos for $\mathrm{Ra} > 28000$
- Hysteresis behaviour of the chaotic branch resulting in frozen states with tetrahedral symmetry

Outlook

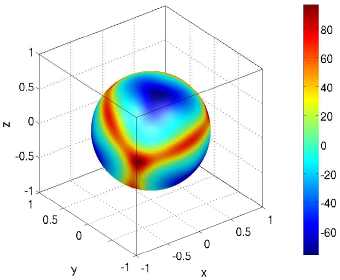
- Further refinement of the stability regions of the steady states
- Application of continuation for the rotating case ($\mathrm{Ta} \neq 0$)
- Comparison of experimental results and theoretical predictions

$E(t)$ for tetrahedral state



Tetrahedral state at Ra $= 18600$

t=30:



t=130: