Noise in Linear And Non-Linear Circuits And Systems

by

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Topics

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Linear and Non-Linear Systems

- A circuit is considered **linear** if the output of the circuit has a linear relationship with the input signal, and the circuit/system response follows **the law of superposition theorem** so that output response can be expressed as a linear combination of their responses to their individual inputs, which is described by Equations (1)-(3) for all values of constants k_1 and k_2 as

$$x_1(t) \rightarrow y_1(t) \qquad (1)$$

$$x_2(t) \rightarrow y_2(t) \qquad (2)$$

$$k_1 x_1(t) + k_2 x_2(t) \rightarrow k_1 y_1(t) + k_2 y_2(t) \qquad (3)$$

- **Non-linear** systems does not obey the law of superposition !

Linear and Non-Linear Systems, Cont'd.

Example:

- 1 dB increase at the input results exactly in a 1 dB increase at the output. For a fixed input frequency, the phase of the output signal does not change with the amplitude of the input signal. Up to very large signals, this relationship is valid for all passive circuits.
- At very large signals above 100 Watts the metal connectors or cables can become non-linear due to the junctions of different metals.
- For extremely small input signals, typically slightly above the noise, active circuits such as amplifiers or mixers will also be linear and remains quasi-linear till it reaches the saturation.

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Linear and Non-Linear Systems, Cont'd.

- Once the input signal or input signals come close to 1/10 of the magnitude of the operating DC bias of the active device, non-linearity typically begins and the law of superposition does not hold any longer.

- In general, non-linearity for both single tone and multi-tone condition start where the beginning of gain compression starts (i.e. 1dB compression point)

- A standard measurement that shows very subtle compression occurrences is used for television signals, the measurement of differential gain and differential phase.

Linear and Non-Linear Systems, Cont'd.

- Large signal conditions caused from one or more large signals at the input overload result in saturation, intermodulation, and mixing of the various input signals.

- If the numbers and magnitude of the signals at the input of an active device are high enough, the resulting energy will also cause a operating DC bias shift, cross-modulation (a special form of inter-modulation) and an increase of the noise figure.

- The signals themselves then may become noisy as the noise currents and voltages in the active device mix with spectral pure/cleaner signals. A special form of this occurs in systems it is called blocking or reciprocal mixing. These combinations of non-linear effects are noticeable for one or more signals (tones) and occur in amplifiers, mixers and oscillators.

Noise in Linear Circuits/Systems

- Noise comes in various forms. Sitting at the beach and listening to the waves is listening to an acoustic form of noise.

- Likewise, water running from a faucet also generates noise. These are forms of mechanical noise, which the ear realizes based on input acoustic wave vibrations. When addressing "noise" it is typically referred to as white noise. White noise means that there are amplitude components from very low frequencies to very high frequencies (DC to light) of equal amplitude.

- Colored noise means that the noise has a random tilt in the frequency response. This can mean that there are more low frequency (LF) or more high frequency (HF) components or peaking.

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Noise in Linear Circuits, Cont'd.

•In electrical systems there are different types of noise. The most familiar noise is the Johnson noise (thermal noise), which is due to the movement of molecules in a solid device, for instance, a carbon resistor or a metal film resistor.

•The energy delivered from such a resistor based on the molecular movement is kT, where k is the Boltzmann constant, 1.38 exp –23 Watt/Kelvin, T is the Temperature in Kelvin. A typical value at room temperature is 290K, resulting in $290 \times 1.38 \times 10^{-23} = 4 \times 10^{-18}$ Watts, or –174dBm in a 1-Hz bandwidth.

•This equation can be solved for the quadratic voltage as $V_{noise} = \sqrt{4kTBR}$

With R being the resistor value, B = integrated bandwidth relative to 1Hz, the resulting voltage or more precisely the means square noise voltage (the non-terminated voltage, typically referred to as electrical motorical force, EMF).

•When properly terminated, the voltage is half of the open voltage.
Assume B = 10 MHz ,T = 290K, K=1.38 exp-23 and R = 75Ω, The resulting voltage is 1.73 uV (EMF).

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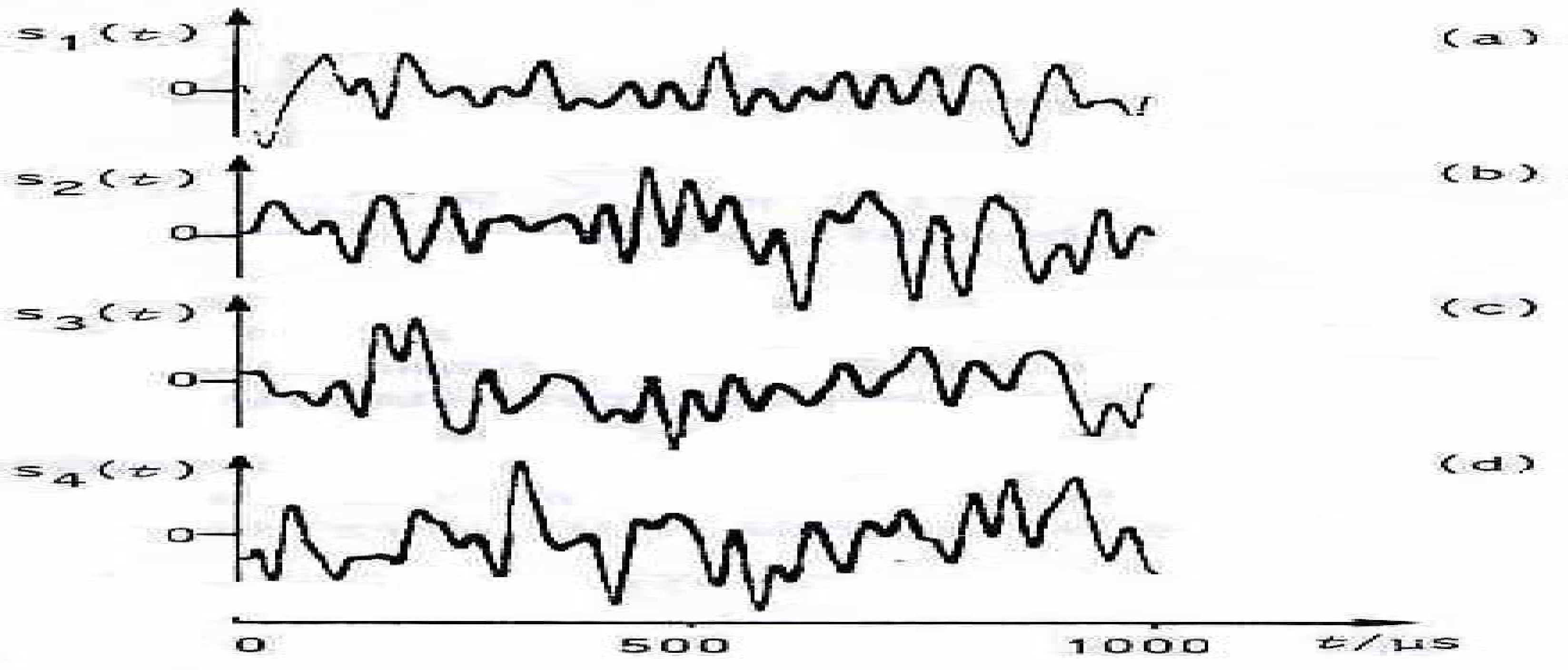
Characteristics of Noise Signal

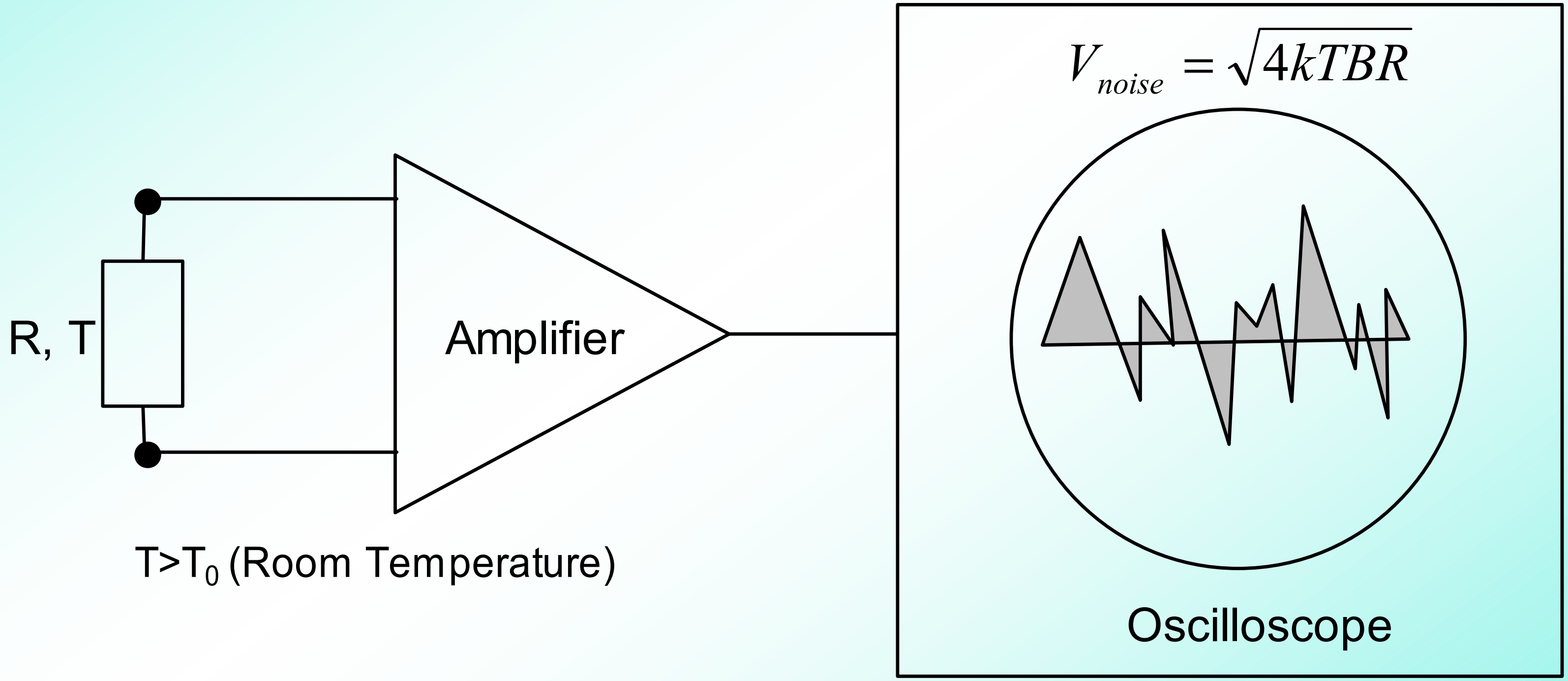
A white noise source at any given time or time intervals will show random signals that never repeat. If we have an oscilloscope with sufficient sensitivity to look at the noise at any given time, we would see a different random waveform presenting the noise voltage. Figure below shows four different voltages as a function of time ($S_1(t)$, $S_2(t)$, $S_3(t)$, and $S_4(t)$) that are totally random or stochastic.



S_1 (t), S_2 (t), S_3(t), and S_4(t) illustrate the display from a oscilloscope displaying a random noise generator.

Characteristics of Noise Signal, Cont'd.

The time characteristic of noise signals can be displayed with the help of an oscilloscope, if the thermal noise voltage of resistor is amplified sufficiently. The energy delivered from such a resistor based on the molecular movement is kT.



Where R is resistor value, B = bandwidth relative to 1Hz, k is the Boltzmann constant.

Concept of Noise Correlation

• Signals resulting from the same origin are always 100 % correlated. Thermal noise of resistor can be described by $V_{noise} = \sqrt{4kTBR}$. Let us assume two 50 Ω resistors, they can be connected either parallel or in series.

•The resulting noise power P ($P = \frac{[\sum_{i=1}^{n} V_i]^2}{R}$) from those two resistors will be the same.

• If we put two resistors in parallel we obtain 25 Ω, which would give the same noise as a single 25 Ω resister.

• If we put them in series, the resulting noise will be the same as a single 100 resistor.

• However, if each resistor has a different temperature then the noise energy from each resistor has to be specially calculated because noise level is temperature dependent, hence in this case noise generated from both the resistors are partially correlated.

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Concept of Noise Correlation

Another noise, which occurs in semiconductors, is the Shottky noise. The Shottky noise occurs in conducting PN junctions where electrons are freely moving. The root mean square (rms) noise current is given by

$$\overline{i_n^2} = 2 \times q \times I_{dc}; \qquad P = \frac{\overline{i_n^2}}{R}$$

Where q is the charge, P is power, and I_{dc} is the dc bias current.

Since the origin of this noise generated is totally different then the thermal noise, therefore, there is no correlation.

To describe the correlation of noise sources, we introduce a noise correlation coefficient. The correlation coefficient "C_r" is zero for 2-totally different noise sources (Thermal noise and Shottky noise) as discussed above, it can be 0.5 for partially correlated sources or is 1 for 100% correlated noise sources.

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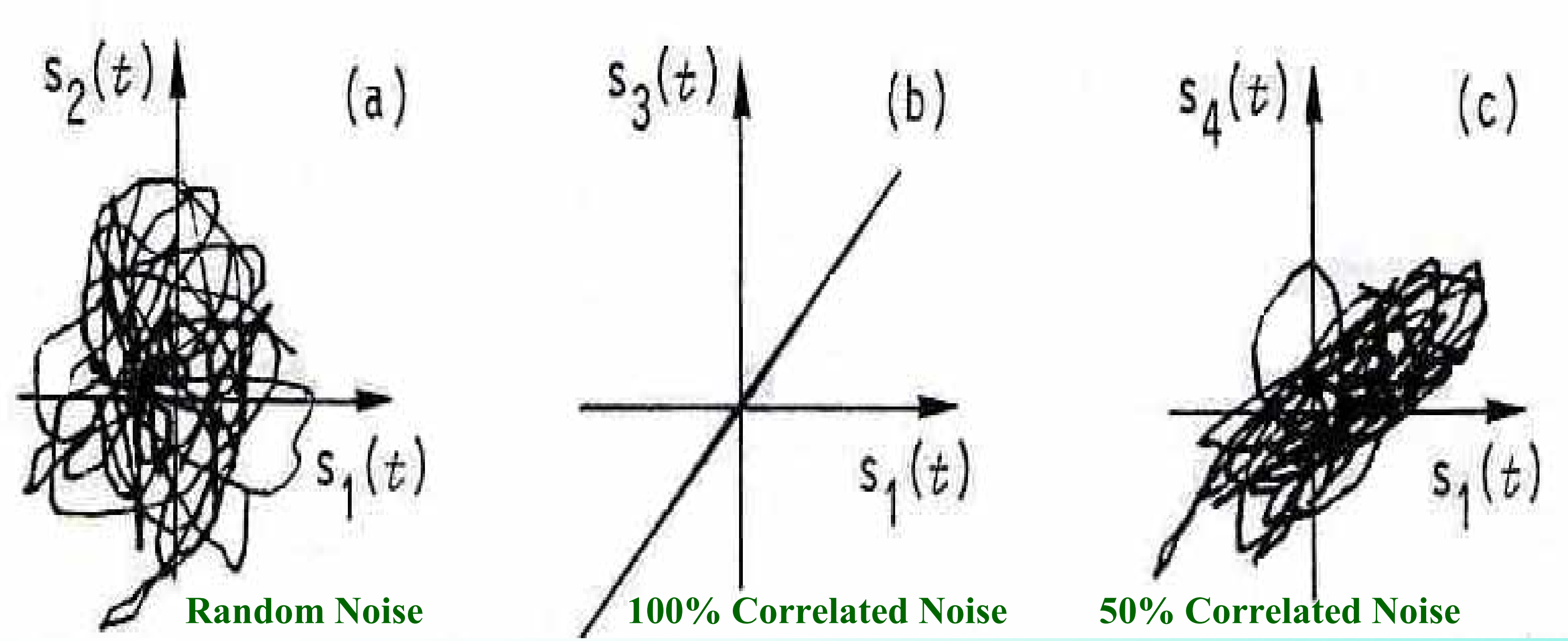
Concept of Noise Correlation,Cont'd.

Using an oscilloscope signal S_1 is connected to the X input and signal 2 is connected to the Y input. The left picture as shown in Figure 2(a), signal display with zero correlation between the two signals $S_1(t)$ and $S_2(t)$. The center figure shows the two signals $S_1(t)$ and $S_3(t)$ that are 100% correlated. The picture on the right side shows signal $S_1(t)$ and $S_4(t)$ that are 50% correlated.



Random Noise | **100% Correlated Noise** | **50% Correlated Noise**

Noise in Electronic Circuits

- **Noise is associated with all the components of the electronic circuits, however the major contribution of the noise in amplifier, mixer, and oscillator circuits are from the active device, which introduces AM (amplitude modulation) noise and PM (phase modulation) noise.**

- **There are mainly two types of noise sources in electronic circuit: broadband noise due to thermal and shot noise effects and the low-frequency noise source due to 1/*f* (flicker noise effects) characteristics.**

- **The resulting DC current flow in a transistor is not a continuous process but is made up of the diffusive flow of large number of discrete carriers and the motions of these carriers are random, and explain the noise phenomena and modulates the DC current.**

- **The thermal fluctuation in the carrier flow and generation-recombination processes in the semiconductor device generates thermal noise, shot noise, partition-noise, burst noise and 1/*f* noise.**

Noise in Electronic Circuits

Thermal noise, which always exists at non-zero temperatures, originates from variations of the lattice atoms, which are transferred to the free electrons. The electrons are thus performing an unsteady movement, being interrupted by collisions. These unsteady movements lead to an irregularly fluctuating voltage between both ends of the conductor. The available noise power of a resistor only depends on the absolute temperature of the resistor. Thermal noise is a relatively weak noise phenomenon, which can be further reduced by cooling.

Due to the fluctuation of the electrons around a time average, thereby, the flow of DC current cause to generate shot noise in the semiconductor devices.

The electrical properties of surfaces or boundary layers are influenced energetically by so-called boundary layer states, which are also subject to statistical fluctuations and therefore, lead to the so-called flicker noise or 1/*f* noise for the current flow. 1/*f* noise is observable at low frequencies and generally decreases with increasing frequency *f* according to 1/*f* -law until it will be covered by frequency independent mechanism, e.g. thermal noise or shot noise.

Noise in Linear Electronic Components, Cont'd.

Resistor:

The thermal movement of the electrons or holes in metals or semiconductors causes the noise in resistor, and this phenomenon is called Johnson noise or thermal noise.

The mean square values of the noise generator short circuit current in a narrow frequency interval Δf is given by

$$\overline{i^2(t)} = \frac{4kT}{R}\Delta f = 4kTG\Delta f$$

where resistance is denoted by R and the conductance by $G=1/R$.

Similarly, it can be found by a voltage measurement of the open circuit noise voltage as

$$\overline{v^2(t)} = 4kTR\Delta f$$

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Noise in Linear Electronic Components, Cont'd.

Resistor, Cont'd.

The spectral density function $S(f)$ represents the mean square value of the voltage or current, respectively, in 1Hz bandwidth, and can be given by

$$S_v(f) = \frac{\overline{v^2}}{\Delta f} = \frac{4kTR\Delta f}{\Delta f} = 4kTR; \qquad S_i(f) = \frac{\overline{i^2}}{\Delta f} = \frac{4kT\Delta f}{R\Delta f} = 4kTG$$

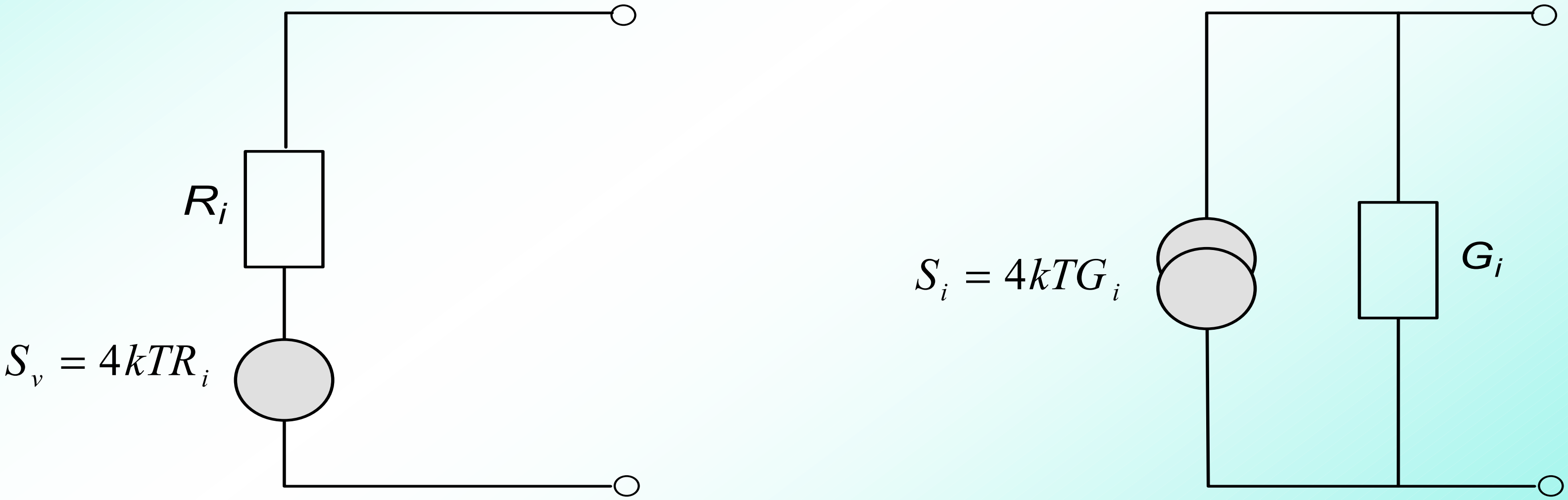
$$\overline{v^2}(t) = \int_{f_1}^{f_2} S_v(f)df; \qquad \overline{i^2}(t) = \int_{f_1}^{f_2} S_i(f)df$$

The spectral density function is also called the spectral distribution or spectrum or power spectrum. For thermal noise the spectral density function does not depend on the frequency, if the frequency is not too high and if the temperature is not too low.

Noise in Linear Electronic Components, Cont'd.

Resistor, Cont'd.

The Figure shows the noise equivalent circuits of a thermally noisy resistor with a voltage and current source. As depicted in the Figure the internal resistance R_i and the internal conductance G_i are noiseless. The voltage source is assumed to have zero internal resistance and the current source has infinite resistance.



(a) Voltage source

(b) Current source

Noise in Linear Electronic Components, Cont'd.

Resistor, Cont'd.

If several resistors at the same temperature are combined, then an equivalent circuit can be defined for the resulting circuit. The overall resistance is determined first and then an equivalent noise source is calculated or the equivalent noise source of all individual resistors is determined first and subsequently are combined. The necessary condition for this approach is that the noise sources are have zero cross correlation , i.e. their mean square values can be added [$\overline{(a+b)^2} = \overline{a^2} + \overline{b^2} + \overline{2ab^{\bullet}} = \overline{a^2} + \overline{b^2}; \qquad \overline{2ab^{\bullet}} = 0$].

The equivalent noise spectral density for a series combination of resistances R_1 and R_2 for a given identical temperature T_1:

$$[S_v(f)]_{series-combination} = [S_v(f)]_{R_1,T_1} + [S_v(f)]_{R_2,T_1} = \frac{\overline{v_{R_1}^2}}{\Delta f} + \frac{\overline{v_{R_2}^2}}{\Delta f} = \frac{4kTR_1\Delta f}{\Delta f} + \frac{4kTR_2\Delta f}{\Delta f} = 4kT_1(R_1 + R_2)$$

The equivalent noise spectral density for a parallel combination of resistances R_1 and R_2 for a given identical temperature T_1:

$$[S_v(f)]_{parallel-combination} = [S_v(f)]_{R_1,T_1} + [S_v(f)]_{R_2,T_1} = 4kT_1\left[\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right]\frac{\Delta f}{\Delta f} = 4kT_1\frac{R_1 R_2}{(R_1 + R_2)}$$

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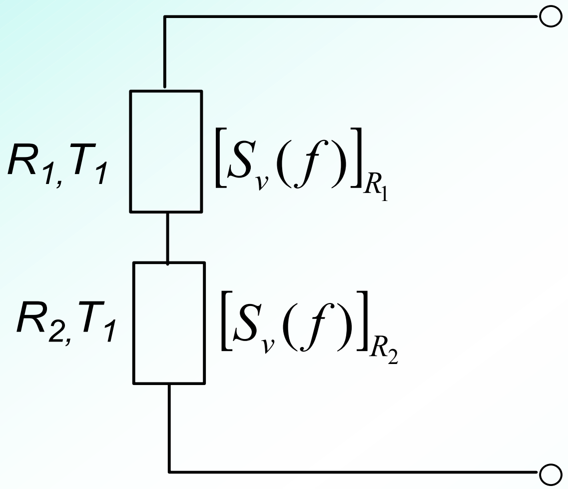
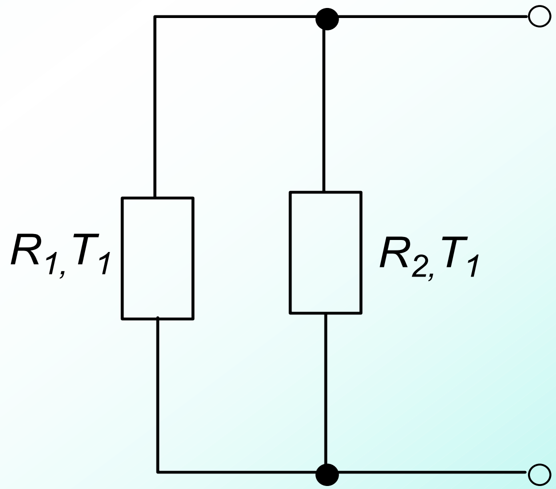
Noise in Linear Electronic Components, Cont'd.

Resistor, Cont'd.

The Figure below shows the series and parallel combination of resistors for the purpose of the calculation of the equivalent spectral density functions.

Identical Temperature $T=T_1$





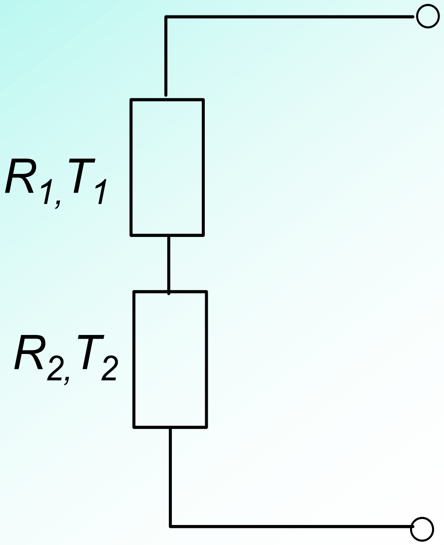
$$[S_v(f)]_{series-combination} = 4kT_1(R_1 + R_2)$$

$$[S_v(f)]_{parallel-combination} = 4kT_1 \frac{R_1 R_2}{(R_1 + R_2)}$$

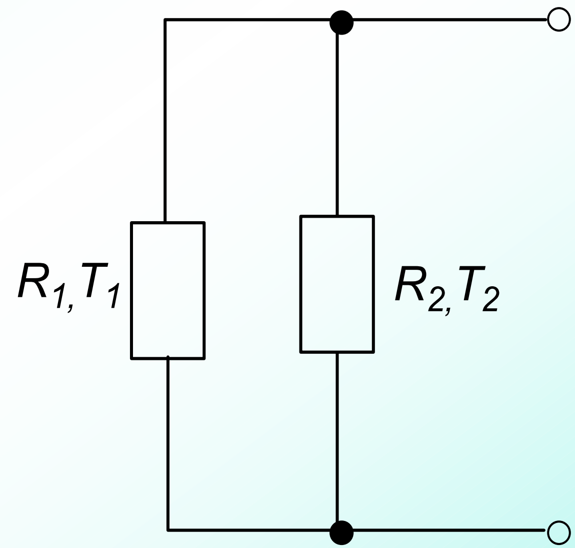
Noise in Linear Electronic Components, Cont'd.

Resistor, Cont'd.

The equivalent noise spectral density for a series combination of resistances R_1 and R_2 for a given temperature T_1 and T_2:



Series



Parallel

$$[S_v(f)]_{series} = 4k(R_1T_1 + R_2T_2)$$

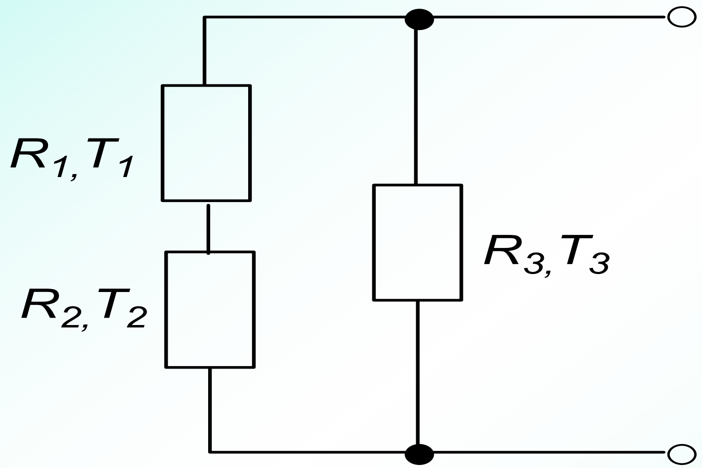
$$[S_v(f)]_{parallel-combination} = 4k\frac{R_1R_2(T_1R_2 + T_2R_1)}{(R_1 + R_2)^2}$$

Noise in Linear Electronic Components, Cont'd.

Resistor, Cont'd.

The Figure below shows the typical representation of the hybrid combination, such as series (R_1 and R_2) and parallel (R_3) combination for a given temperature T_1, T_2, and T_3.

Hybrid (serial and parallel)



$$[S_v(f)]'_{R_1,T_1} = [S_v(f)]_{R_1,T_1}\left[\frac{R_3}{R_1+R_2+R_3}\right]^2$$

$$[S_v(f)]'_{R_2,T_2} = [S_v(f)]_{R_2,T_2}\left[\frac{R_3}{R_1+R_2+R_3}\right]^2$$

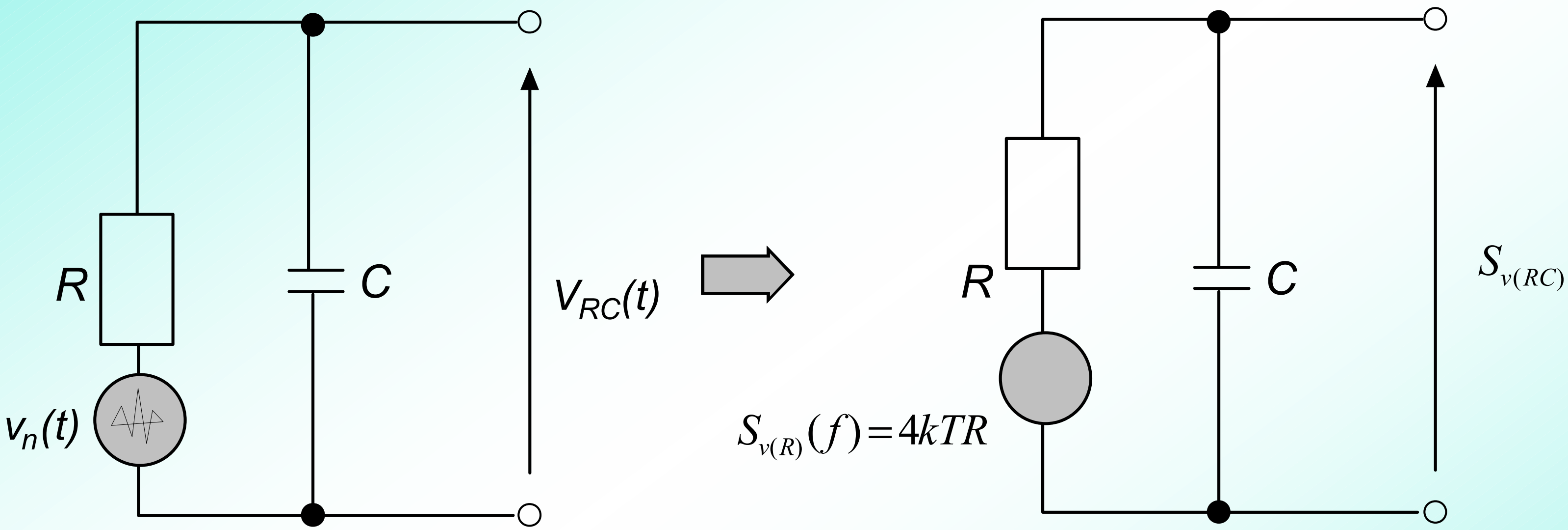
$$[S_v(f)]'_{R_3,T_3} = [S_v(f)]_{R_3,T_3}\left[\frac{R_1+R_2}{R_1+R_2+R_3}\right]^2$$

$$[S_v(f)]_{hybrid-combination} = [S_v(f)]'_{R_1,T_1} + [S_v(f)]'_{R_2,T_2} + [S_v(f)]'_{R_3,T_3}$$

$$[S_v(f)]_{hybrid-combination} = 4k\frac{R_1R_2(T_1R_1R_3^2 + T_2R_{21}R_3^2 + T_3R_3[R_1+R_2]^2)}{(R_1+R_2+R_3)^2}$$

Noise in Linear Electronic Components, Cont'd.

Thermal Noise of a Low pass RC circuit



The equivalent noise spectral density for low pass RC circuit is given by $S_{v(RC)}$ as

$$S_{v(RC)}(f) = S_{v(R)}(f)\left|\frac{1/(j\omega C)}{R+1/(j\omega C)}\right|^2 = 4kTR\left|\frac{1/(j\omega C)}{R+1/(j\omega C)}\right|^2 = \frac{4kTR}{1+(\omega CR)^2}$$

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Noise in Linear Electronic Components, Cont'd.

Thermal Noise of Low pass RC circuit, Cont'd.

The spectral density becomes frequency dependent because of the capacitor across the resistor. The mean square value of the voltage at the capacitor can be calculated by integration over the entire frequency range as

$$\overline{v_{RC}^2(t)} = \int_0^\infty S_{v(RC)}(f)\,df = \int_0^\infty \frac{4kTR}{1+(\omega CR)^2}\,df$$

$$\overline{v_{RC}^2(t)} = \frac{2kT}{\pi C}\int_0^\infty \frac{1}{1+x^2}\,dx \quad with \quad x = \omega CR$$

$$\overline{v_{RC}^2(t)} = \frac{2kT}{\pi C}[\tan^{-1} x]_0^\infty = \frac{kT}{C}$$

Fluctuation energy stored in the capacitor is given by $\frac{1}{2}C[\overline{v_{RC}^2(t)}] = \frac{kT}{2}$

The mean square voltage across the capacitor C is finite for an infinite frequency range and does not depend upon the value of resistor R. The resistance R does not affect the total energy kT/2 per degree of freedom (DOF), but R determines the magnitude and the bandwidth of the spectral density.

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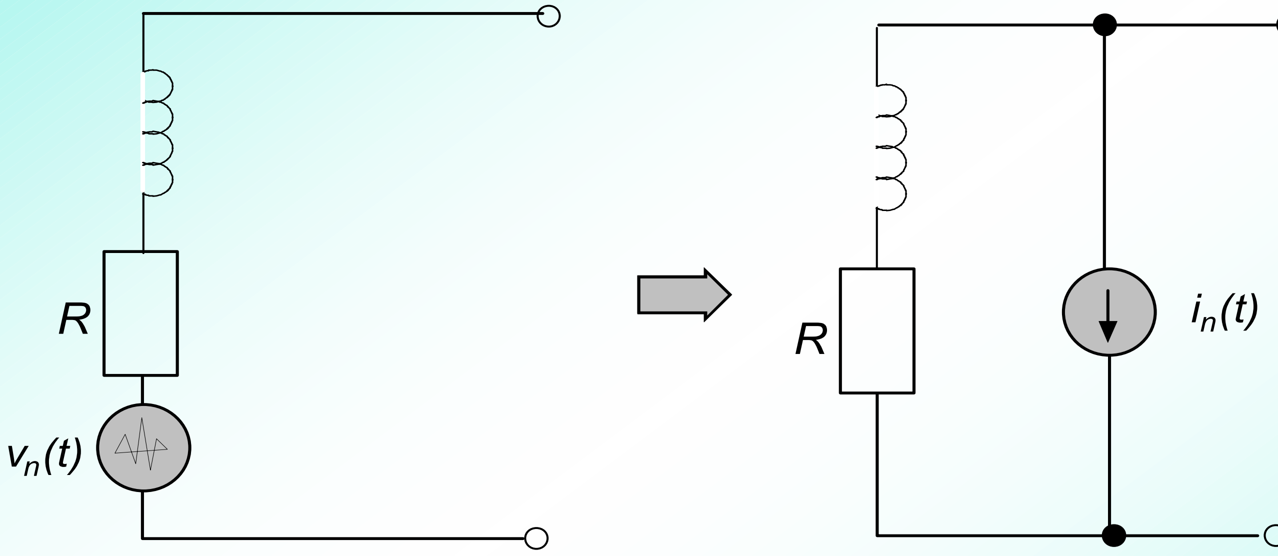
Noise in Linear Electronic Components, Cont'd.

Thermal Noise of RL circuit



The equivalent noise spectral density for series RL circuit is given by $S_{v(RL)}$

$$S_{v(RL)}(f) = S_{v(R)}(f) = 4kTR$$

$$\Rightarrow S_{i(RL)}(f) = 4kT \bullet \mathrm{Re}[Y(f)] = 4kT\left[\frac{R}{R^2+\omega^2 L^2}\right] = S_{v(R)}(f)\left[\frac{1}{R^2+\omega^2 L^2}\right]$$

Noise in Linear Electronic Components, Cont'd.

Thermal Noise of RL circuit, Cont'd.

The spectral density becomes frequency dependent because of the inductor L. The mean square value of the noise current generator can be calculated by integration over the entire frequency range as

$$\overline{i_{RL}^2(t)} = \int_0^\infty S_{v(RC)}(f)df = \int_0^\infty \frac{4kTR}{[1+(\omega L/R)^2]R^2}df = \frac{kT}{L}$$

Fluctuation energy stored in the inductor is given by

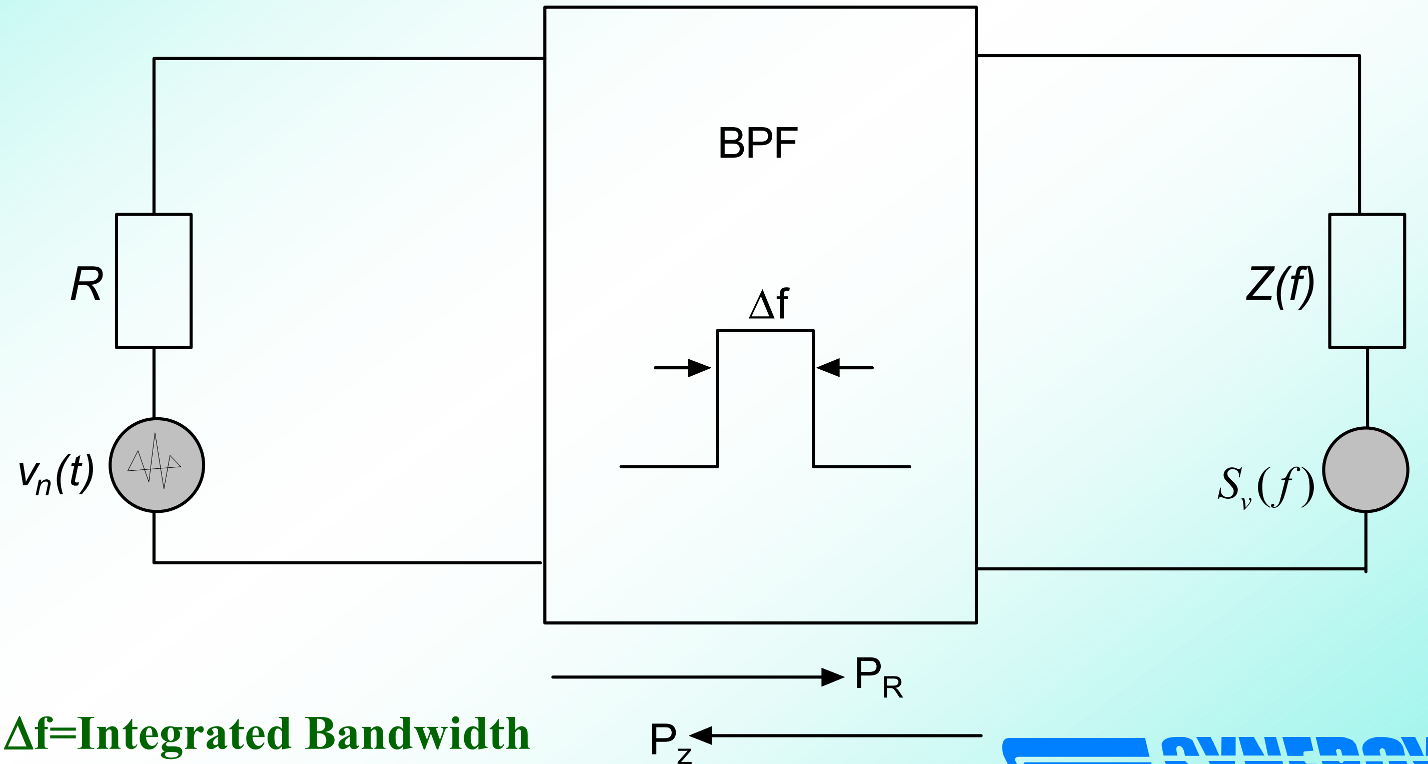
$$\frac{1}{2}L[\overline{i_{RL}^2(t)}] = \frac{kT}{2}$$

The resistance R does not affect the total energy kT/2 per degree of freedom (DOF), but R determines the magnitude and the bandwidth of the spectral density.

Noise in Linear Electronic Components, Cont'd.

Thermal Noise of Complex Impedances

Resistor R and a complex impedance Z(f) are connected by a band pass filter (BPF). Resistance R and complex impedance Z (f) are assumed to be at the same temperature T.



Δf=Integrated Bandwidth

Noise in Linear Electronic Components, Cont'd.

Thermal Noise of Complex Impedances, Cont'd.

The BPF filter is assumed to be lossless; therefore, it does not contribute to the noise. From the thermodynamic equilibrium theorem, the noise power P_R, which is transmitted by the resistor R to the load Z(f), must be equal to the noise power P_Z, which is transmitted by the complex impedance Z(f) to the load R, i.e. $P_R \equiv P_Z$. The noise powers (P_R and P_Z) are given by

$$P_R = \frac{4kTR}{\left|R + Z(f)\right|^2} \bullet \mathrm{Re}\{Z\} \bullet \Delta f$$

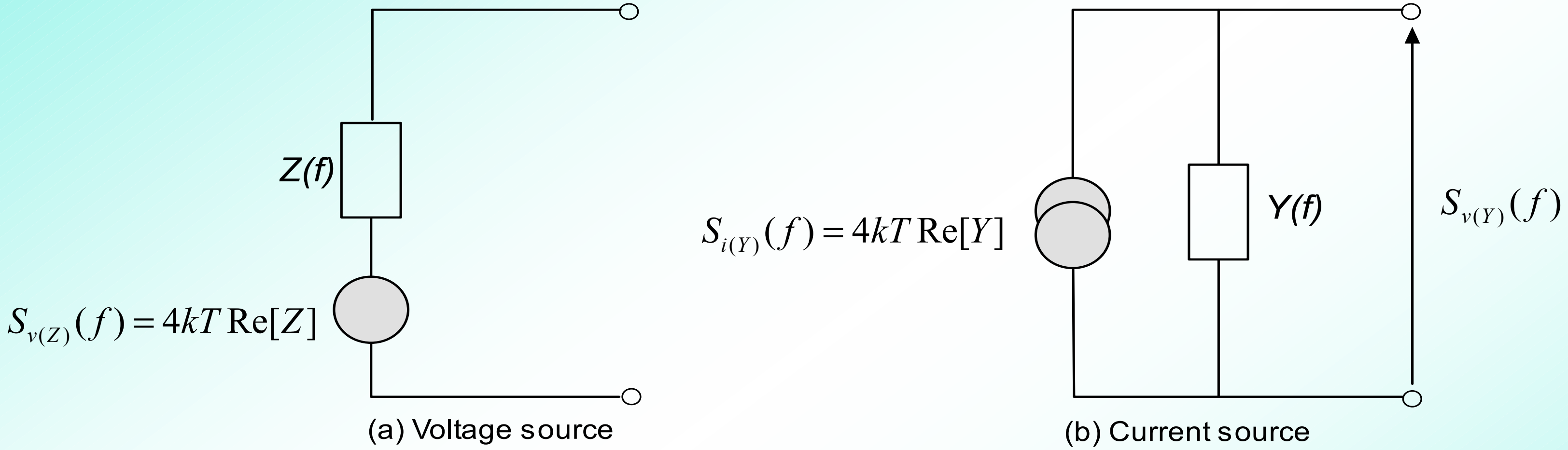
$$\Rightarrow P_Z = \frac{S_v}{\left|R + Z(f)\right|^2} \bullet R \bullet \Delta f$$

Since $P_R = P_Z$, one can determine noise spectral density as

$$S_v(f) = 4kT \bullet \mathrm{Re}\{Z(f)\}; \qquad S_i(f) = 4kT \bullet \mathrm{Re}\{Y(f)\}$$

Noise in Linear Electronic Components, Cont'd.

Thermal Noise of Complex Impedances, Cont'd.



$$S_{v(Y)}(f) = S_{i(Y)}(f)\frac{1}{|Y(f)|^2}$$

$$\Rightarrow S_{i(Y)}(f)\frac{1}{|Y(f)|^2} = [4kT \bullet \mathrm{Re}\{Y\}] \bullet \left[\frac{1}{Y(f) \bullet Y^*(f)}\right]$$

$$\Rightarrow S_{v(Y)}(f) = 4kT \bullet \left[\frac{Y + Y^*}{2 \bullet Y \bullet Y^*}\right] = 4kT \bullet \frac{1}{2} \bullet \left[\frac{1}{Y} + \frac{1}{Y^*}\right] = 4kT \bullet \mathrm{Re}\{Z(f)\}$$

$$\Rightarrow S_{v(Y)}(f) = S_{v(z)}(f)$$

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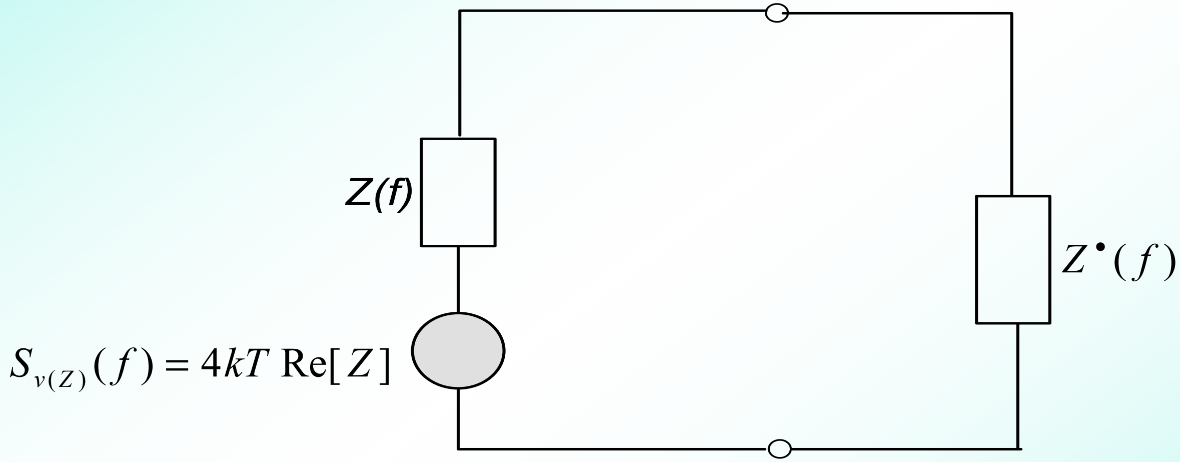
Noise in Linear Electronic Components, Cont'd.

Available Noise Power and Equivalent Noise Temperature:

The maximum available noise power P_{av} is obtained if a circuit is terminated by the complex conjugate of the generator source impedance.



The transmitted power P_t to $Z^{\bullet}(f)$ can be described by

$$P_t = \frac{S_v}{\left|Z + Z^{\bullet}\right|^2} \bullet \mathrm{Re}\{Z^{\bullet}(f)\} \bullet \Delta f = \frac{S_v}{4 \bullet \mathrm{Re}^2\{Z\}} \bullet \mathrm{Re}\{Z\} \bullet \Delta f$$

$$\Rightarrow P_t = \frac{4KT \bullet \mathrm{Re}\{Z\} \bullet \mathrm{Re}\{Z\} \bullet \Delta f}{4 \bullet \mathrm{Re}^2\{Z\}} = kT \bullet \Delta f = P_{av}$$

Noise in Linear Electronic Components, Cont'd.

Available Noise Power and Equivalent Noise Temperature, Cont'd.

The available noise power does not depend on the value of resistor but it is a function of temperature T. The noise temperature can thus be used as a quantity to describe the noise behavior of a general lossy one-port network.

For high frequencies and/or low temperature a quantum mechanical correction factor has to be incorporated for the validation of equation. This correction term results from Planck's radiation law, which applies to blackbody radiation. In general case, $P_{av} = kT \bullet \Delta f$ is replaced by

$$P_{av} = kT\Delta f \bullet p(f,T); \quad with \quad p(f,T)\left[= \frac{hf/kT}{e^{\left(\frac{hf}{kT}\right)} - 1} \right]$$

$h = 6.626 \bullet 10^{-34} J/s$ Where '*h*' is *Planck constant*

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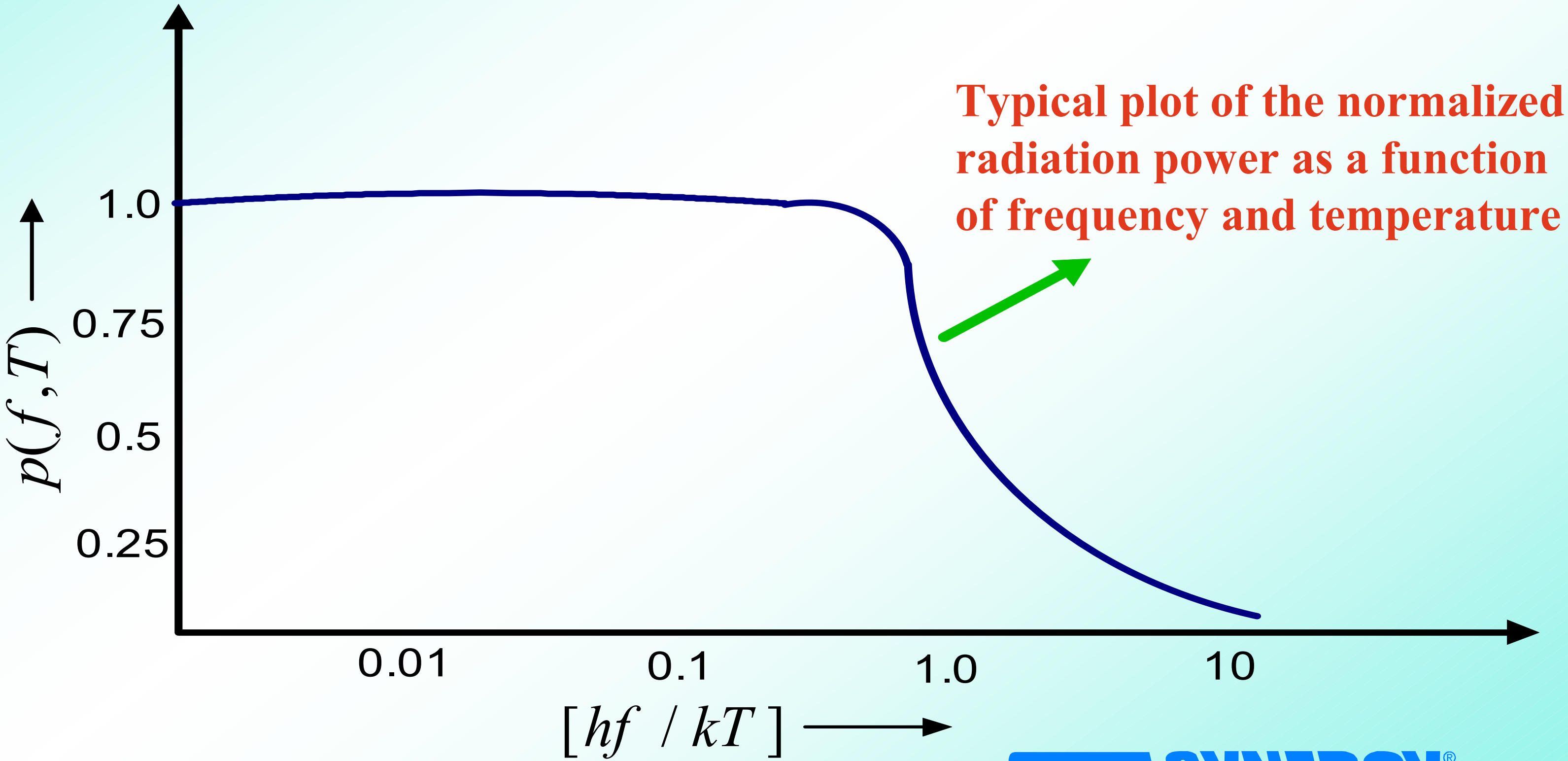
Noise in Linear Electronic Components, Cont'd.

Available Noise Power and Equivalent Noise Temperature, Cont'd.

The Planck correction factor $p(f,T)$ prevents (keeps within the finite limit) from the noise power becomes infinite for arbitrarily large bandwidths.



3-Terminal Active Device (Transistor) Models

3-terminal active device

There are many form of 3-terminal active device, the commonly used are discussed here for large signal consideration:

1. **BJT: BJT is a current controlled transistor which is a minority carrier device in the base region; this a bipolar device because there are 2 junctions, the emitter-base junction which is forward biased to inject the minority carriers into the base and the collector-base junction which is reverse biased to collect all of the base minority carriers into the collector. The Gummel-Poon model is most commonly used, followed by the VBIC and MEXTRAM models. The VBIC is an extension of the Gummel-Poon model, and the MEXTRAM model uses fewer nodes (5 vs. 7) and therefore converges faster than other models in nonlinear situations (developed by Philips).**
2. **MOSFET:Modern MOSFETs have become important at frequencies below 6 GHz. Some of the history begins with DMOS transistors which were developed at Signetics in the early 70's, the high frequency performance of CMOS transistors, and the development of the high-power LDMOS transistor. The nonlinear models come from SPICE developments, including Bi-CMOS models among others. Bi-CMOS implies there are BJTs, n-channel MOSFETs, and p-channel MOSFETs are on the same silicon chip.**

3-Terminal Active Device Models, Cont'd.

3-terminal active device, Cont'd.

3. **MESFET**: This transistor came about in 1965 with the development of Schottky diodes and ohmic contacts simultaneously on GaAs. It is a majority carrier device which is voltage controlled at the gate. The name means Metal-Semiconductor-Field-Effect-Transistor. The MESFET/HEMT models constitute a long list including: Curtice Quadratic, Curtice Cubic, Statz-Pucel, Materka and modified Materka (Raytheon/Ansoft), Tajima, Root (HP/Agilent), Angelov, Parker, EEFET3, EEHEMT1, TOM3 (Triquent's Model) etc. with more to come.
4. **HEMT** (PHEMT and MHEMT): This is replacing MESFETs in many applications due to superior performance. It is a High Electron Mobility Transistor first introduced about 1980 by Fujitsu. It has progressed to PHEMT and MHEMT structures, with even better performance. A PHEMT is a lattice matched pseudo-morphic HEMT, while a MHEMT is a meta-morphic HEMT, where graded layers of doping are employed.
5. **HBT**: The Heterojunction bipolar transistor was originally developed to improve emitter injection efficiency in GaAs BJTs, which has been a long standing problem (since 1965). In addition, the SiGe HBT has been added to the list about 1985, which offers a very low cost process with excellent microwave performance limited only by the low value of $T_{j,max}$ of 155°C.

3-Terminal Active Device (Transistor) Models, Cont'd.

Transistor Classification

Microwave transistors can be presently classified into seven groups:

1. **Silicon BJTs**
2. **Silicon MOSFETs**
3. **Gallium Arsenide MESFETs**
4. **InGaAs/InP etc. PHEMTs**
5. **InAlAs/InGaAs MHEMTs**
6. **InGaP/InGaAs and SiGe HBTs**
7. **SiC and GaN (Next developments, only lab sample!)**

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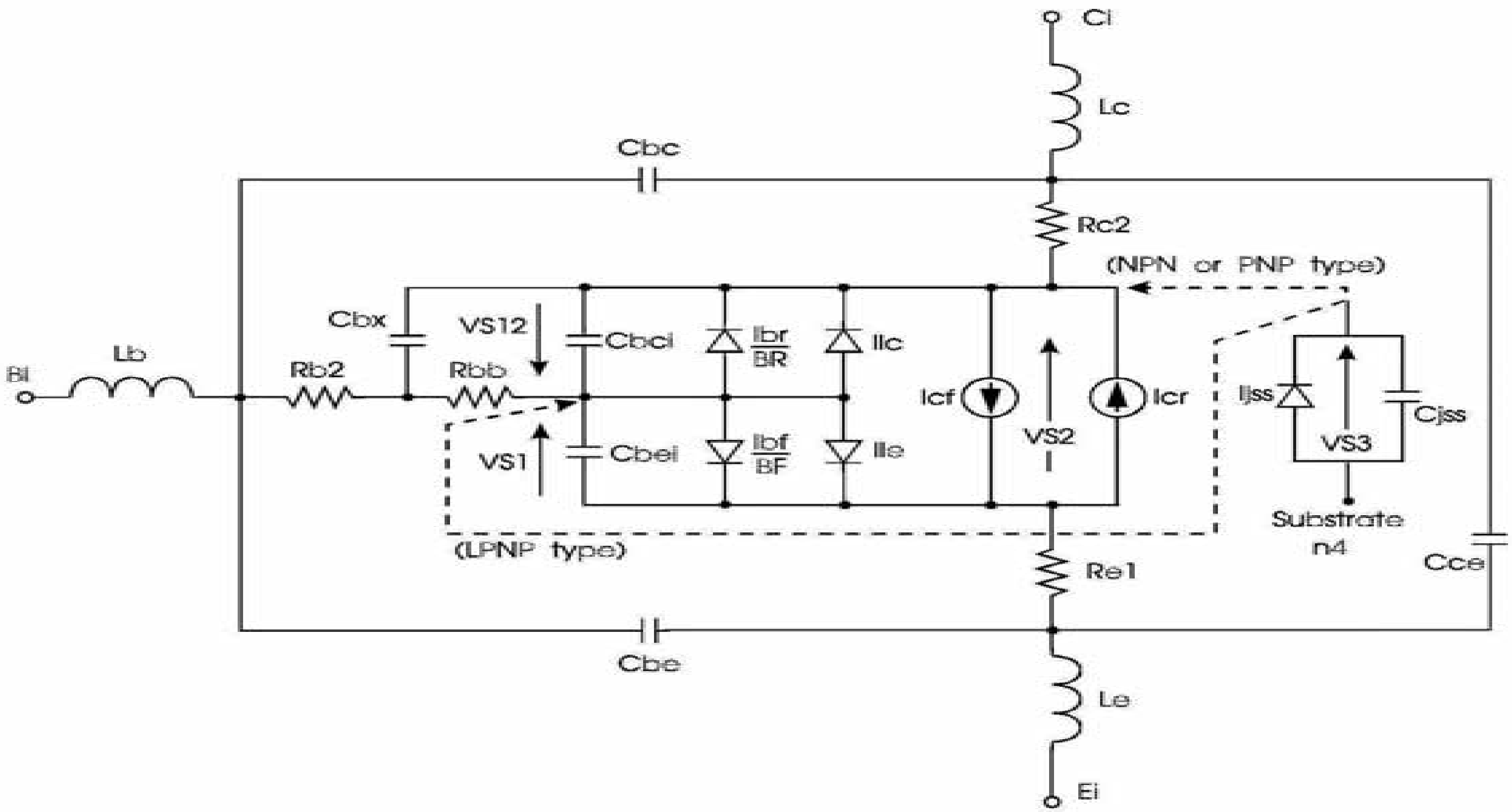
3-Terminal Active Device (Transistor) Models

Bipolar Transistor Model



Equivalent circuit for a microwave bipolar transistor (Ansoft Designer Model)

3-Terminal Active Device (Transistor) Models

Spice parameters

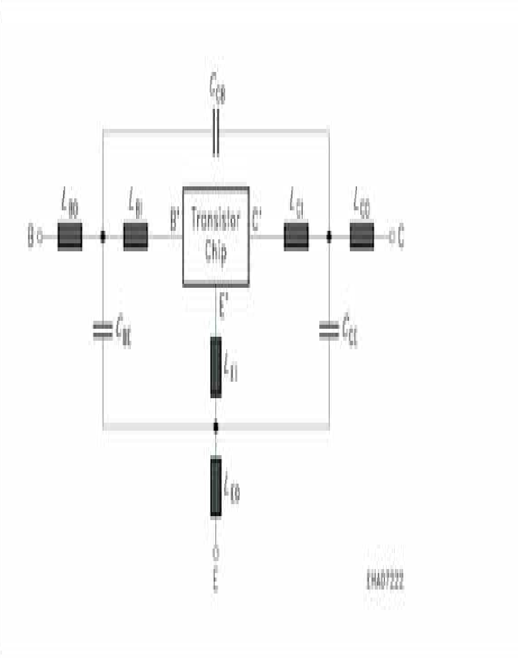
SPICE parameters and package equivalent circuit of the Infineon transistor BFP 520

SPICE Parameters (Gummel-Poon Model, Berkley-SPICE 2G.6 Syntax) :

Transistor Chip Data

IS =	15	aA	BF =	235	-	NF =	1	-
VAF =	25	V	IKF =	0.4	A	ISE =	25	fA
NE =	2	-	BR =	1.5	-	NR =	1	-
VAR =	2	V	IKR =	0.01	A	ISC =	20	fA
NC =	2	-	RB =	11	Ω	IRB =	-	A
RBM =	7.5	Ω	RE =	0.6		RC =	7.6	Ω
CJE =	235	fF	VJE =	0.958	V	MJE =	0.335	-
TF =	1.7	ps	XTF =	10	-	VTF =	5	V
ITF =	0.7	A	PTF =	50	deg	CJC =	93	fF
VJC =	0.661	V	MJC =	0.236	-	XCJC =	1	-
TR =	50	ns	CJS =	0	fF	VJS =	0.75	V
MJS =	0.333	-	XTB =	-0.25	-	EG =	1.11	eV
XTI =	0.035	-	FC =	0.5	-	TNOM	298	K

Package Equivalent Circuit:

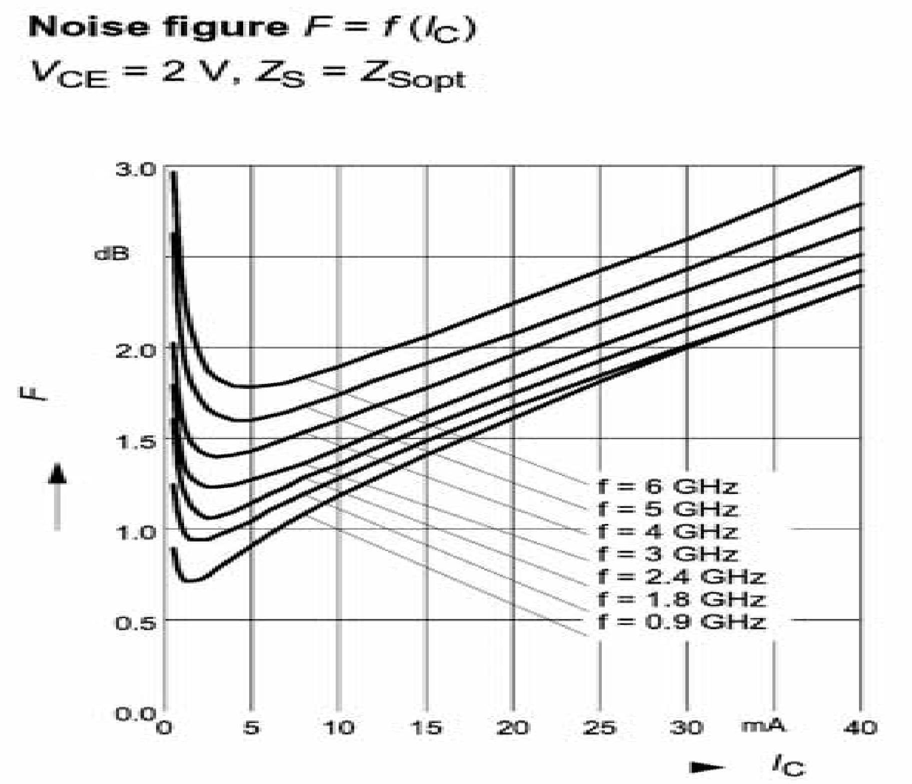


L_{BI} =	0.47	nH
L_{BO} =	0.53	nH
L_{EI} =	0.23	nH
L_{EO} =	0.05	nH
L_{CI} =	0.56	nH
L_{CO} =	0.58	nH
C_{BE} =	136	fF
C_{CB} =	6.9	fF
C_{CE} =	134	fF

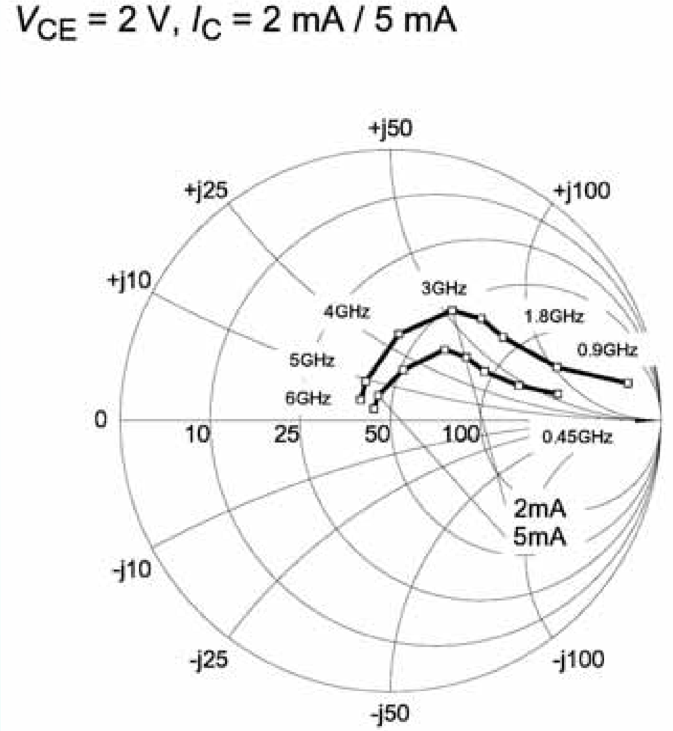
Valid up to 6GHz

3-Terminal Active Device (Transistor) Models

Noise figure and source impedance for best noise figure as a function of current and frequency of the Infineon transistor BFP 520



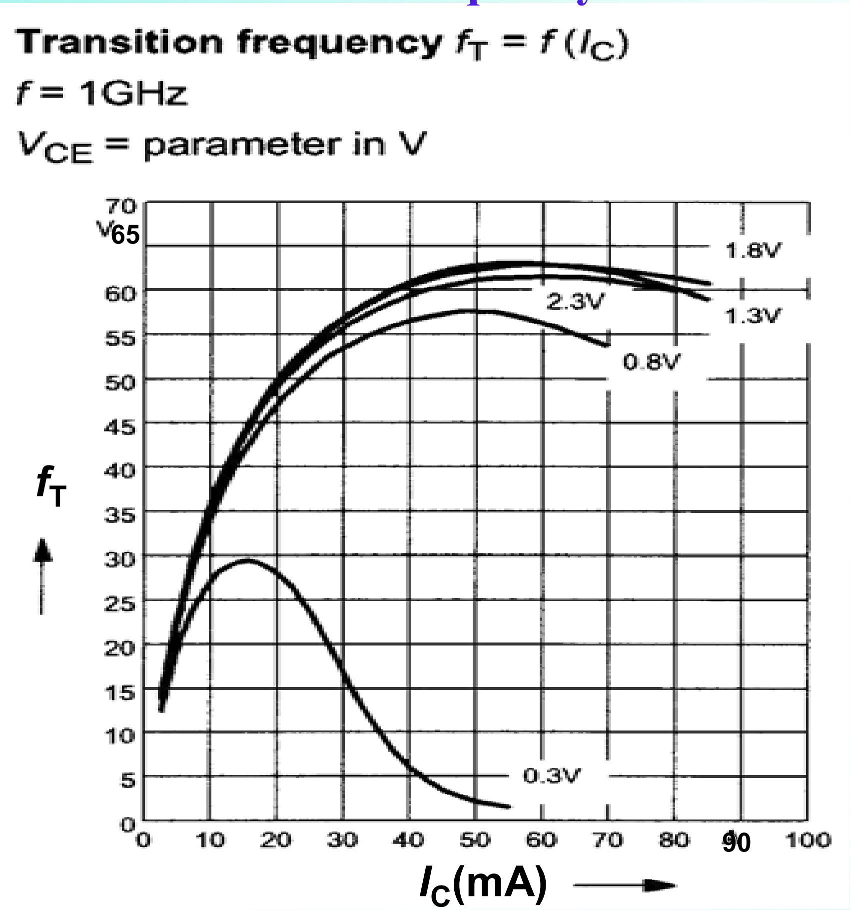
Noise Figure vs current



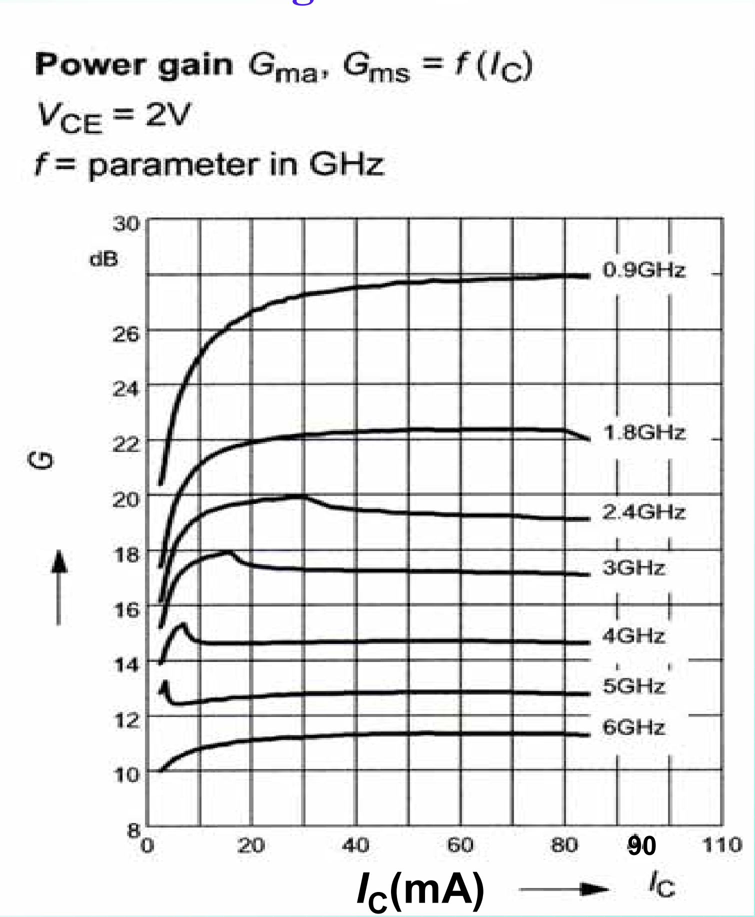
Source impedance Γ_{opt} for minimum noise Figure vs frequency

3-Terminal Active Device (Transistor) Models

Transistor cut-off frequency vs. current



Power gain vs. current



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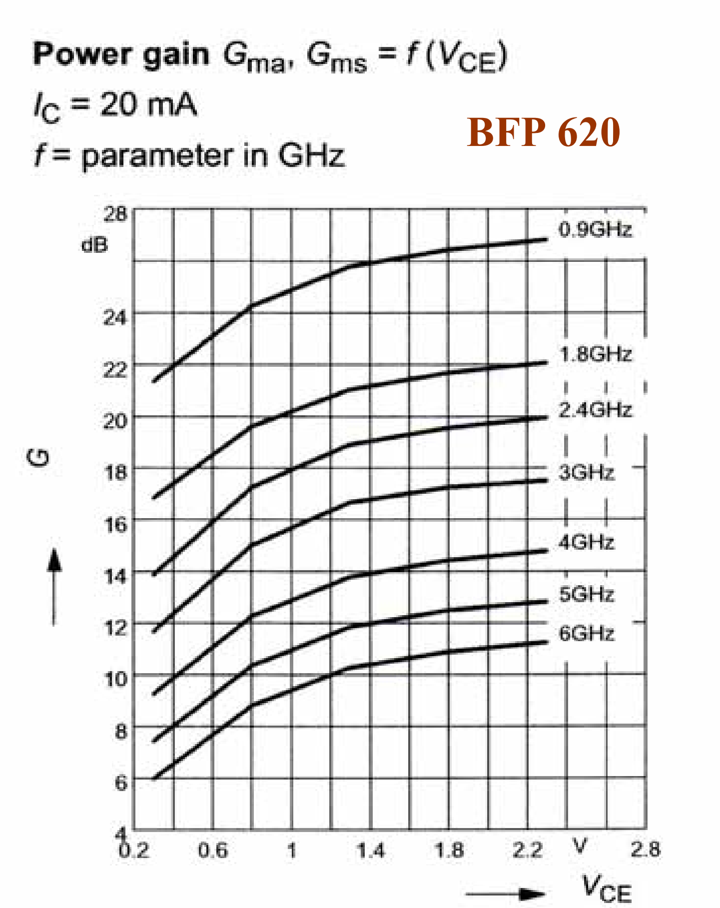
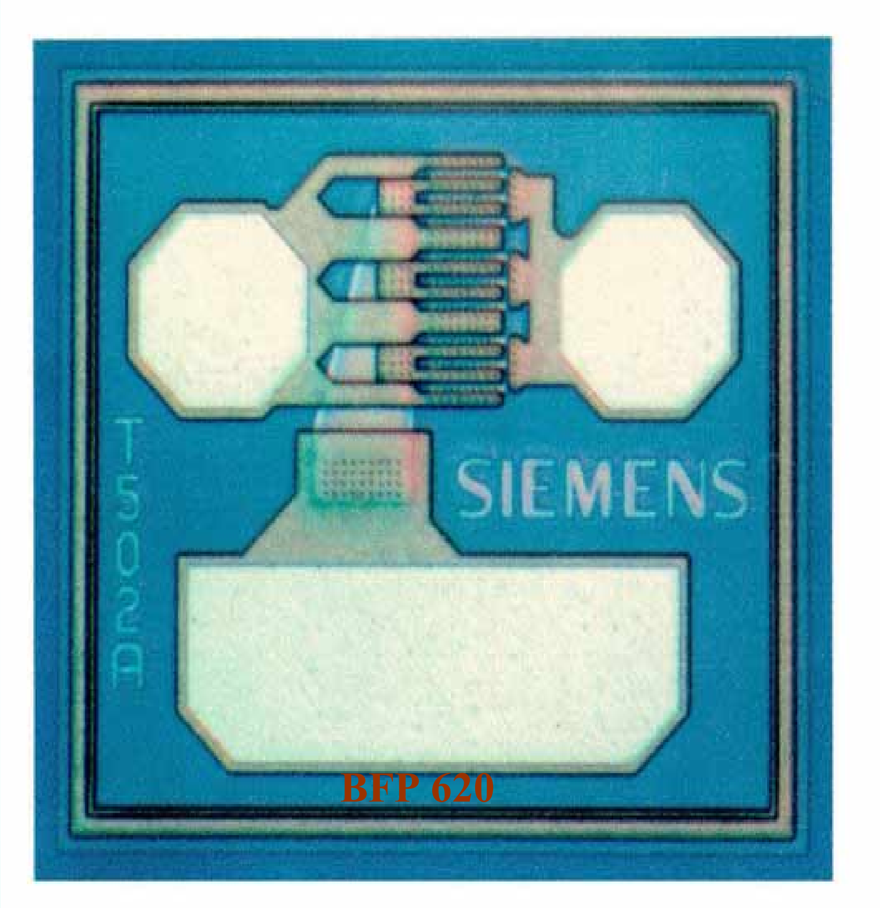
3-Terminal Active Device (Transistor) Models

Power gain vs. V_{CE}



Infineon BFP 620 Microwave Transistor die



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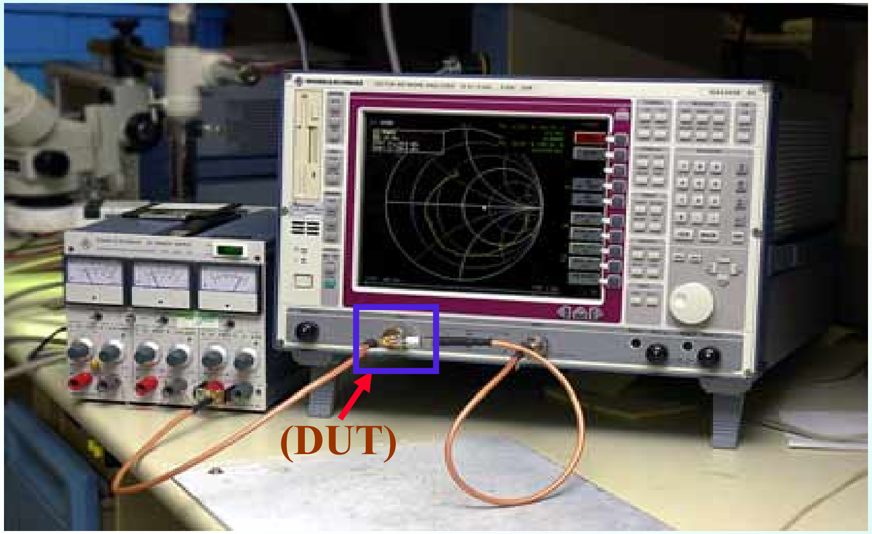
3-Terminal Active Device (Transistor) Models

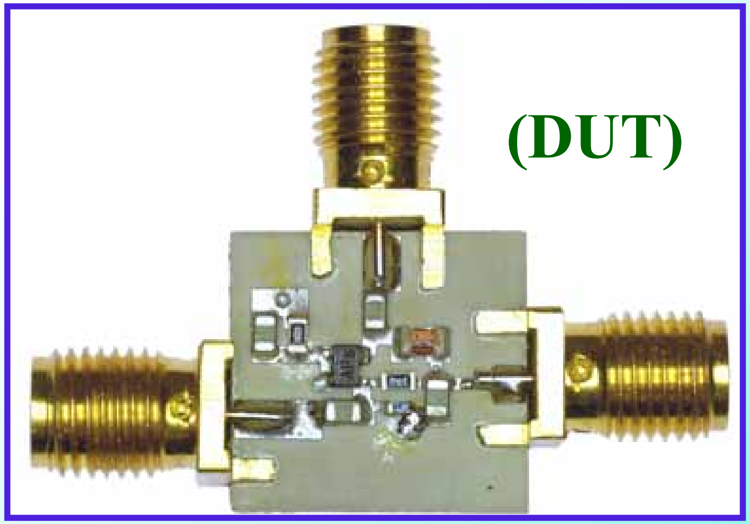
Large Signal Measurements

While the datasheets provided by the manufacturer are given under small signal conditions, small signal conditions mean power levels in the vicinity of –40 dBm. The network analyzers used to measure these S-parameters, have bias tees built-in and have 90 dB dynamic ranges. Figure below shows the test fixture, which was generated to measure the large signal *S*-parameters for the device under test (DUT). The test fixture was calibrated to provide 50Ω to the transistor leads and a proper de-embedding has been done.



Rohde & Schwarz 3 GHz network analyzer to measure the large signal *S*-parameters at different drive levels.

Test fixture to measure large signal *S*-parameters



3-Terminal Active Device (Transistor) Models

Large Signal Measurements (Bipolar)

The definition of S-parameters in a large-signal environment is ambiguous compared to that of small-signal S-parameters. When an active device is driven with an increasingly higher level, the output current consists of a DC current and RF current, the fundamental frequency, and its harmonics. When the drive level is increased, the harmonic content rapidly increases. S_{12}, mostly defined by the feedback capacitance, now reflects harmonics back to the input.

If these measurements are done in a 50 Ω system, which has no reactive components, then we have an ideal system for termination. In practical applications, however, the output is a tuned circuit or matching network, which is frequency selective. Depending on the type of circuit, it typically presents either a short- circuit or an open-circuit for the harmonic.

For example, suppose that the matching network has a resonant condition at the fundamental and second harmonic frequencies or at the fundamental and third harmonic frequencies (quarter-wave resonator). Then a high voltage occurs at the third harmonic, which affects the input impedance and, therefore, S_{11} (Miller effect).

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3-Terminal Active Device (Transistor) Models

Large Signal Measurements (Bipolar)

Currents and voltages follow Kirchoff's law in a linear system. A linear system implies that there is a linear relationship between currents and voltages. All transistors, when driven at larger levels, show nonlinear characteristics. The FET shows a square law characteristic, while the bipolar transistor has an exponential transfer characteristic. It is important to note that the output impedances of FETs are much less RF voltage-dependent or power dependent than those of the bipolar transistor. The generation of large-signal S-parameters is, therefore, much more important for bipolar transistors than for FETs.

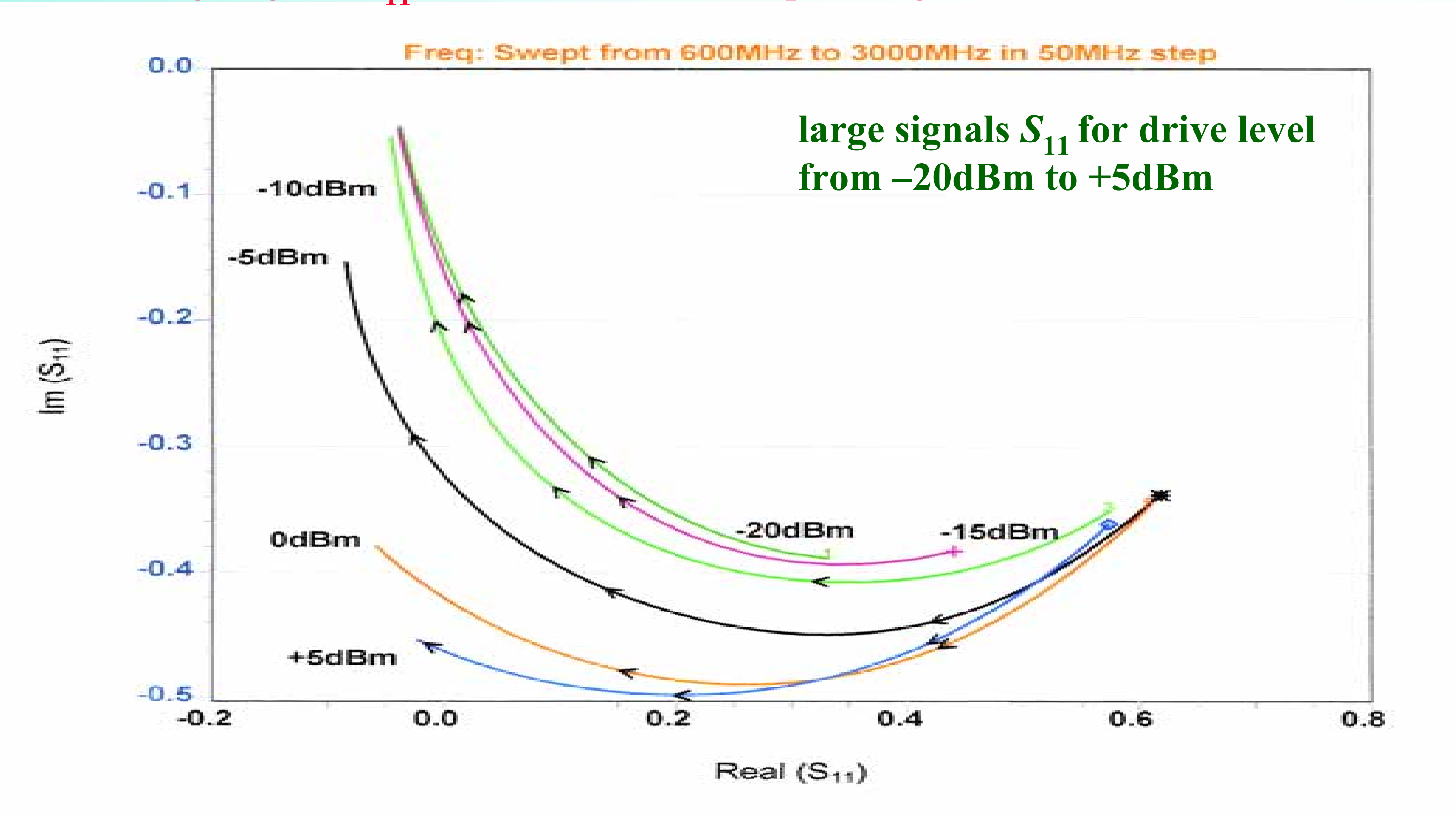
This indicates that *S*-parameters measured under large-signal conditions in an ideal 50 Ω systems may not correctly predict device behavior when used in a non-50 Ω environment. A method called load pulling, which includes fundamental harmonics, has been developed to deal with this issue.

The following four plots, show S_{11}, S_{12}, S_{21}, and S_{22} measured from 50 MHz to 3000 MHz with driving levels from –20 dBm to 5 dBm. The DC operation conditions were 1.9 V and 20 mA,

3-Terminal Active Device (Transistor) Models

Large Signal Measurements (Bipolar)

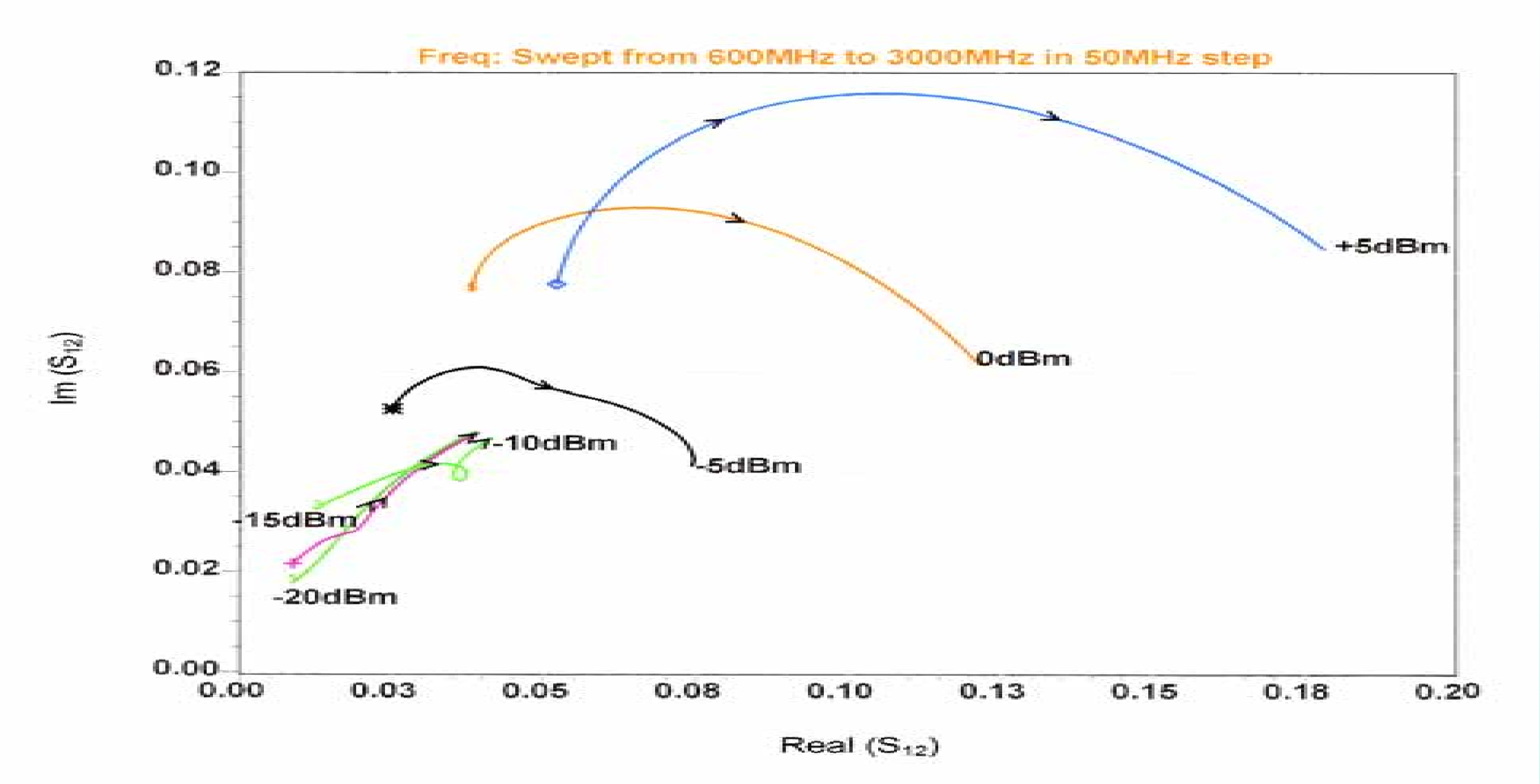
Measured large signals S_{11} of the BFP 520 (DC operating conditions were 1.9 V and 20 mA).



3-Terminal Active Device (Transistor) Models

Large Signal Measurements (Bipolar)

Measured large signals S_{12} of the BFP 520 (DC operating conditions were 1.9 V and 20 mA)



large signals S_{12} for drive level from –20dBm to +5dBm

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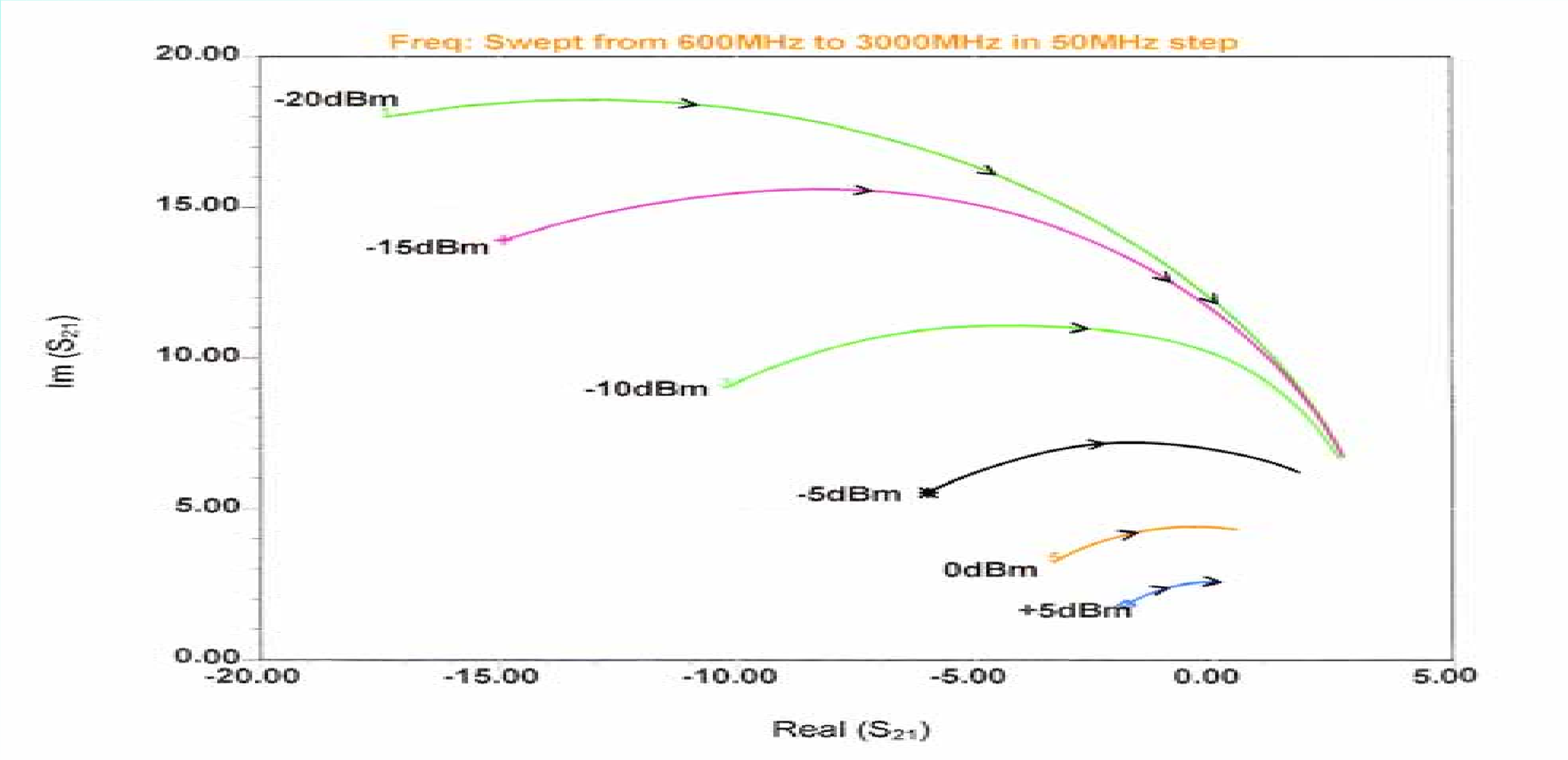
3-Terminal Active Device (Transistor) Models

Large Signal Measurements (Bipolar)

Measured large signals S_{21} of the BFP 520 (DC operating conditions were 1.9 V and 20 mA)

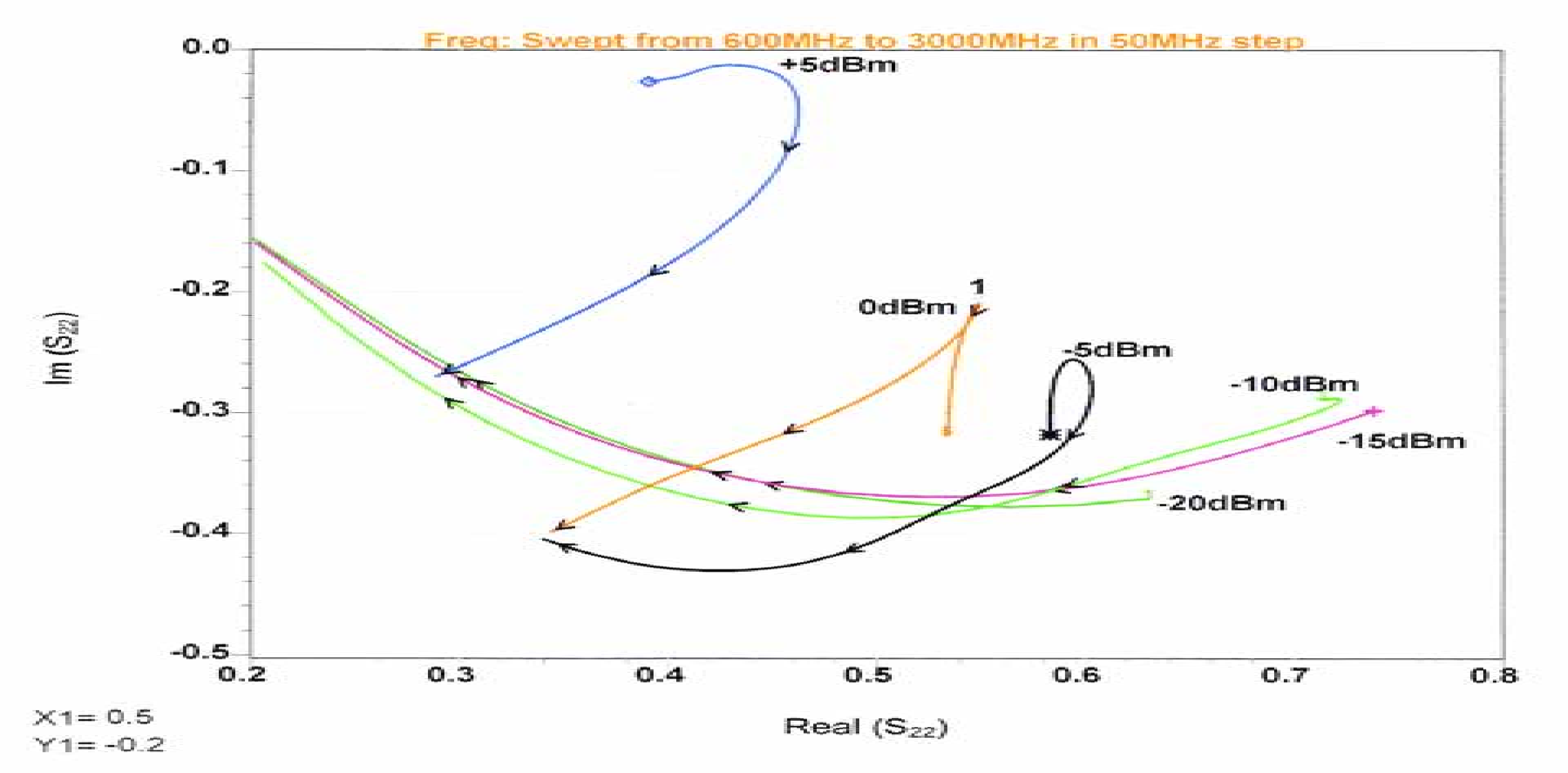


large signals S_{21} for drive level from –20dBm to +5dBm

3-Terminal Active Device (Transistor) Models

Large Signal Measurements (Bipolar)

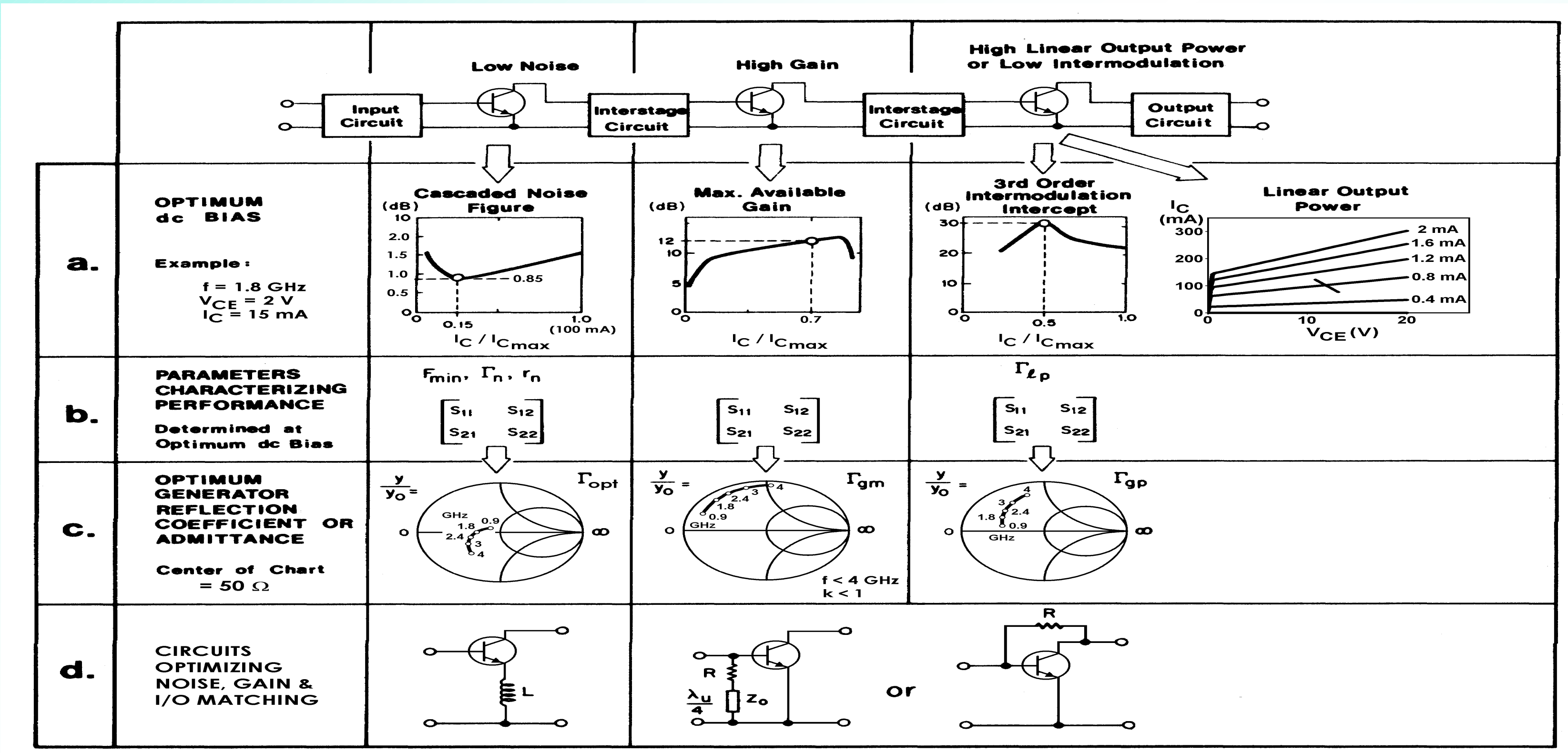
Measured large signals S_{22} of the BFP 520 (DC operating conditions were 1.9 V and 20 mA)



large signals S_{22} for drive level from –20dBm to +5dBm

3-Terminal Active Device

Key parameters in applying a BJT in low-noise front-end, high-gain and linear-powers stages.The example is based on the Siemens BJTs BFP420 (low-noise stage), BFP450 (high-gain stage), and BFG235 (output stage).



a) Optimum dc bias for a low-noise front end
b) Characterizing device performance
c) Γ_{opt} for the first stage
d) Optimize stage noise, gain and I/O matching

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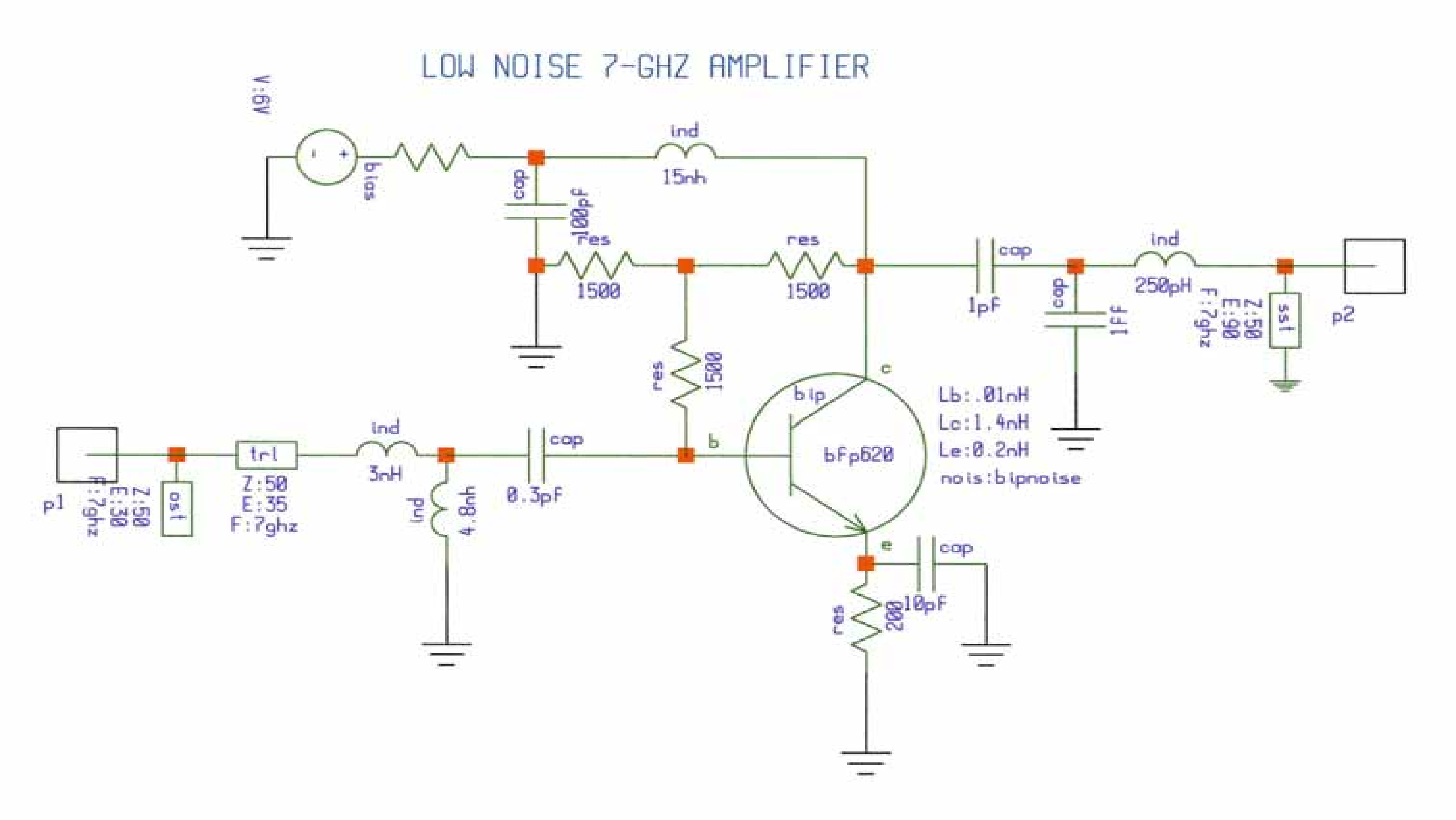
LNA (Low Noise Amplifier)

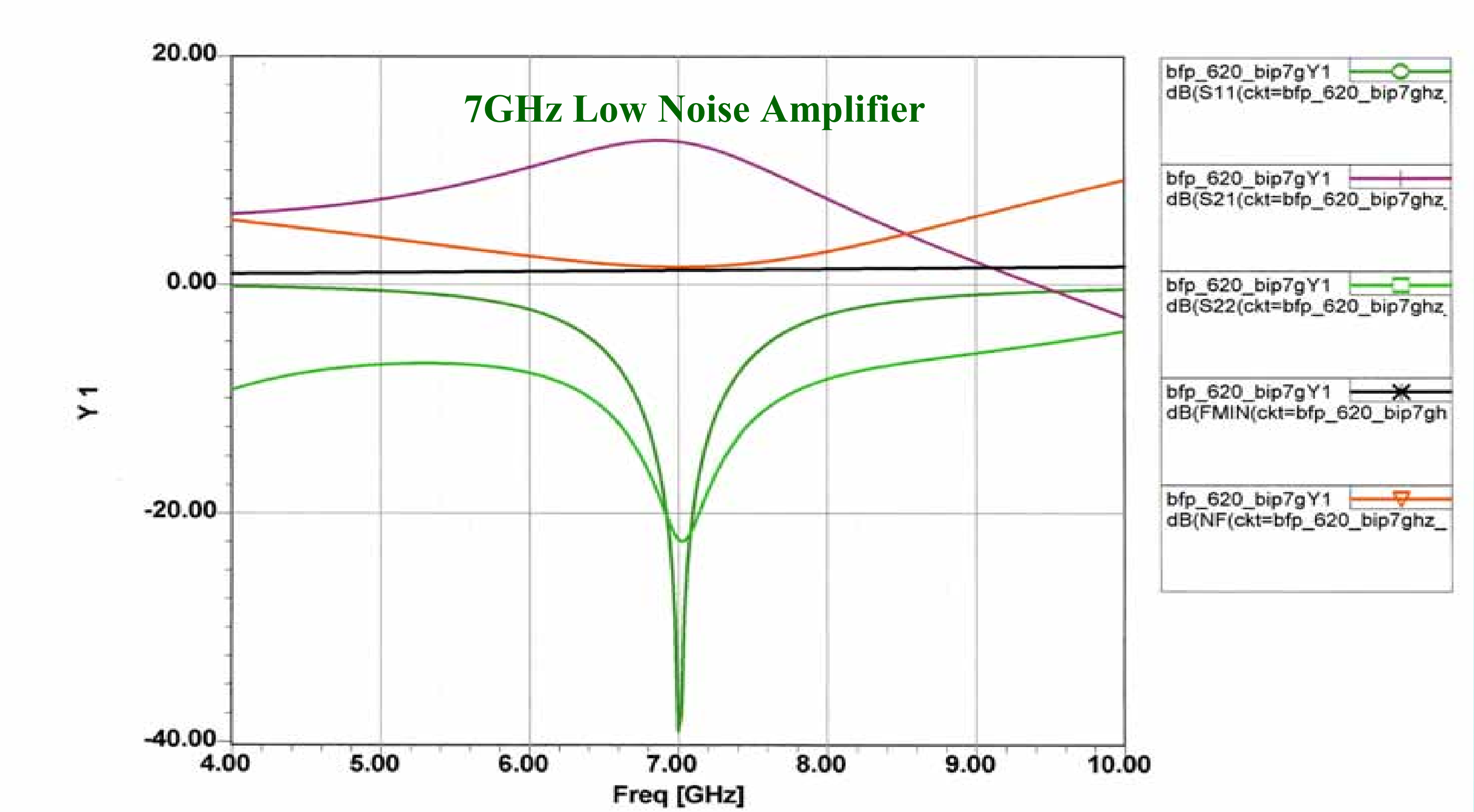
Low Noise Amplifier at 7GHz (Infineon BFP 620)



Schematic of 7GHz Low Noise Amplifier

LNA (Low Noise Amplifier), Cont'd.

CAD Simulation (Ansoft Designer)



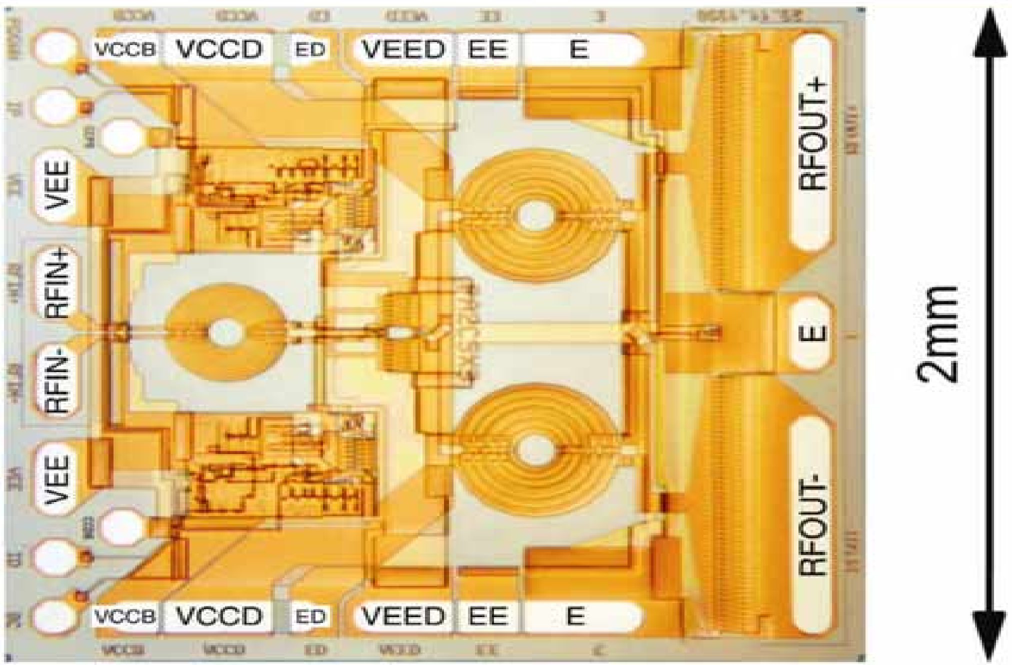
Simulated S-parameters and noise figure

Monolithic Si-Bipolar Power Amplifier

High efficiency 900 MHz Amplifier

Infineon technologies

A Monolithic 2.8 V, 3.2 W Si-Bipolar Power Amplifier
with 54 % PAE at 900 MHz



Ref.: A. Heinz et al., RFIC2000, pp. 117-120, June 2000, Boston

Never stop thinking

W. Simbürger
CPR HF
Apr-2002, p. 17

Monolithic Si-Bipolar power amplifier

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3-Terminal Active Device (FETs)

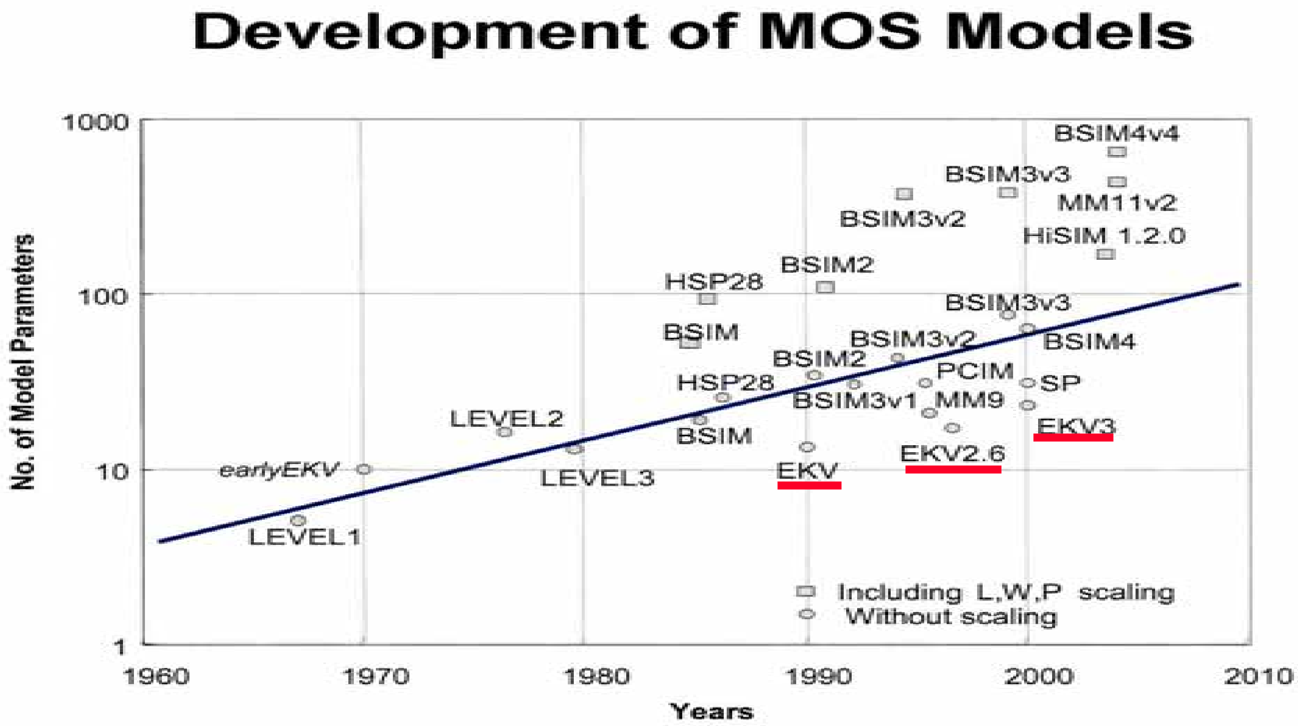
There are several members of the FET family that can be used to high frequencies. The Si junction FET, which has been used for many years, is limited to about 500 MHz for reasonable performance, the most 1 GHz. Their fairly high input capacitance of about 1pF and large feedback capacitance of about 0.1pF limits their use. However, coming from the bipolar process, CMOS transistors have become a strong competitor to GaAs in the RFIC world.

Modern RF and microwave integrated circuits are based increasingly on MOS technology. The reason for this is low cost, low power consumption, and higher integration density. Similarly, as with the bipolar transistor and GaAs FET transistors, the transistors are being described by using a model and model parameters. Drawbacks are low breakdown voltage, leakage currents, and high flicker noise.

CMOS transistors with 0.35-micron technology are used in many applications and even 0.06-micron devices are now available. Operating frequencies above 50 GHz have been shown. The general circuit design rules, however, are the same as for GaAs FETs; one needs to know the measured S parameters or the SPICE parameters. Besides the RFIC MOS and D-MOS transistors, the LDMOS transistors have become very popular for power application.

3-Terminal Active Device (FETs)

Overview of the MOS models developed since 1960 and the number of required parameters. From C. Enz, MOS Transistor Modeling for RF IC Design, June 2004



- Number of DC model parameters vs. the year of the introduction of the model
  Most recent versions of the EKV, HiSIM, MM11and SP models are included
- Significant growth of the parameter number that includes geometry (W/L) scaling

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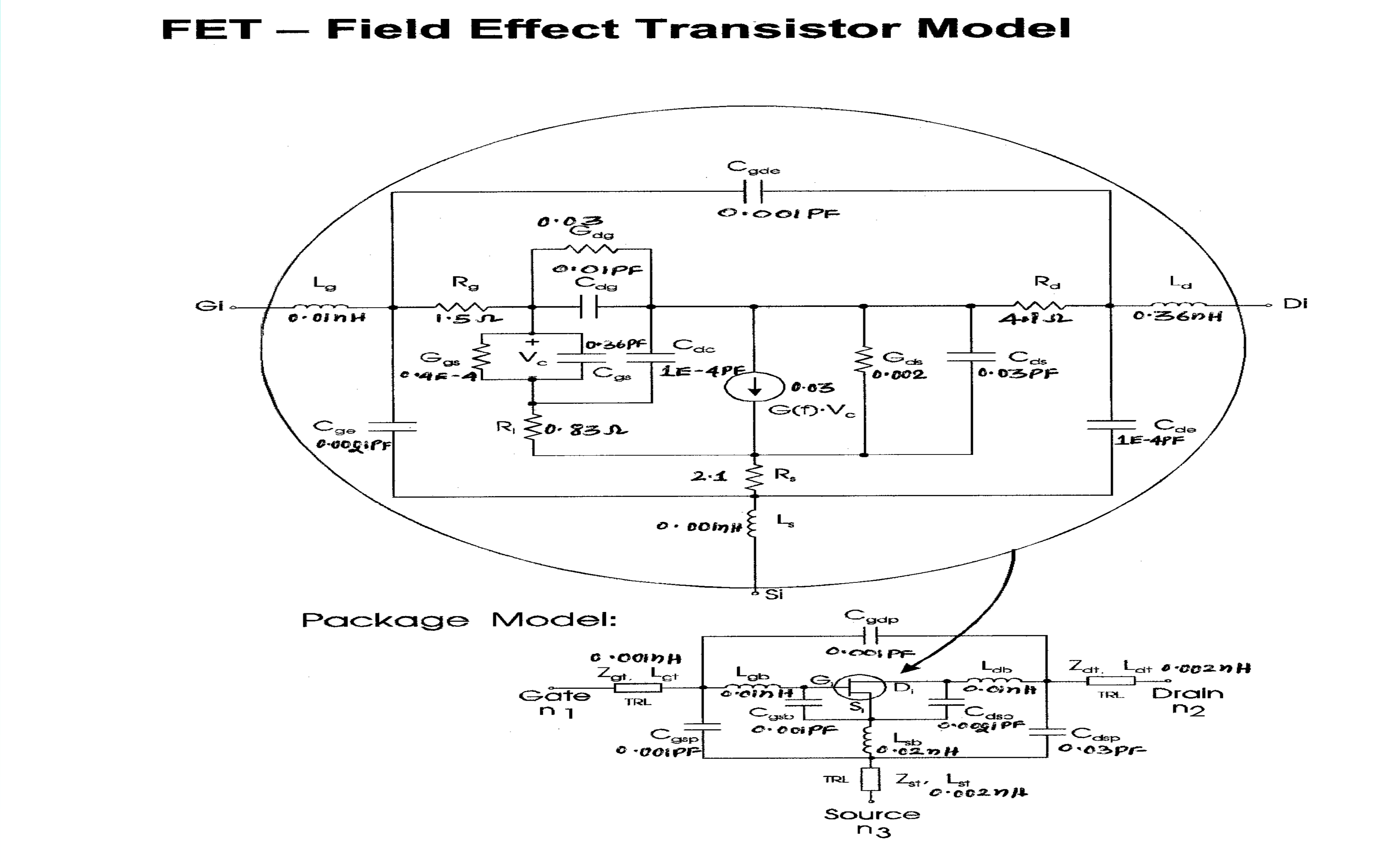
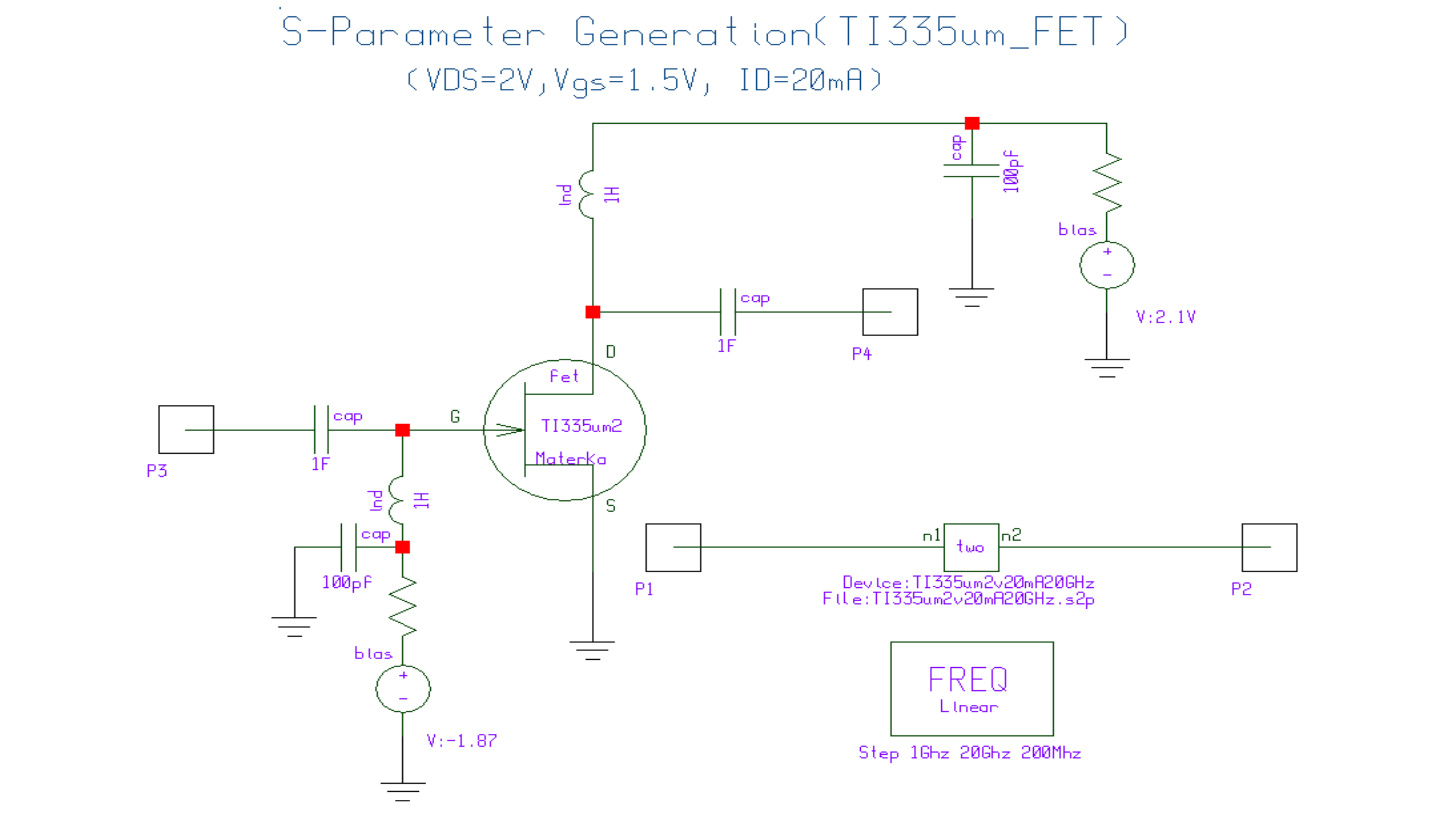
3-Terminal Active Device (FETs)



Texas Instruments 335µm MESFET Model

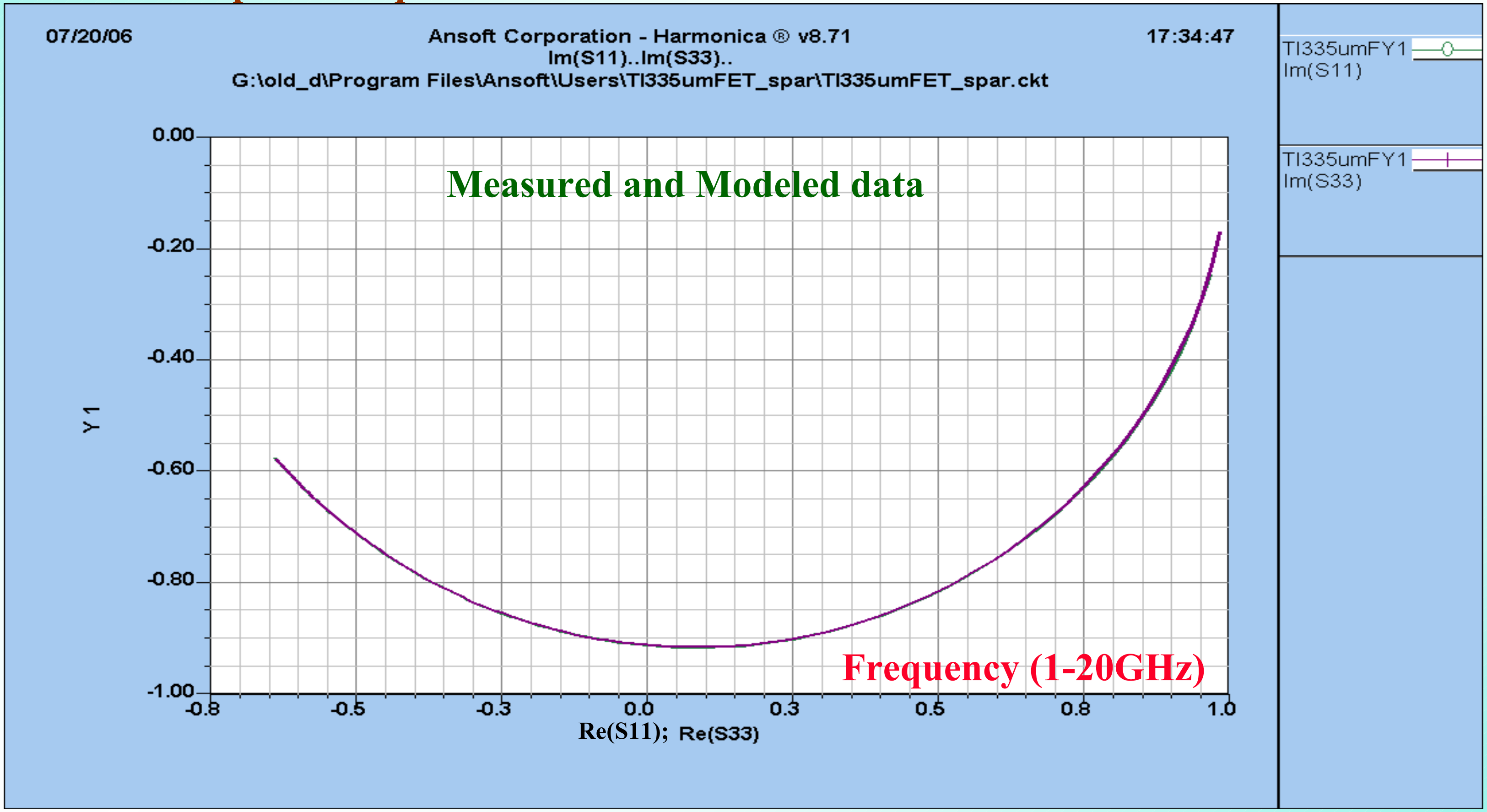
3-Terminal Active Device (FETs)

Typical schematic for S-Parameter Generation (TI335um MESFET)



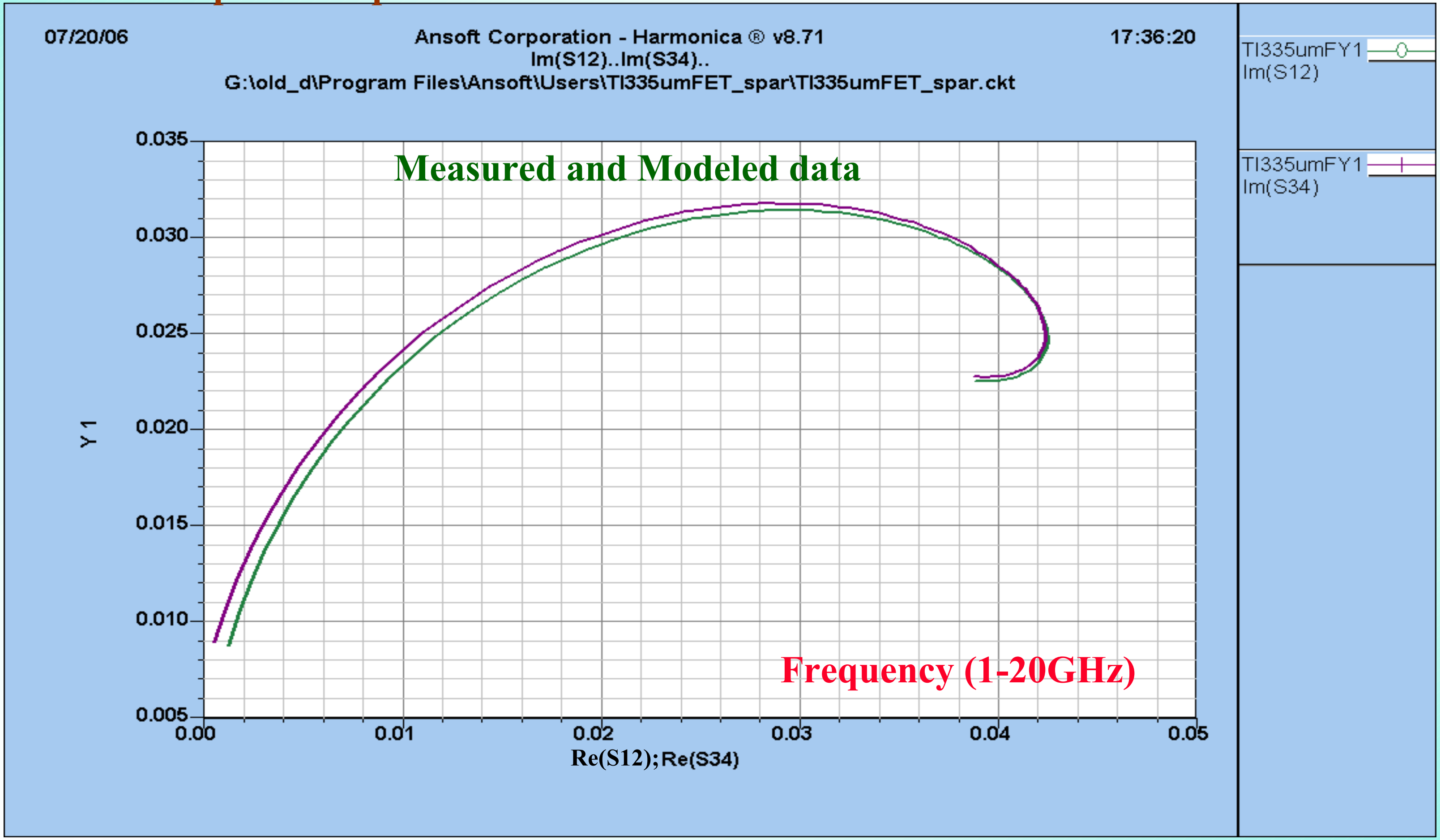
3-Terminal Active Device (FETs)

The curve (S11 and S33) show very good agreement between measured and modeled data based upon the equivalent circuit for a TI 335-um FET.



3-Terminal Active Device (FETs)

The curve (S12 and S34) show very good agreement between measured and modeled data based upon the equivalent circuit for a TI 335-um FET.



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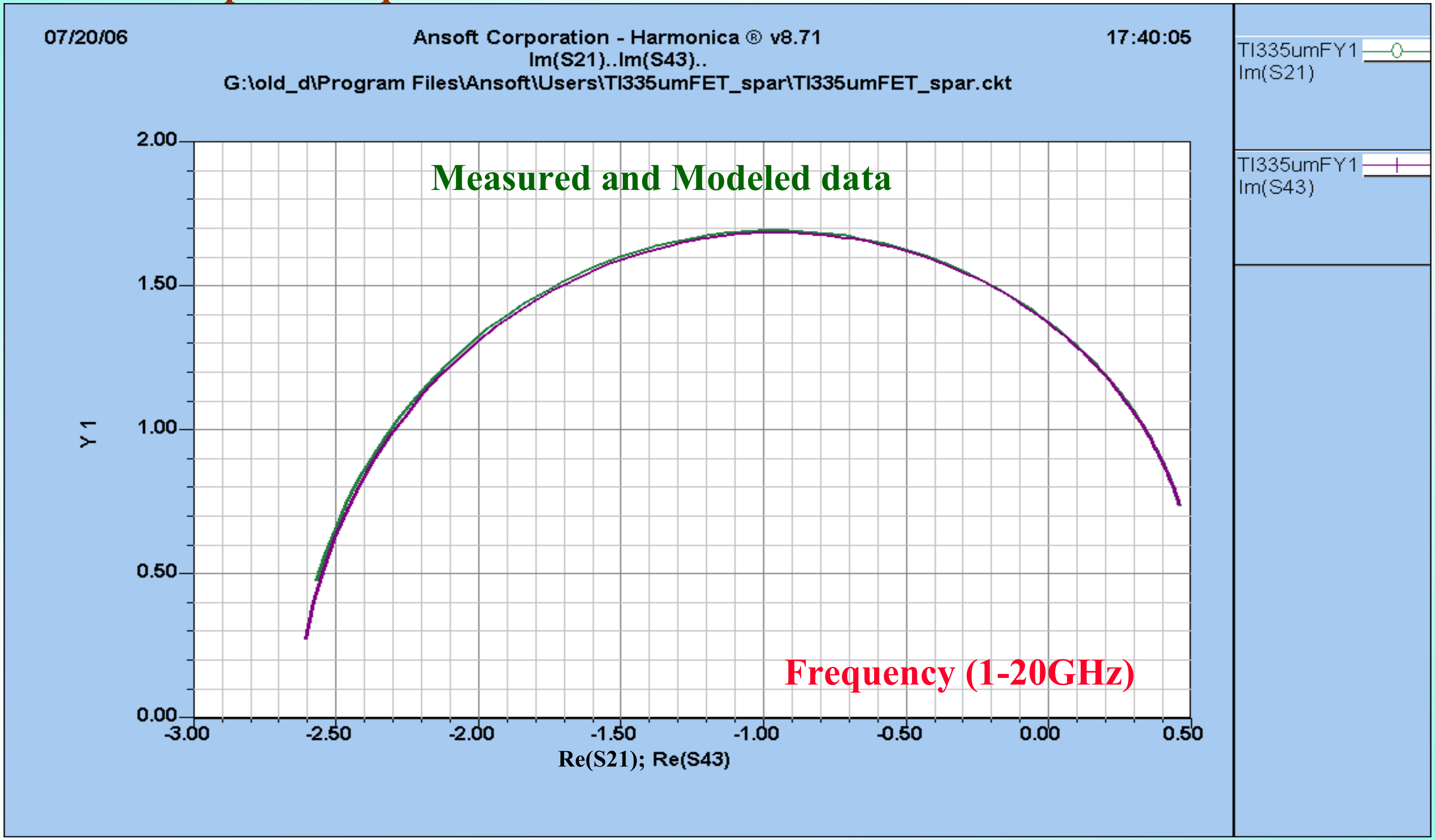
3-Terminal Active Device (FETs)

The curve (S21 and S43) show very good agreement between measured and modeled data based upon the equivalent circuit for a TI 335-um FET.



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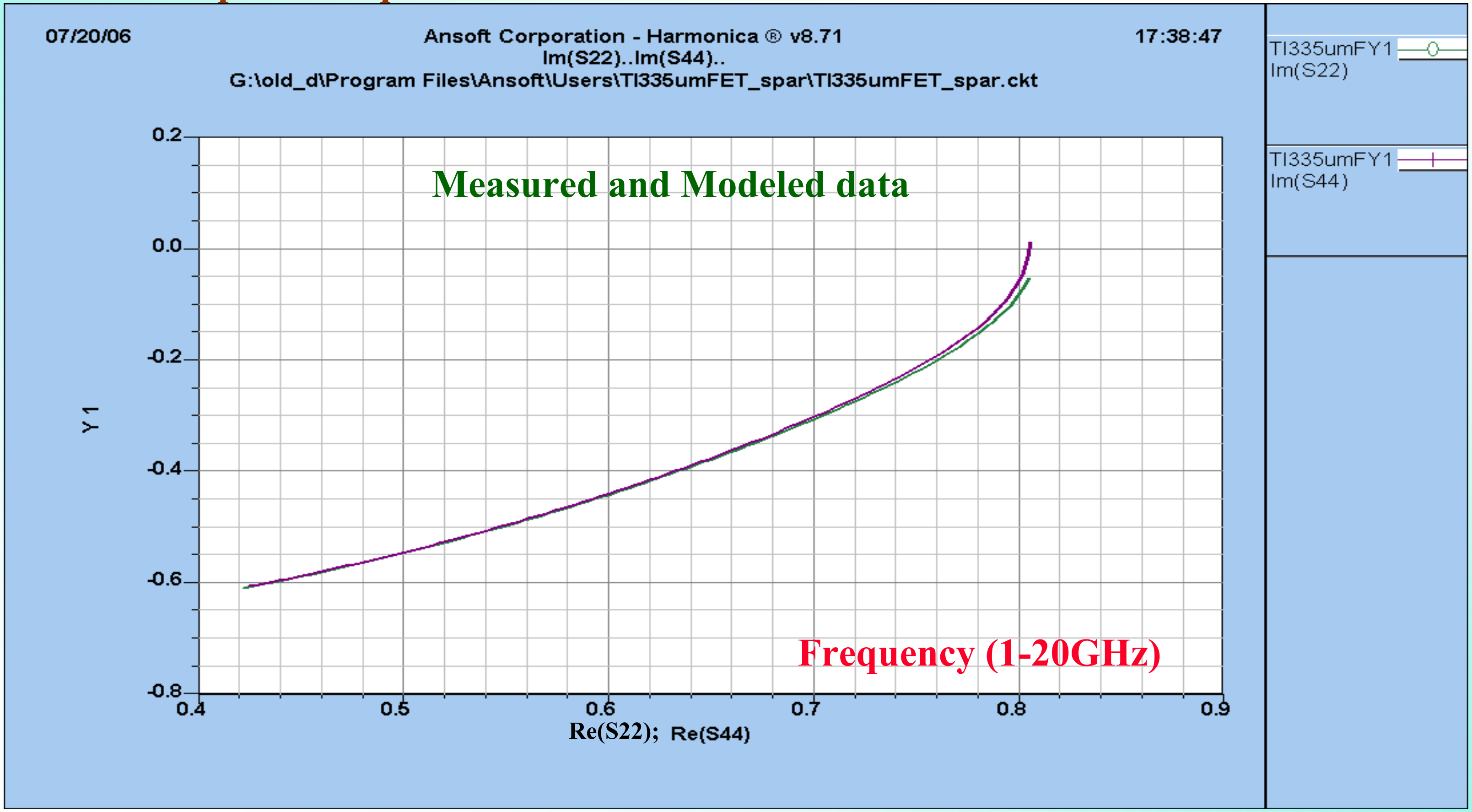
3-Terminal Active Device (FETs)

The curve (S22 and S44) show very good agreement between measured and modeled data based upon the equivalent circuit for a TI 335-um FET.



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Noise in 2-Port (Linear & Non-Linear Circuits/Systems)

2-Port (Linear):

Even when a two-port is linear, the output waveform may differ from the input, because of the failure to transmit all spectral components with equal gain (or attenuation) and delay (group delay is a form of linear distortion). By careful design of the two-port, or by limitation of the bandwidth of the input waveform, such distortions can largely be avoided. However, noise generated within the two-port can still change the waveform of the output signal. In a linear passive two-port, noise arises only from the losses in the two-port; thermodynamic considerations indicate that such losses result in the random changes that we call noise.

2-Port (Non-Linear):

When the two-port contains active devices, such as transistors, there are other noise mechanisms that are present. A very important consideration in a system is the amount of noise that it adds to the transmitted signal. This is often judged by the ratio of the output signal power to the output noise power (S/N). The ratio of signal plus noise power to noise power [$(S + N)/N$] is generally easier to measure, and approaches S/N when the signal is large.

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Noise in 2-Port (Linear & Non-Linear Circuits), Cont'd.

Noise Factor

The noise factor of a system is defined as the ratio of signal-to-noise ratios available at input and output as

$$F = \frac{(S/N)_{\text{input}}}{(S/N)_{\text{output}}} \geq 1$$

When this ratio of powers is converted to decibels, it is generally referred to as the noise figure rather than noise factor. Various conventions are used to distinguish the symbols used for noise factor and noise symbol. Here we use *F* to represent the noise factor and NF to represent the noise figure, although the terms are usually used interchangeably. For an amplifier with the power gain G, the noise factor can be rearranged as

$$F = \frac{S_i/N_i}{GS_i/G(N_i + N_a)} \qquad F = 1 + N_a/N_i \qquad \mathrm{NF} = 10 \log_{10} F$$

where N_a is the additional noise power added by the amplifier referred to the input.

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Noise in 2-Port
(Linear & Non-Linear Circuits), Cont'd.

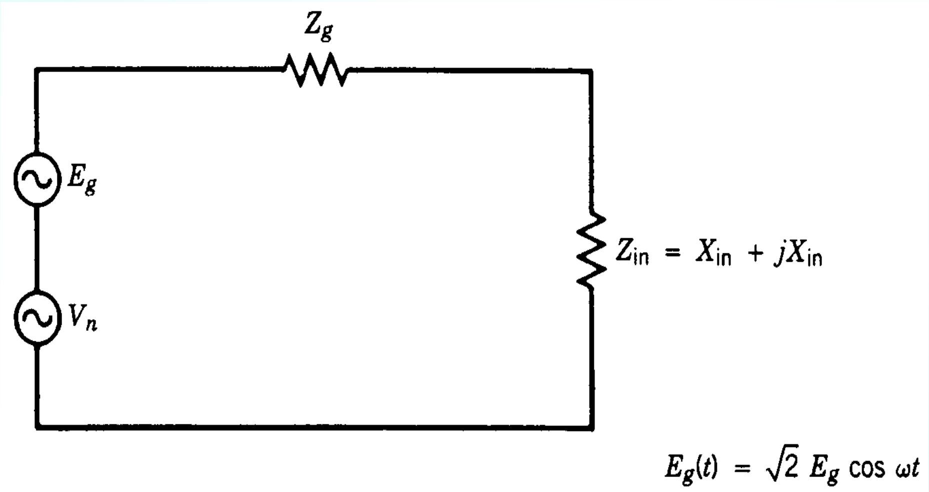
Signal-To-Noise Ratio

The signal power delivered to the input is given by

$$S_{\text{in}} = P_{\text{in}} = \frac{E_g^2 \operatorname{Re}(Z_{\text{in}})}{|Z_g + Z_{\text{in}}|^2}$$

where E_g is the rms voltage of the input signal supplied to the system, and the noise power supplied to the input is expressed by

$$N_{\text{in}} = \frac{\overline{v_n^2} \operatorname{Re}(Z_{\text{in}})}{|Z_g + Z_{\text{in}}|^2}$$



where the noise power at the input is provided by the noise energy of the real part of Z_g, The input impedance Z of the system in the form $Z = R_{in} + jX_{in}$ is assumed to be complex.

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Noise in 2-Port
(Linear & Non-Linear Circuits), Cont'd.

The Johnson noise (Thermal noise) of a resistor [here R= Re(Z_g)] is given by the mean-square voltage $\overline{v_n^2} = 4kTRB$

with k (Boltzman's constant) = 1.38×10^{-23} J/K, T the absolute temperature of the resistor, and B the bandwidth, is sufficiently small that the resistive component of impedance does not change. For an ambient temperature of 290 K, $kT = 4 \times 10^{-21}$ W/Hz. This expression is also given as as kT = –204 dBW/Hz = –174 dBm/Hz = –114 dBm/MHz

The generator resistor acts as a Johnson noise generator, its maximum available power is given by

$$P_A = \frac{4kTRB}{4R} = kTB$$

The value of S/N contributed by the generator is given by

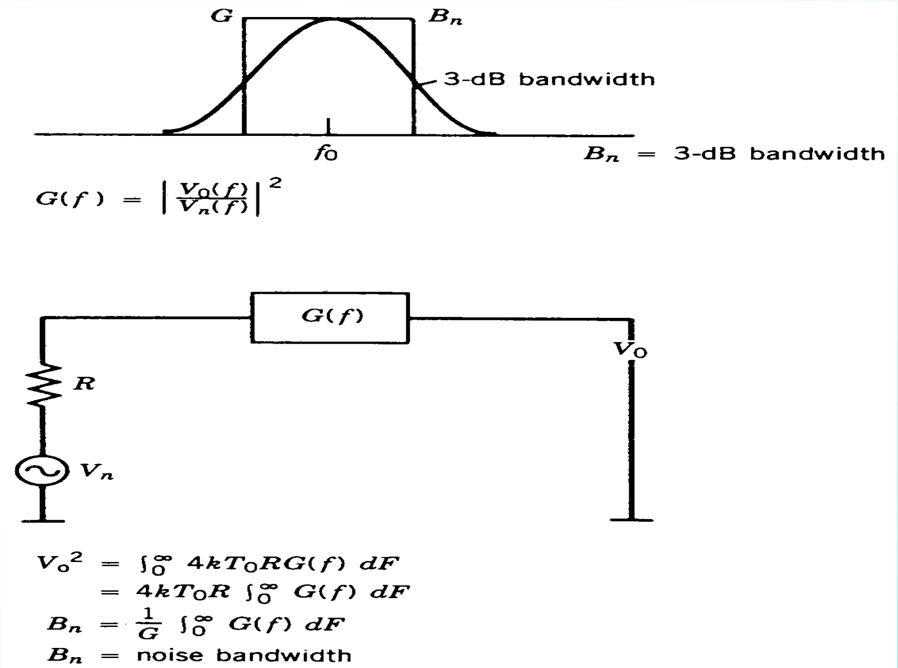
$$\left(\frac{S}{N}\right)_{\text{in}} = \frac{E_g^2}{4kT\,\mathrm{Re}(Z_g)B}$$

Noise in 2-Port
(Linear & Non-Linear Circuits), Cont'd.

Noise Bandwidth

Noise bandwidth, B_n, is defined as the equivalent integrated bandwidth, as shown in Figure below. For reasons of group delay correction, most practical filters have round Gaussian response rather than sharp comers (Chebyshev- type).

Graphical and mathematical explanation of the noise bandwidth from a comparison of the Gaussian-shaped bandwidth to the rectangular filter response.



Noise in 2-Port
(Linear & Non-Linear Circuits), Cont'd.

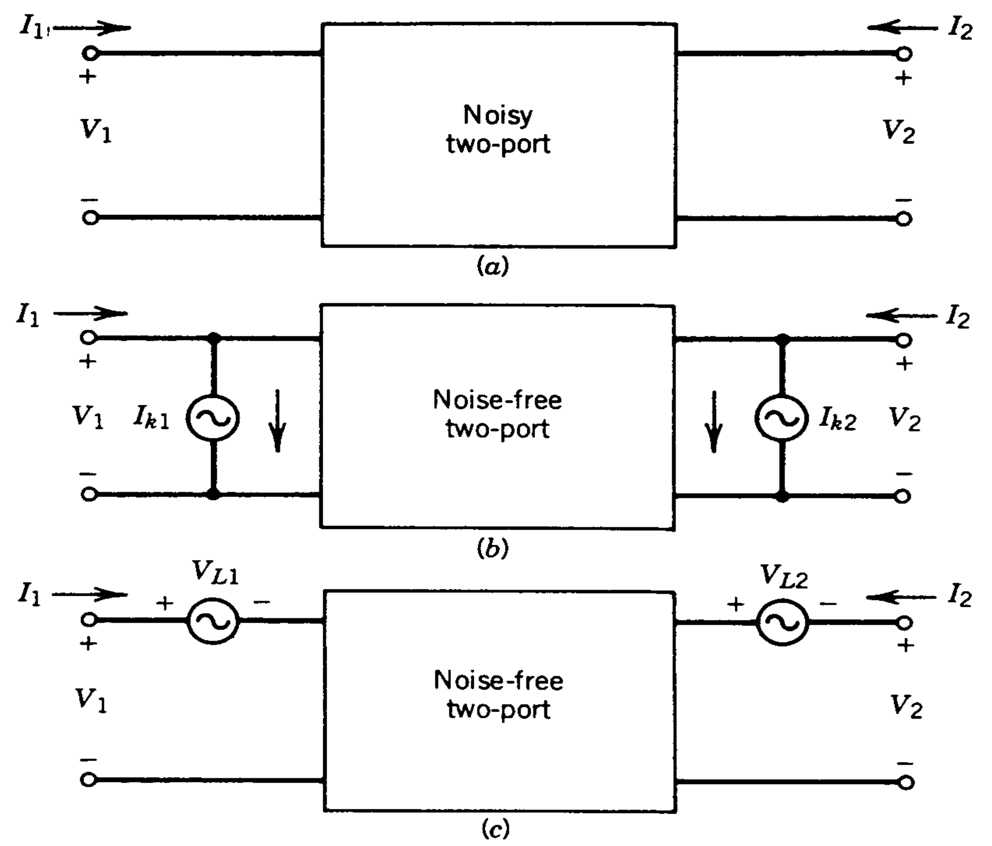
An active system such as a combination of amplifiers and mixers will add noise to the input signals, and the noise factor that describes this is defined as the *S*/*N* ratio at the input to the *SIN* ratio at the output, which is always greater than unity. In practice, a certain minimum signal-to-noise ratio leveled on the node is required for operation.

For example, in a communication system such a minimum is required for intelligible transmission, either voice or data. For high-performance TV reception, to provide a picture noise-free to the eye, a typical requirement is for a 60-dB *S*/*N*.

In the case of a TV system, a large dynamic range is required, as well as a very large bandwidth to reproduce all colors truthfully and all shades from high-intensity white to black. Good systems will have 8-MHz bandwidth or more.

Noise in 2-Port
(Linear & Non-Linear Circuits), Cont'd.

Noise in two-ports: (*a*) general form; (*b*) admittance form; (*c*) impedance form



$$I_1 = y_{11}V_1 + y_{12}V_2 + I_{K1}$$
$$I_2 = y_{21}V_1 + y_{22}V_2 + I_{K2}$$

$$V_1 = z_{11}I_1 + z_{12}I_2 + V_{L1}$$
$$V_2 = z_{21}I_1 + z_{22}I_2 + V_{L2}$$

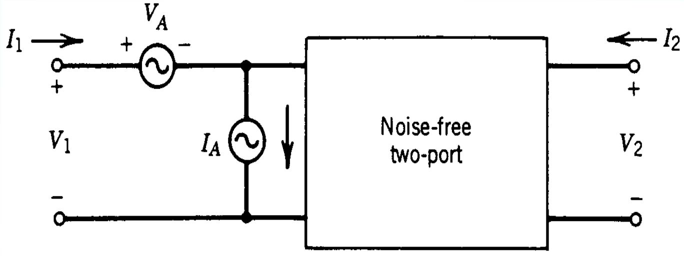
where the external noise sources are I_{K1}, I_{K2}, V_{L1}, and V_{L2}.

Noise in 2-Port
(Linear & Non-Linear Circuits), Cont'd.

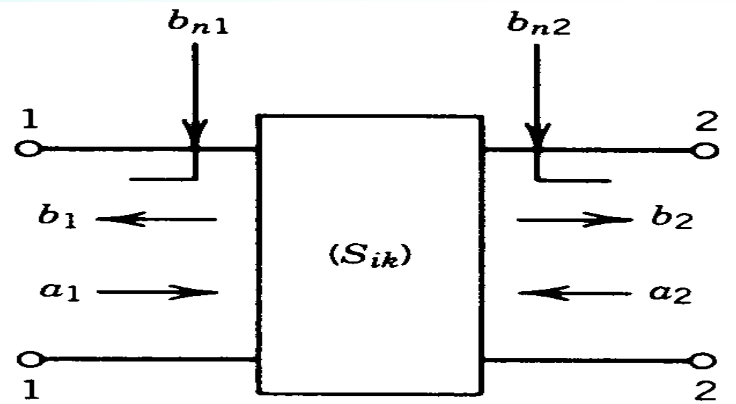
S-parameters of 2-port circuit

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_{n1} \\ b_{n2} \end{bmatrix}$$

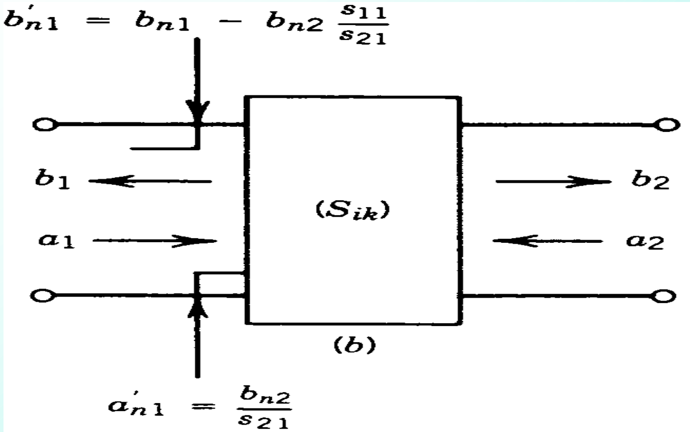
Chain Matrix form of 2-port circuit



S-parameters representation of noisy 2-port circuit



(a)



(b)

Noise in 2-Port
(Linear & Non-Linear Circuits), Cont'd.

There are different physical origins for the various sources of noise. Typically, thermal noise is generated by resistances and loss in the circuit or transistor, whereas, shot noise is generated by current flowing through semiconductor junctions and vacuum tubes. Since these many sources of noise are represented by only two noise sources at the device input, the two equivalent input noise sources are often a complicated combination of the circuit internal noise sources. Often, some fraction of V_A and I_A is related to the same noise source. This means that V_A and I_A are not independent in general.

Before we can use V_A and I_A to calculate the noise factor/noise figure of the two-port, we must calculate the correlation between the V_A and I_A .The noise source V_A represents all of the device noise referred to the input when the generator impedance is zero; that is, the input is short-circuited. The noise source I_A represents all of the device noise referred to the input when the generator admittance is zero; that is, the input is open circuited.

Noise in 2-Port
(Linear & Non-Linear Circuits), Cont'd.

The correlation of these two noise sources considerably complicates the analysis. By defining correlation admittance, we can simplify the mathematics and get some physical intuition for the relationship between noise figure and generator admittance. Since some fraction of I_A will be correlated with V_A, we split I_A into correlated and uncorrelated parts as follows:

$$I_A = I_n + I_u$$

I_u is the part of I_A uncorrelated with V_A. Since I_n is correlated with V_A, we can say that I_n is proportional to V_A and the constant of proportionality is the correlation Admittance Y_{cor} . This leads to:

$$I_n = Y_{cor}V_A \qquad I_A = Y_{cor}V_A + I_u$$

Y_{cor} is not a physical component located somewhere in the circuit. Y_{cor} is a complex number derived by correlating the random variables I_A and V_A.

Noise in 2-Port
(Linear & Non-Linear Circuits), Cont'd.

Calculation of Y_{cor}:

$$I_A = Y_{cor} V_A + I_u$$

Multiply each side of by V_A^* and average the result

$$V_A^* I_A = Y_{cor} V_A^* V_A + V_A^* I_u \Rightarrow \overline{V_A^* I_A} = Y_{cor} \overline{V_A^2}$$ **where** $V_A^* = complex - conjugate \quad of \quad V_A$

where the I_u term averaged to zero since it is uncorrelated with V_A. The correlation admittance is thus given by

$$Y_{cor} = \frac{\overline{V_A^* I_A}}{\overline{V_A^2}}$$

Correlation Coefficient 'c_r':

"Correlation coefficient" is a normalized quantity, which is is defined as

$$c_r = \frac{\overline{V_A^* I_A}}{\sqrt{\overline{V_A^2}\,\overline{I_A^2}}} \Rightarrow c_r = Y_{cor} \sqrt{\frac{\overline{V_A^2}}{\overline{I_A^2}}}; \quad c_r = Z_{cor} \sqrt{\frac{\overline{I_A^2}}{\overline{V_A^2}}}$$

Note that the dual of this admittance description is the impedance description. Thus the impedance representation has the same equations as above with *Y* replaced by *Z*, *I* replaced by *V*, and *V* replaced by *I*.

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Noise in 2-Port :Noise Circle

Noise tuning

Noise tuning is the method to change the values of the input admittance to obtain the best noise performance. There is a range of values of input reflection coefficients over which the noise figure is constant. In plotting these points of constant noise figure, we obtain noise circles, which can be drawn on the Smith chart Γ_G plane. The center of the noise circle can be given by

$$C_i = \frac{\Gamma_{0n}}{1 + N_i}$$

where $T_{on} = \dfrac{Z_{on} - Z_o}{Z_{on} + Z_o}$

The radius of the noise circle can be given by

$$r_i = \frac{\sqrt{N_i^2 + N_i\left(1 - |\Gamma_{0n}|^2\right)}}{1 + N_i}$$

where N_i is the input noise

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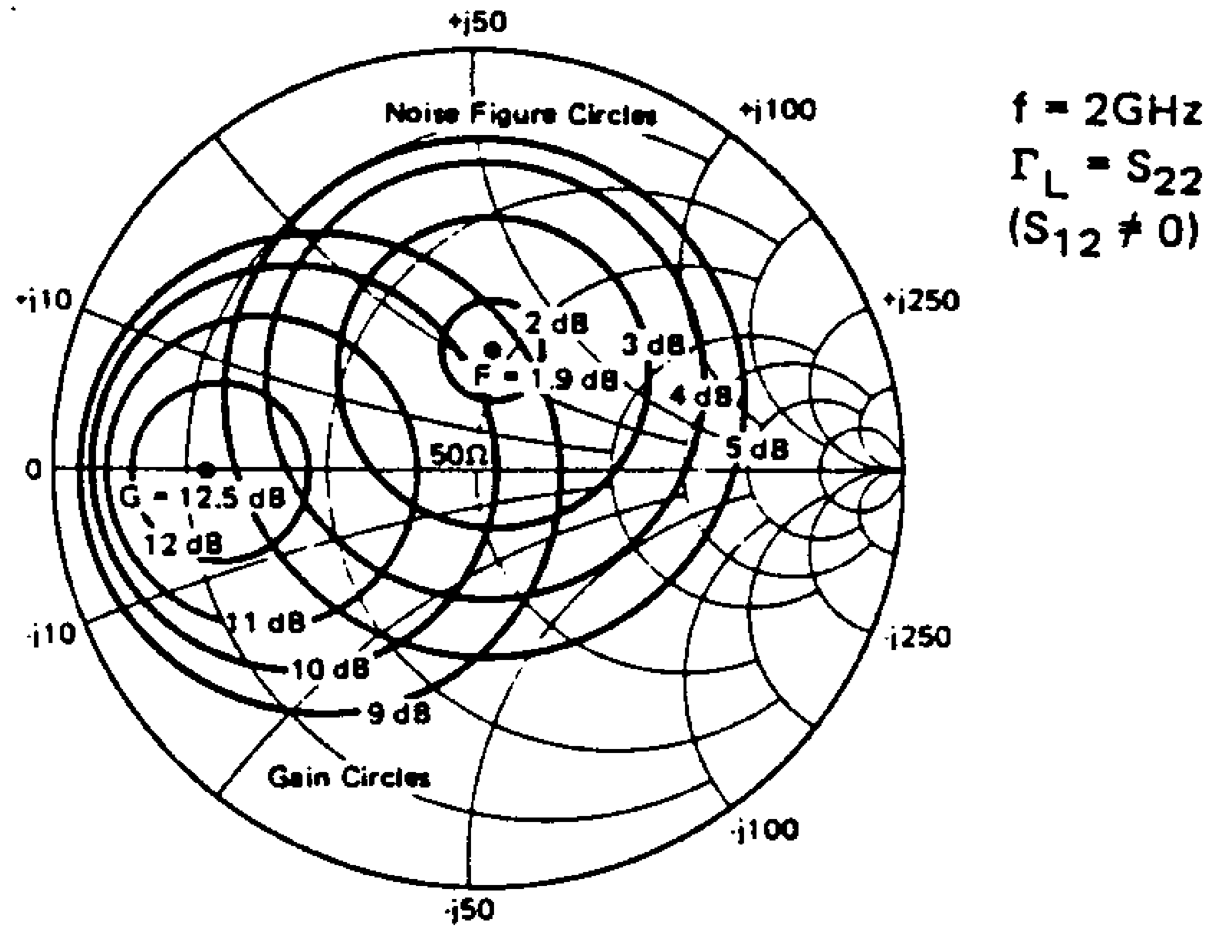
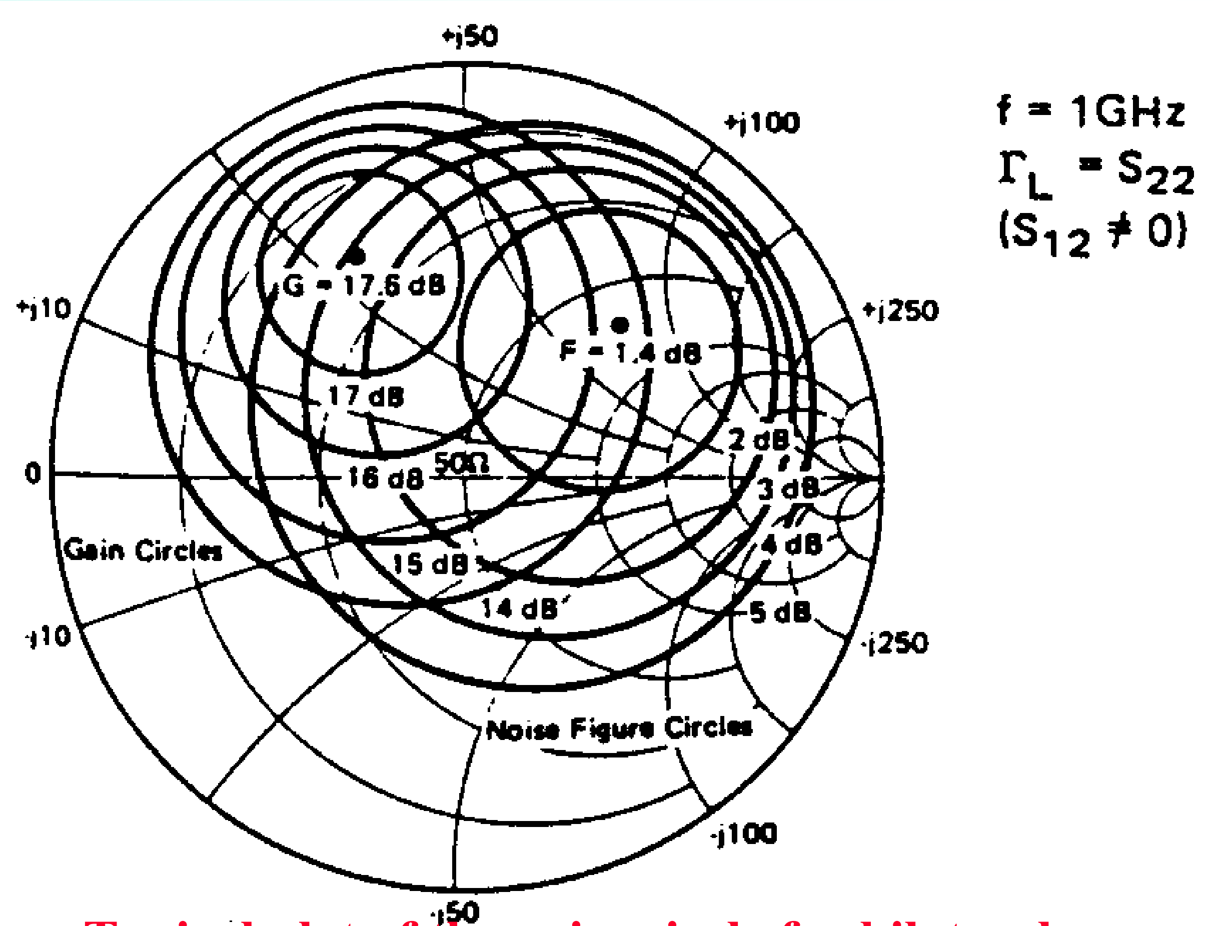
Noise in 2-Port : Noise Circle, Cont'd.



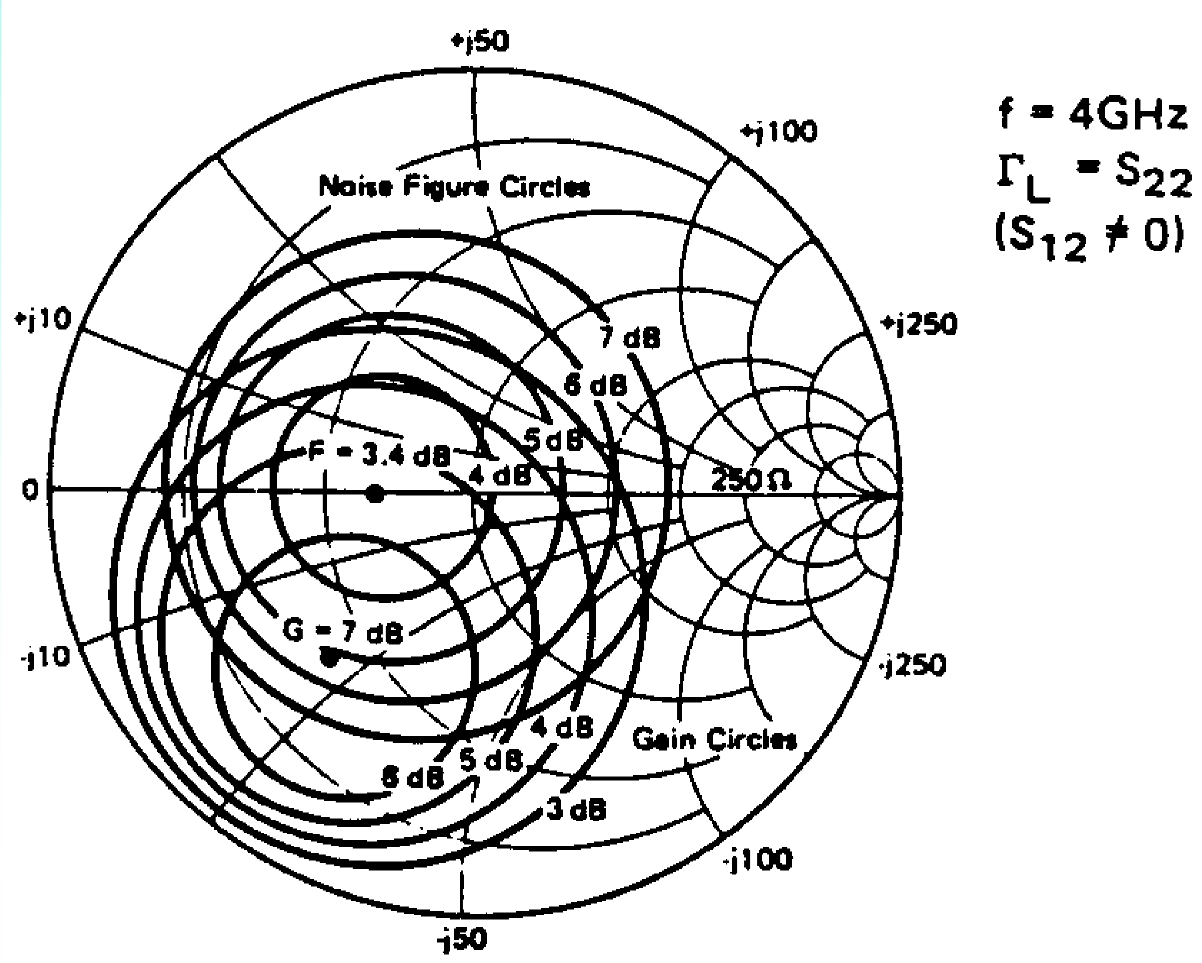
Typical plot of the noise circle for bilateral case

Noise in 2-Port : Noise Circle, Cont'd.



Typical plot of the noise circle for bilateral case

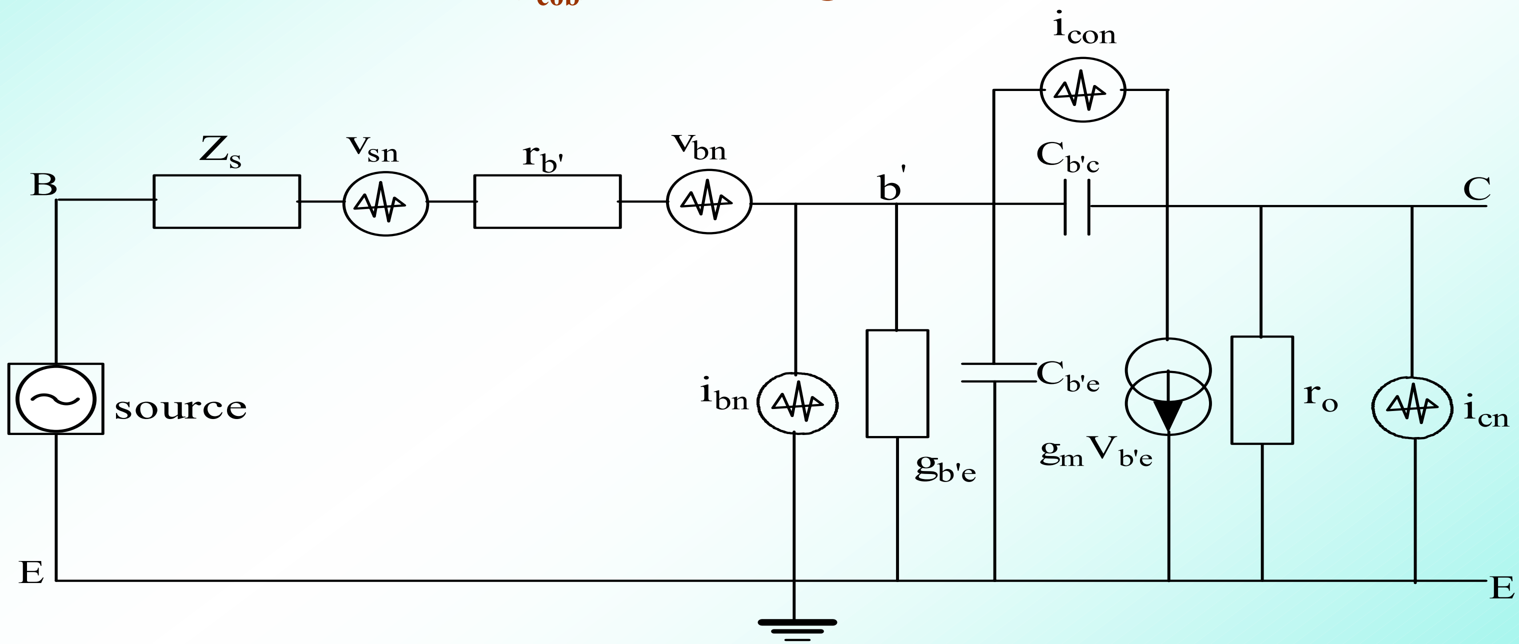
Noise in 2-Port : Noise Circle, Cont'd.



Typical plot of the noise circle for bilateral case

Noise Parameter of Bipolar Transistor

The high frequency noise of a bipolar transistor in grounded emitter configuration can be modeled by using the three noise sources. The emitter junction in this case is conductive and this generates shot noise on the emitter. The emitter current is divided in to a base (I_b) and a collector current (I_c) and both these currents generate shot noise. The collector reverse current (I_{cob}), which also generates shot noise.



Hybrid-π Configuration (Grounded Emitter)

Noise Parameter of Bipolar, Cont'd.

The emitter, base and collector are made of semiconductor material and have finite value of resistance associated with them, which generates thermal noise.

The value of base resistor is relatively high in comparison to resistance associated with emitter and collector, so the noise contribution of these resistors can be neglected.

For noise analysis three sources are introduced in a noiseless transistor and these noise generators are due to fluctuation in DC bias current (i_{bn}), DC collector current (i_{cn}) and thermal noise of the base resistance.

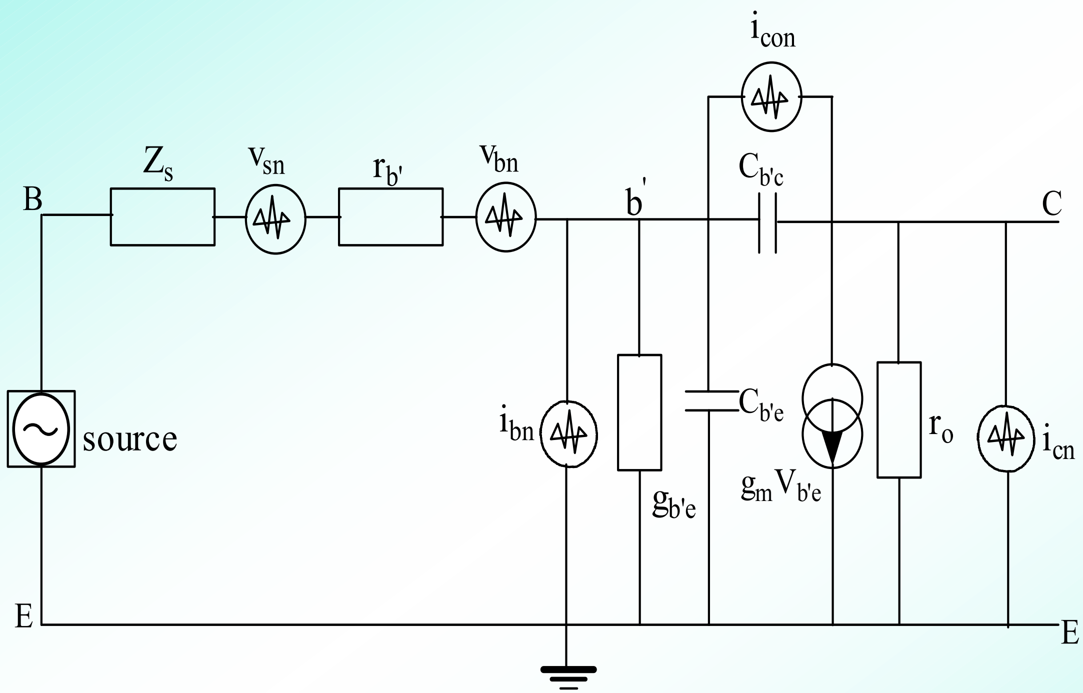
In Silicon transistor the collector reverse current (I_{cob}) is very small and noise (i_{con}) generated due to this can be neglected.

Note:

For the evaluation of the noise performances, the signal-driving source should also be taken into consideration because its internal conductance generates noise and its susceptance affects the noise figure through noise tuning.

Noise Parameter of Bipolar, Cont'd.

The mean square value of above noise generator in a narrow frequency interval Δf are given by



$$\overline{i_{bn}^{2}} = 2qI_{b}\Delta f$$

$$\overline{v_{sn}^{2}} = 4kTR_{b}\Delta f$$

$$\overline{i_{cn}^{2}} = 2qI_{c}\Delta f$$

$$\overline{i_{con}^{2}} = 2qI_{cob}\Delta f$$

$$\overline{v_{bn}^{2}} = 4kTR_{b}\Delta f$$

I_b, I_c and I_{cob} are average DC current over Δf noise bandwidth.

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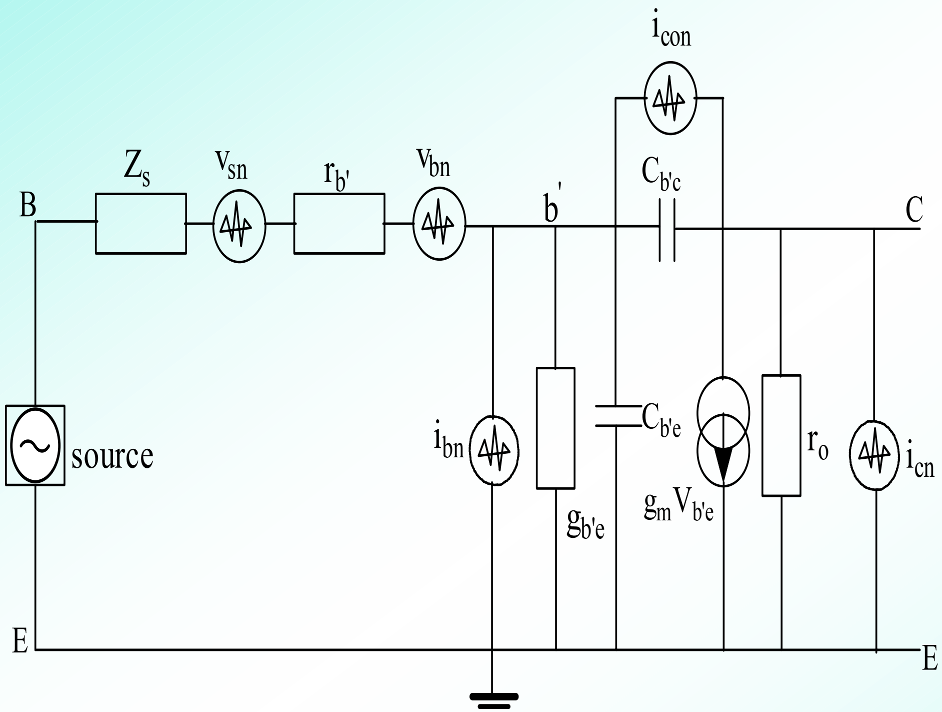
Noise Parameter of Bipolar, Cont'd.

The noise power spectral densities due to noise sources is given as



$$S(i_{bn}) = \frac{\overline{i_{bn}^2}}{\Delta f} = 2qI_b = \frac{2KTg_m}{\beta}$$

$$S(i_{cn}) = \frac{\overline{i_{cn}^2}}{\Delta f} = 2qI_c = 2KTg_m$$

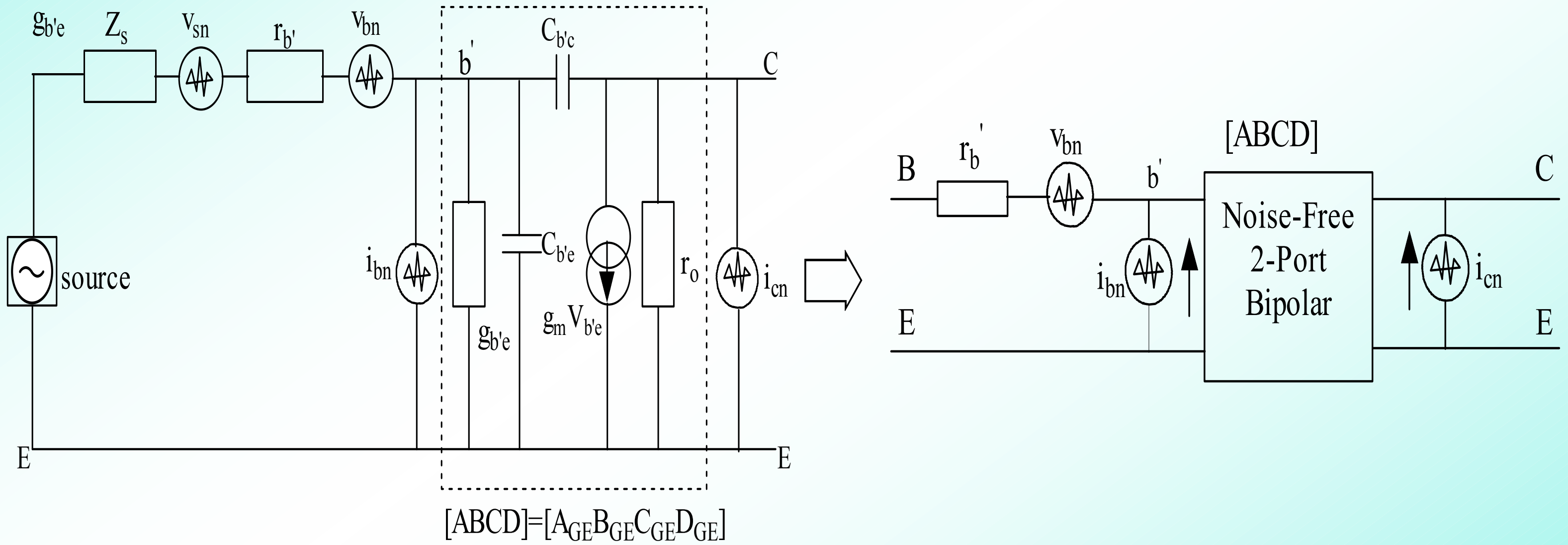
$$S(v_{sn}) = \frac{\overline{v_{sn}^2}}{\Delta f} = 4KTR_s$$

$$S(v_{bn}) = \frac{\overline{v_{bn}^2}}{\Delta f} = 4KTR_b$$

r_b' **and R_s are base and source resistance and Z_s is the complex source impedance**.

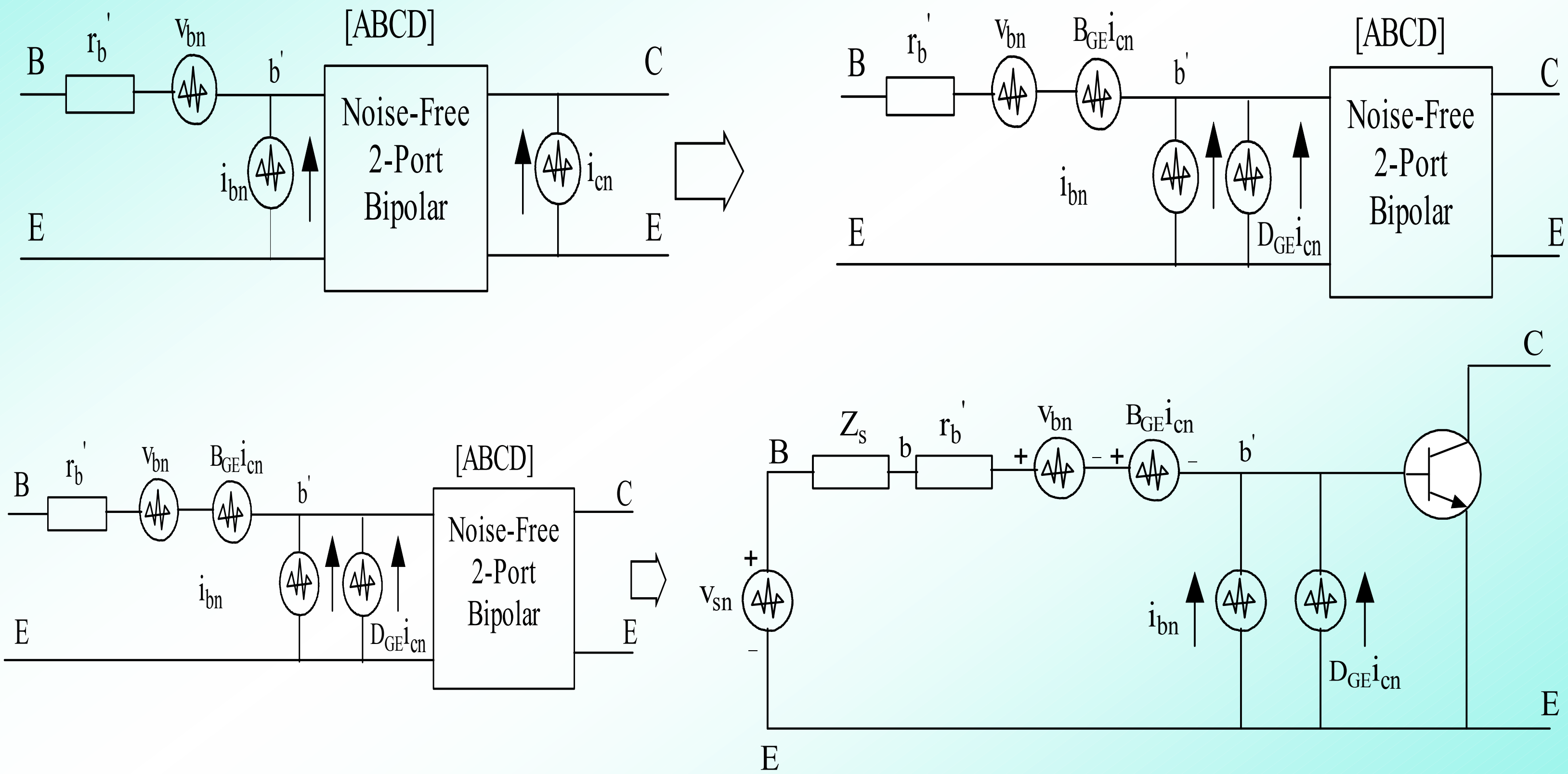
Noise Parameter of Bipolar, Cont'd.

2-port [ABCD] and the GE bipolar transistor presentation for the calculation of the Noise Factor:



π-configuration of the bipolar transistor with noise sources and equivalent [ABCD] representation of the intrinsic transistor.

Noise Parameter of Bipolar, Cont'd.



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Noise Parameter of Bipolar, Cont'd.

The noise factor F is the ratio of the total mean square noise current and the thermal noise generated from the source resistance.

$$F = \frac{\overline{v_{n(total)}^{2}}}{\overline{v_{sn}^{2}}} = \frac{\overline{V_{sn}^{2}} + \overline{V_{network}^{2}}}{\overline{V_{sn}^{2}}} \Rightarrow F = \frac{\overline{V_{sn}^{2}}}{\overline{V_{sn}^{2}}} + \frac{\overline{V_{network}^{2}}}{\overline{V_{sn}^{2}}} = 1 + \frac{\overline{V_{network}^{2}}}{\overline{V_{sn}^{2}}}$$

$$V_{n(total)} = V_{sn} + V_{n(network)}$$

Where $V_{n(total)}$ **= Total noise voltage**

V_{sn} **= Noise due to source**

$V_{n(network)}$ **= Noise due to network**

$$F = 1 + \left[\frac{[\overline{V_{bn}^{2}} + \overline{I_{bn}^{2}}(R_s + r_b')^2 + \overline{I_{cn}^{2}} r_e^2 + \overline{I_{cn}^{2}}(R_s + r_b')^2 (\frac{f^2}{f_T^2}) + \overline{I_{cn}^{2}}(R_s + r_b')^2 (\frac{1}{\beta^2}) + \overline{I_{bn}^{2}}(R_s + r_b')(R_s + r_b' + 2r_e)]}{4kT\Delta f R_s} + \left[\frac{[\overline{I_{bn}^{2}} X_s^2 + \overline{I_{cn}^{2}}(\frac{1}{\beta^2}) - \overline{I_{cn}^{2}}(\frac{f^2}{f_T^2})]}{4kT\Delta f R_s} \right] \right]$$

Noise Parameter of Bipolar, Cont'd.

For Real Source Impedance:

In the case of real source impedance, $X_s = 0$, and noise factor F can be expressed as

$$F = 1 + \frac{r_b^{'}}{R_s} + \frac{r_e}{2R_s} + \frac{(r_b^{'} + R_s)(r_b^{'} + R_s + 2r_e)}{2r_e \beta R_s} + \frac{(r_b^{'} + R_s)^2}{2r_e R_s \beta^2} + \frac{(r_b^{'} + R_s)^2}{2r_e R_s}\left(\frac{f}{f_T}\right)^2$$

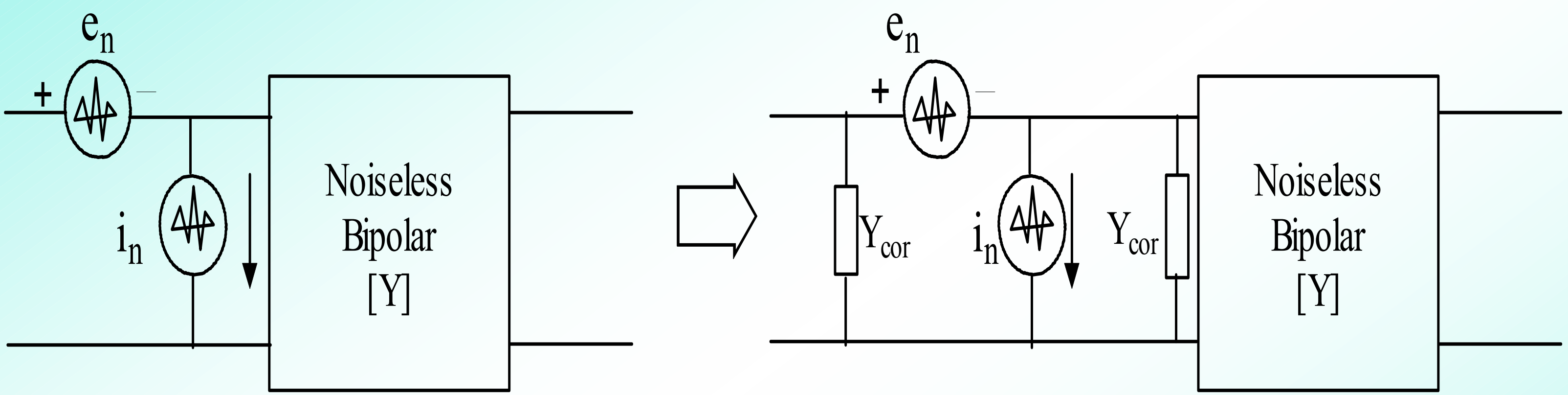
If $wr_e C_{b'c} << 1$ and $\beta >> 1$ then noise factor can be further simplified as

$$F = 1 + \frac{1}{R_s}\left[\left\langle r_b^{'}\right\rangle + \left\langle \frac{(r_b^{'} + R_s)^2}{2r_e\beta}\right\rangle + \left\langle \frac{r_e}{2} + \frac{(r_b^{'} + R_s)^2}{2r_e}\left(\frac{f^2}{f_T^2}\right)\right\rangle\right]$$

where the contribution of the first term is due to the base resistance, the second term is due to the base current and the last term is due to the collector current.

Noise Parameter of Bipolar, Cont'd.

Noise Correlation Matrix of the Bipolar Transistor



$$C_r = \frac{\overline{|e_n^{\bullet} i_n|}}{\sqrt{\overline{|e_n^2|}\,\overline{|i_n^2|}}} \qquad Y_{cor} = \frac{\overline{|e_n^{\bullet} i_n|}}{\sqrt{\overline{|e_n^2|}}}$$

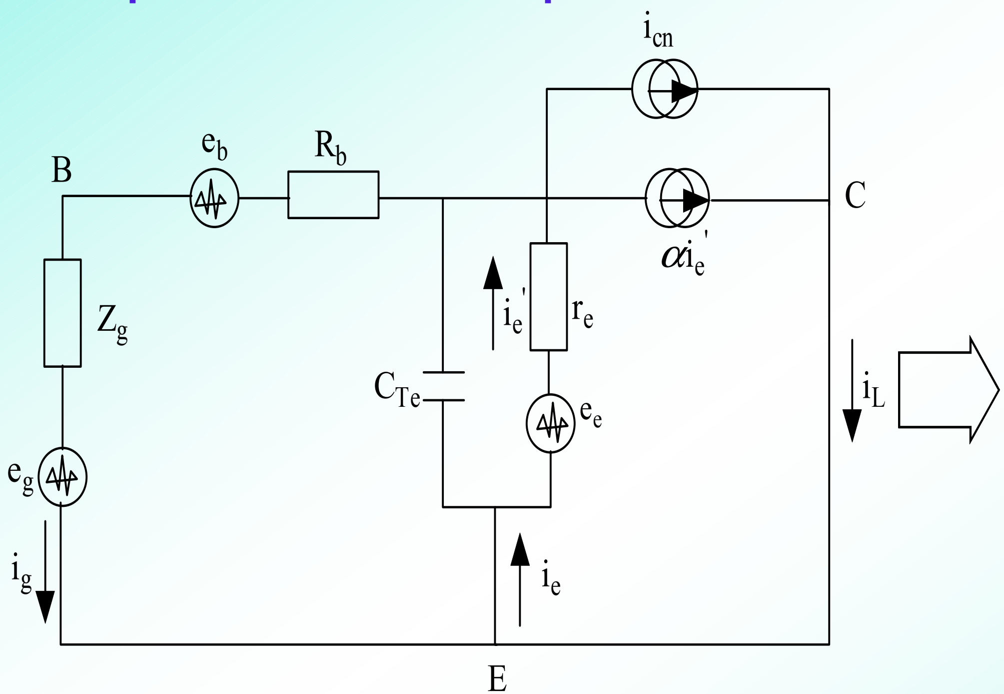
$$C_r = \frac{\overline{|e_n^{\bullet} i_n|}}{\sqrt{\overline{|e_n^2|}\,\overline{|i_n^2|}}} = Y_{cor}\sqrt{\frac{\overline{|e_n^2|}}{\overline{|i_n^2|}}}$$

Where $e_n^{\bullet}$ is the conjugate of e_n

Noise Parameter of Bipolar, Cont'd.

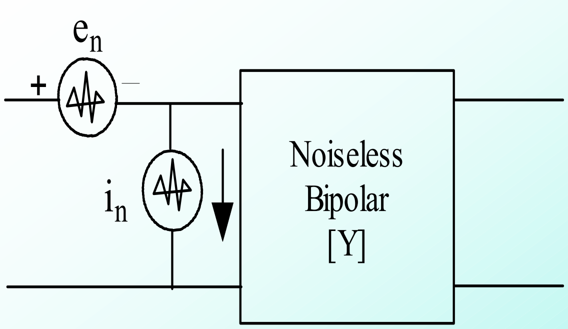
Noise Correlation Matrix of the Bipolar Transistor in T-Equivalent Configuration:

T-equivalent circuit of bipolar noise model



$$[Y]_{tr} = \begin{bmatrix} \{(1-\alpha)g_e + jwC_e + Y_c & -Y_c \\ \alpha g_e - Y_c & Y_c \end{bmatrix}$$

Transformed bipolar transistor noise model represented as a 2-port admittance [Y] matrix.



$$[Y]_{tr} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

Noise Parameter of Bipolar, Cont'd.

The matrix for the intrinsic device is defined as [*N*] and for the transformed noise circuit as [*C*].

$$[N]_{\text{int}\,rinsic} = \frac{1}{4KT\Delta f}\begin{bmatrix} \overline{e_e e_e^*} & \overline{e_e i_{cp}^*} \\ \overline{i_{cp} e_e^*} & \overline{i_{cp} i_{cp}^*} \end{bmatrix} = \begin{bmatrix} \frac{1}{2g_e} & 0 \\ 0 & \frac{g_e(\alpha_0 - |\alpha|^2)}{2} \end{bmatrix}$$

$$[C]_{transformed} = \frac{1}{4KT\Delta f}\begin{bmatrix} \overline{e_n e_n^*} & \overline{e_n i_n^*} \\ \overline{i_n e_n^*} & \overline{i_n i_n^*} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

The noise correlation matrix *C* in terms of *N* can be obtained by a straightforward application of the steps outlined as

$$C = AZTN(AZT)^{\oplus} + ARA^{\oplus}$$

The sign ⊕ denotes the Hermitian conjugate (Hermitian matrix is defined as self-adjoint matrix)

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Noise Parameter of Bipolar, Cont'd.

The matrix *Z* is the inverse of the admittance matrix *Y* for the intrinsic portion of the model and *T* is a transformation matrix, which converts the noise sources e_e and i_{cp} to shunt current sources, respectively, across the base-emitter and collector-emitter ports of the transistor. The matrix *A* is a circuit transformation matrix and the matrix *R* is a noise correlation matrix representing thermal noise of the extrinsic base resistance.

$$T = \begin{bmatrix} -(1-\alpha)g_e & 1 \\ -\alpha g_e & -1 \end{bmatrix}; \quad A = \begin{bmatrix} 1 & \dfrac{Z_{11}+r_b}{Z_{21}} \\ 0 & -\dfrac{1}{Z_{11}} \end{bmatrix}; \quad R = \frac{1}{4KT\Delta f}\begin{bmatrix} \overline{e_b e_b^*} & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} r_b & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} C_{uu^\bullet} & C_{ui^\bullet} \\ C_{u^\bullet i} & C_{ii^\bullet} \end{bmatrix} = \begin{bmatrix} R_n & \dfrac{F_{\min}-1}{2} - R_n Y_{opt}^\bullet \\ \dfrac{F_{\min}-1}{2} - R_n Y_{opt} & R_n \left|Y_{opt}\right|^2 \end{bmatrix}; \quad R_n = \frac{C_{uu^\bullet}}{2kT}$$

$$Y_{opt} = \sqrt{\frac{C_{ii^\bullet}}{C_{uu^\bullet}} - \left[\mathrm{Im}\left(\frac{C_{ui^\bullet}}{C_{uu^\bullet}}\right)\right]^2} + j\,\mathrm{Im}\left(\frac{C_{ui^\bullet}}{C_{uu^\bullet}}\right); \qquad Y_{opt} = G_{opt} + jB_{opt}$$

Noise Parameter of Bipolar, Cont'd.

The noise correlation matrix *C* contains all necessary information about the four extrinsic noise parameters F_{min}, Rg_{opt}, Xg_{opt} and R_n of the bipolar.

The noise factor *F* is given as

$$F = F_{\min} + \frac{R_n}{G_g}[(G_{opt} - G_g)^2 + (B_{opt} - B_G)^2]$$

where

Y_g (Generator admittance)= $G_g + jB_G$

Y_{opt} (Optimum noise admittance)=$G_{opt} + jB_{opt}$

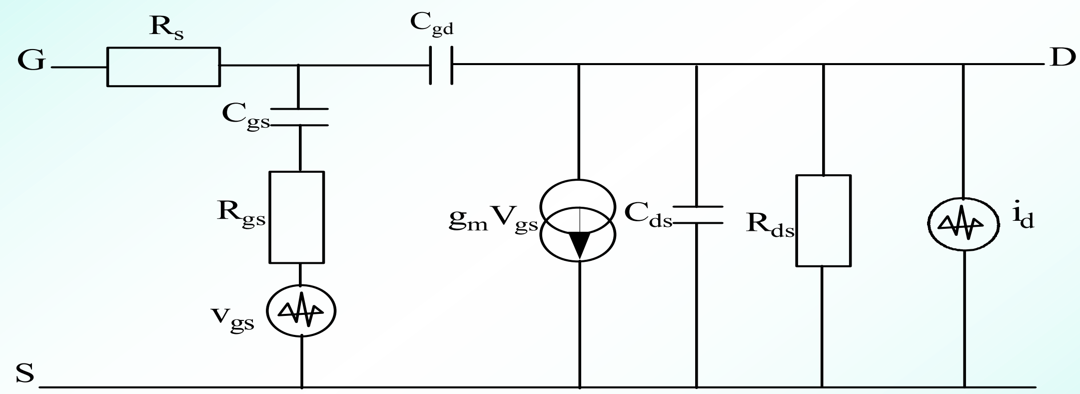
F_{min} (Minimum achievable noise figure)⇒$F = F_{min}$, when $Y_{opt} = Y_g$

R_n (Noise resistance)=Gives the sensitivity of the NF to the source admittance

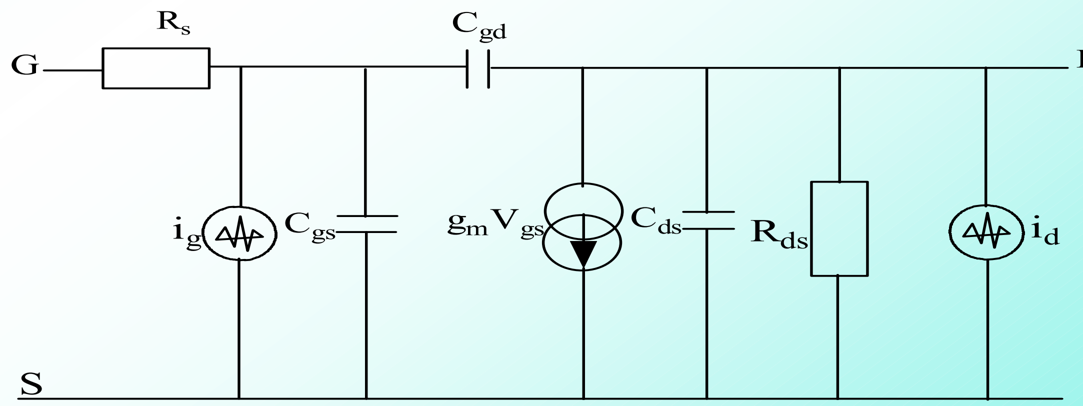
$$R_n = \frac{C_{uu^*}}{2kT} \Rightarrow r_b\left(\frac{1+\left(\frac{f}{f_b}\right)^2}{\alpha_0^2} - \frac{1}{\beta_0}\right) + \frac{r_e}{2}\left[\frac{1+\left(\frac{f}{f_b}\right)^2}{\alpha_0^2} + (g_e r_b)^2\left\{1-\alpha_0 + \left(\frac{f}{f_b}\right)^2 + \left(\frac{f}{f_e}\right)^2 + \left(\frac{1}{\beta_0} - \left(\frac{f}{f_b}\right)\left(\frac{f}{f_e}\right)\right)^2\right\}\right]$$

Noise Parameter of FET

A simplified noise model of a FET with voltage noise source at input and the current-noise source at the output



Noise model of a FET with current noise source at the input and the output



Noise Parameter of FET, Cont'd.

The mean square value of the noise sources in the narrow frequency range Δf are given by

$$\overline{i_g^2} = \frac{4kT(wC_{gs})^2 R}{g_m}\Delta f \qquad S(i_g) = \frac{\overline{i_g^2}}{\Delta f} = \left\langle \left|\overline{i_g^2}\right| \right\rangle = \frac{4kT(wC_{gs})^2 R}{g_m}$$

$$\overline{i_d^2} = 4kTg_m P\Delta f \qquad S(i_d) = \frac{\overline{i_d^2}}{\Delta f} = \left\langle \left|\overline{i_d^2}\right| \right\rangle = 4kTg_m P$$

$$\overline{i_g i_d^{\bullet}} = -jwC_{gs}4kTC\sqrt{PR}\Delta f \qquad S(i_g i_d^{\bullet}) = \left\langle \left|\overline{i_g i_d^{\bullet}}\right| \right\rangle = -jwC_{gs}4kTC\sqrt{PR}$$

Where *P*, *R* and *C* are FET noise coefficients and can be given as

$$P = \left[\frac{1}{4kTg_m}\right]\overline{i_d^2}/Hz$$

; *P* = 0.67 for JFETs and 1.2 for MESFETs

$$R = \left[\frac{g_m}{4kTw^2C_{gs}^{\;2}}\right]\overline{i_g^2}/Hz$$

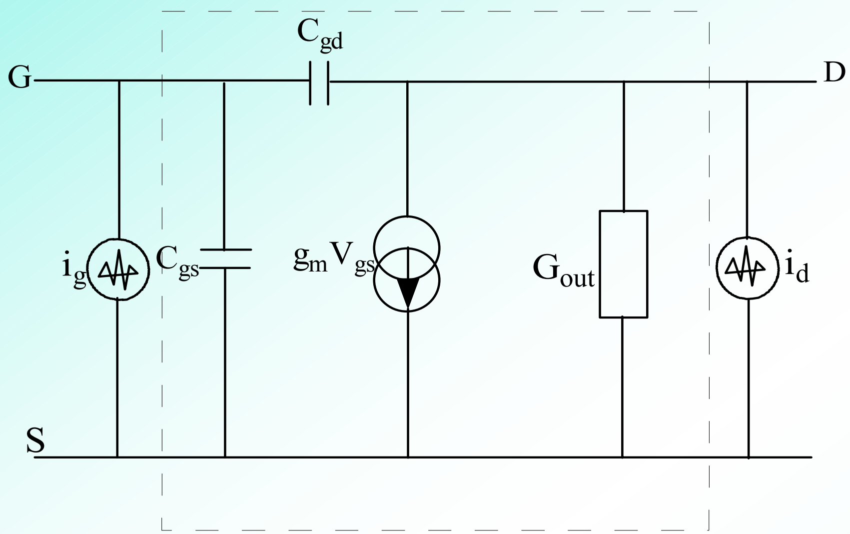
; *R* = 0.2 for JFETs and 0.4 for MESFETs

$$C = -j\left[\frac{\overline{i_g i_d^{\bullet}}}{\sqrt{[\overline{i_d^2}\,\overline{i_g^2}]}}\right]$$

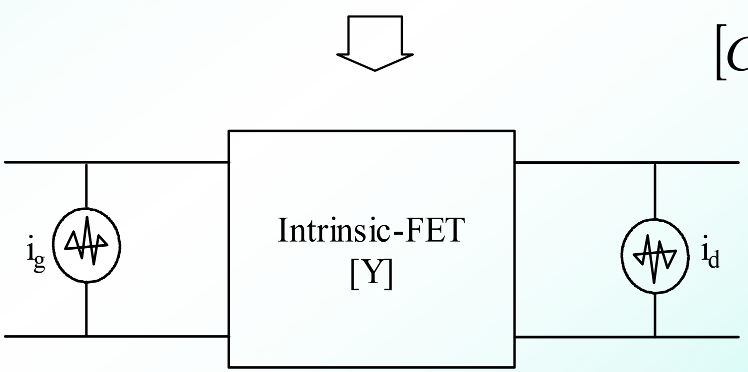
; *C* =0.4 for JFETs and 0.6-0.9 for MESFETs

Noise Parameter of FET, Cont'd.

Intrinsic FET with noise sources at input and output



Intrinsic-FET



$$[Y]_{FET-Intrinsic} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

$$[C_Y] = [N]_{noise\text{-}matrix} = \begin{bmatrix} \overline{i_g i_g^{\bullet}} & \overline{i_g i_d^{\bullet}} \\ \overline{i_d i_g^{\bullet}} & \overline{i_d i_d^{\bullet}} \end{bmatrix}$$

$$[C_Y]_{FET} = 4kT \begin{bmatrix} \dfrac{w^2 c_{gs}^2 R}{g_m} & -jwc_{gs} C\sqrt{PR} \\ jwc_{gs} C\sqrt{PR} & g_m P \end{bmatrix}$$

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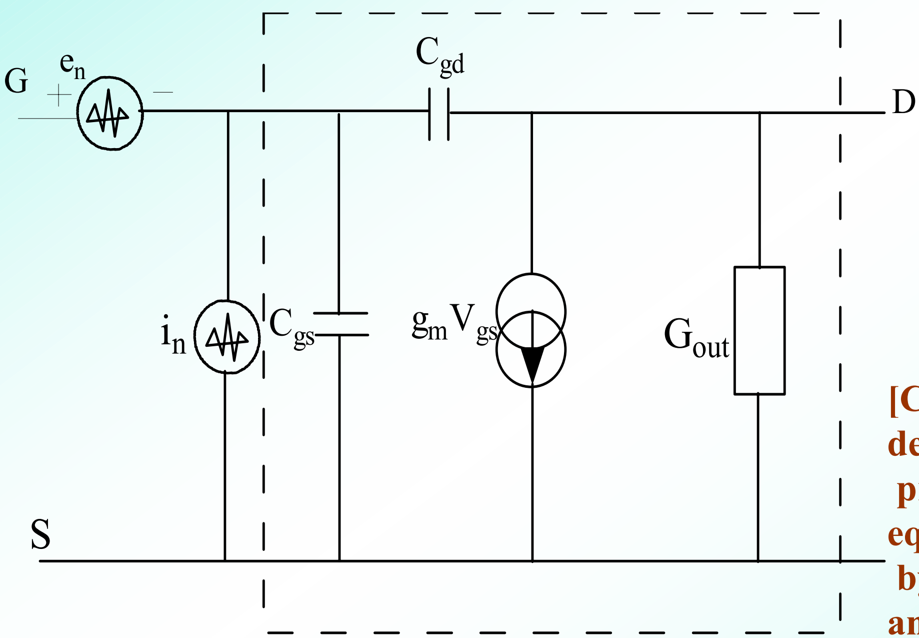
Noise Parameter of FET, Cont'd.

For simplification in analysis, noise transformation from the output to input can be done to calculate the noise parameters:

Equivalent circuit representation of FET with noise sources at input



Intrinsic-FET

$$[C_a]_{tr} = \begin{bmatrix} \overline{e_n e_n^{\bullet}} & \overline{e_n i_n^{\bullet}} \\ \overline{i_n e_n^{\bullet}} & \overline{i_n i_n^{\bullet}} \end{bmatrix}$$

$$[C_a]_{tr} = [T]\,[C_Y]_{tr}[T]^{+}$$

$[C_a]$ is the correlation matrix, which is defined as the mean value of the outer product of the noise vector that is equivalent of multiplying the noise vector by its adjoint (complex conjugate transpose) and averaging the result. [T] is transformation matrix and $[T]^{+}$ is complex conjugate transpose of [T].

Noise Parameter of FET, Cont'd.

$$[C_Y]_{FET} = 4kT\begin{bmatrix} \dfrac{w^2 c_{gs}^2 R}{g_m} & -jwc_{gs}C\sqrt{PR} \\ jwc_{gs}C\sqrt{PR} & g_m P \end{bmatrix}$$

$$[C_a]_{FET} = [T]\,[C_Y]_{tr}[T]^{+}$$

$$[C_a]_{FET} = \begin{bmatrix} 0 & \left(\dfrac{1}{sc_{gd} - g_m} + \dfrac{R_s(sc_{gd} + g_{ds} + sc_{gs} + sc_{ds})}{sc_{gd} - g_m}\right) \\ 1 & \left(\dfrac{(sc_{gd} + g_{ds} + sc_{gs} + sc_{ds})}{sc_{gd} - g_m}\right) \end{bmatrix} * 4kT\begin{bmatrix} \dfrac{w^2 c_{gs}^2 R}{g_m} & -jwc_{gs}C\sqrt{PR} \\ jwc_{gs}C\sqrt{PR} & g_m P \end{bmatrix} * K1$$

$$[C_a]_{FET} = \begin{bmatrix} \overline{e_n e_n^{\bullet}} & \overline{e_n i_n^{\bullet}} \\ \overline{i_n e_n^{\bullet}} & \overline{i_n i_n^{\bullet}} \end{bmatrix} = \begin{bmatrix} C_{uu^{\bullet}} & C_{ui^{\bullet}} \\ C_{u^{\bullet}i} & C_{ii^{\bullet}} \end{bmatrix} = 4kT\begin{bmatrix} R_n & \dfrac{F_{\min} - 1}{2} - R_n Y_{opt}^{\bullet} \\ \dfrac{F_{\min} - 1}{2} - R_n Y_{opt} & R_n\left|Y_{opt}\right|^2 \end{bmatrix}$$

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Noise Parameter of FET, Cont'd.

$$R_n = \frac{C_{uu^\bullet}}{2kT} \qquad F_{\min} = \left[1 + \frac{C_{ui^\bullet} + C_{uu^\bullet} Y_{opt}}{kT} \right]$$

$$Y_{opt} = \sqrt{\frac{C_{ii^\bullet}}{C_{uu^\bullet}} - \left[\operatorname{Im}\left(\frac{C_{ui^\bullet}}{C_{uu^\bullet}} \right) \right]^2} + j \operatorname{Im}\left(\frac{C_{ui^\bullet}}{C_{uu^\bullet}} \right)$$

$$\Gamma_{opt} = \frac{Z_{opt} - Z_o}{Z_{opt} + Z_o} \Rightarrow \frac{Y_{opt} - Y_o}{Y_{opt} + Y_o} \qquad Y_{opt} = G_{opt} + jB_{opt}$$

where

Y_g (Generator admittance)= $G_g + jB_G$

Y_{opt} (Optimum noise admittance)=$G_{opt} + jB_{opt}$

F_{min} (Minimum achievable noise figure)$\Rightarrow F = F_{min}$, when $Y_{opt} = Y_g$

R_n (Noise resistance)=Gives the sensitivity of the NF to the source admittance

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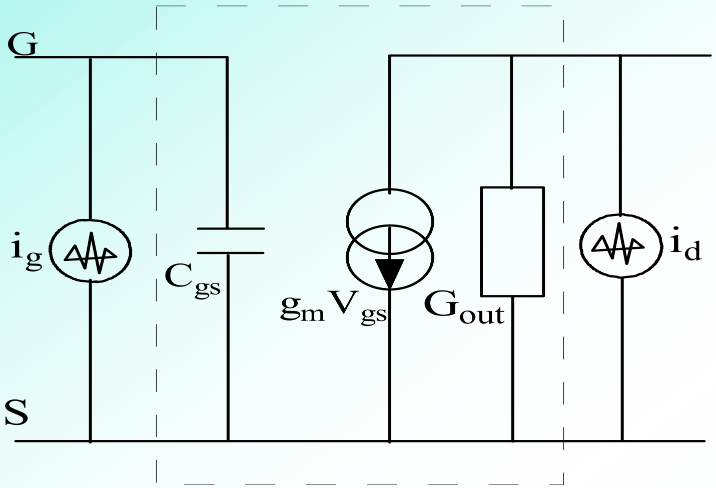
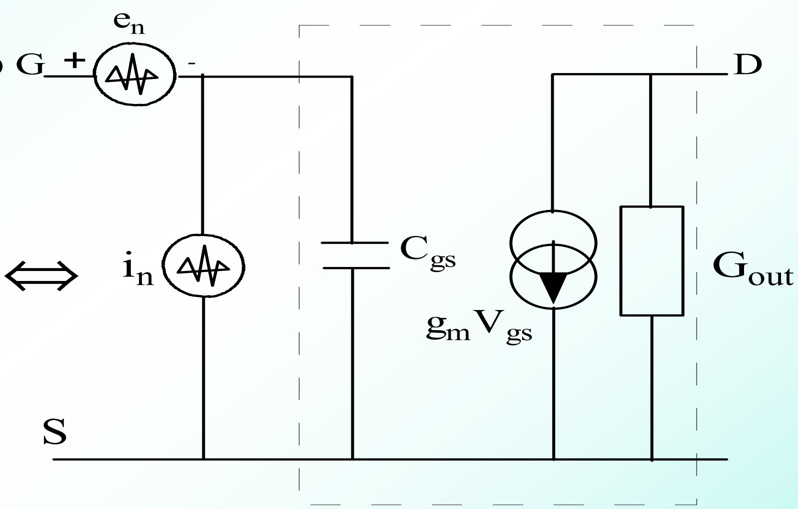
Noise Parameter of FET, Cont'd.

Neglecting the effect of gate leakage current I_{gd} and gate to drain capacitance C_{gd} the models above will be further simplified as shown below:



(a) Intrinsic-FET with current noise sources at input and output



(b) Intrinsic-FET with voltage and current noise sources at input

$$[C_a]_{FET} = \frac{4kT}{g_m}\begin{bmatrix} P & -jwc_{gs}(P+C\sqrt{PR}) \\ jwc_{gs}(P+C\sqrt{PR}) & w^2c_{gs}^2(P+R+2C\sqrt{PR}) \end{bmatrix}$$

$$[C_a]_{FET} = \begin{bmatrix} C_{uu^*} & C_{ui^*} \\ C_{u^*i} & C_{ii^*} \end{bmatrix} = 4kT\begin{bmatrix} R_n & \frac{F_{min}-1}{2} - R_n Y_{opt}^* \\ \frac{F_{min}-1}{2} - R_n Y_{opt} & R_n |Y_{opt}|^2 \end{bmatrix}$$

Noise Parameter of FET, Cont'd.

$$R_n = \frac{C_{uu^\bullet}}{4kT} = \frac{P}{g_m}$$

$$Y_{opt} = \sqrt{\frac{C_{ii^\bullet}}{C_{uu^\bullet}} - \left[\mathrm{Im}\left(\frac{C_{ui^\bullet}}{C_{uu^\bullet}}\right)\right]^2} + j\,\mathrm{Im}\left(\frac{C_{ui^\bullet}}{C_{uu^\bullet}}\right)$$

$$Y_{opt} = G_{opt} + jB_{opt}$$

$$G_{opt} = \frac{wc_{gs}}{P}\sqrt{PR\,(1 - |C|^2)}$$

$$B_{opt} = -wc_{gs}\left(1 + |C|\sqrt{\frac{R}{P}}\right)$$

$$F_{min} = 1 + \frac{C_{ui^\bullet} + C_{uu^\bullet} Y_{opt}}{kT} = 1 + \frac{2wc_{gs}}{g_m}\sqrt{PR\,(1 - |C|^2}$$

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Noise Parameter of FET, Cont'd.

Influence of C_{gd}, R_{gs} and R_s on the Noise Parameters

$$F_{\min} = 1 + 2\left[\left(\frac{w^2 c_{gs}^2}{g_m^2}\right)(R_{gs}+R_s)Pg_m + \sqrt{\frac{w^4 c_{gs}^4}{g_m^4}(R_{gs}+R_s)^2 P^2 g_m^2 + \left(\frac{w^2 c_{gs}^2}{g_m^2}\right)[PR(1-C^2) - Pg_m R_{gs}]}\right]$$

$$R_n = \left|\frac{g_m}{g_m - jwc_{gd}}\right|^2 \left(\frac{P + R - 2C_r\sqrt{RP}}{}\right) + (R_{gs} + R_s)$$

$$R_{opt} = \frac{1}{wc_{gs}}\sqrt{\frac{g_m(R_s+R_{gs}) + R(1-C_r^2)}{P} + w^2 c_{gs}^2 (R_s+R_{gs})^2}$$

$$X_{opt} = \frac{1}{wc_{gs}}(1 - C_r\sqrt{\frac{R}{P}})$$

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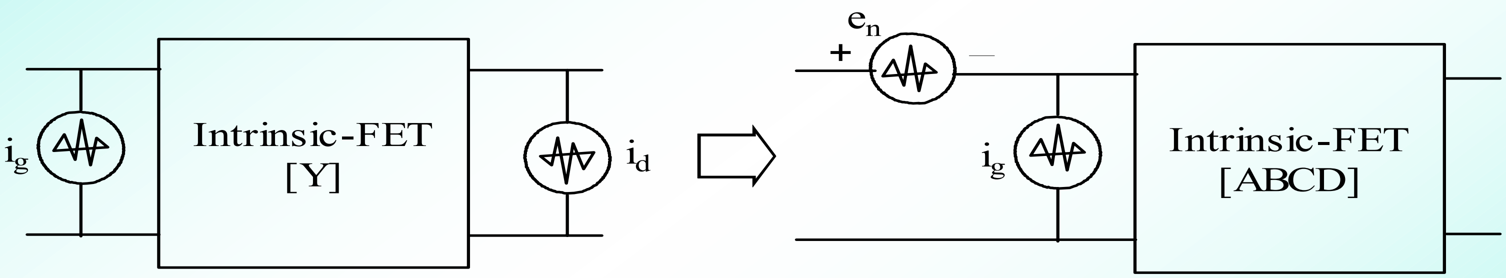
Noise Parameter of FET, Cont'd.

Temperature Dependency of the Noise Parameters of an FET:

We now introduce a minimum noise temperature T_{min} and we will modify the noise parameters previously derived. This equation now will have temperature dependence factors. Figures below show the 2-port representation of the intrinsic FET in admittance and ABCD matrix form.



The ABCD matrix representation and the corresponding noise parameters are

$$R_n = \frac{\overline{|e_n^2|}}{4kT_0\Delta f} \qquad g_n = \frac{\overline{|i_n^2|}}{4kT_0\Delta f} \qquad C_r = \frac{\overline{|e_n i_n^{\bullet}|}}{\sqrt{|e_n^2||i_n^2|}} \qquad N = R_{opt}\, g_n$$

Where k is the Boltzman's constant g_n is noise conductance, T_0 is the standard room temperature (290K), and Δf is the reference bandwidth.

Noise Parameter of FET, Cont'd.

The expression for the noise temperature T_n and a noise measure M of a two-port driven by generator impedance Z_g is expressed as

$$T_n = T_{\min} + T_0 \frac{g_n}{R_g} \left| Z_g - Z_{opt} \right|^2$$

$$T_n = T_{\min} + N T_0 \frac{\left| Z_g - Z_{opt} \right|^2}{R_g R_{opt}}$$

$$T_n = T_{\min} + 4 N T_0 \frac{\left| T_g - T_{opt} \right|^2}{(1 - \left| T_{opt} \right|^2)(1 - \left| T_g \right|^2)}$$

$$T_{opt} = \frac{Z_{opt} - Z_o}{Z_{opt} + Z_o}$$

$$M = \frac{T_n}{T_0} \left(\frac{1}{1 - \frac{1}{G_a}} \right),$$

M is defined as the minimum noise measure; this refers to the lower limit of the noise figure, it is an invariant parameter and is not affected by lossless feedback.

where Z_0 is the reference impedance and G_a is the available gain

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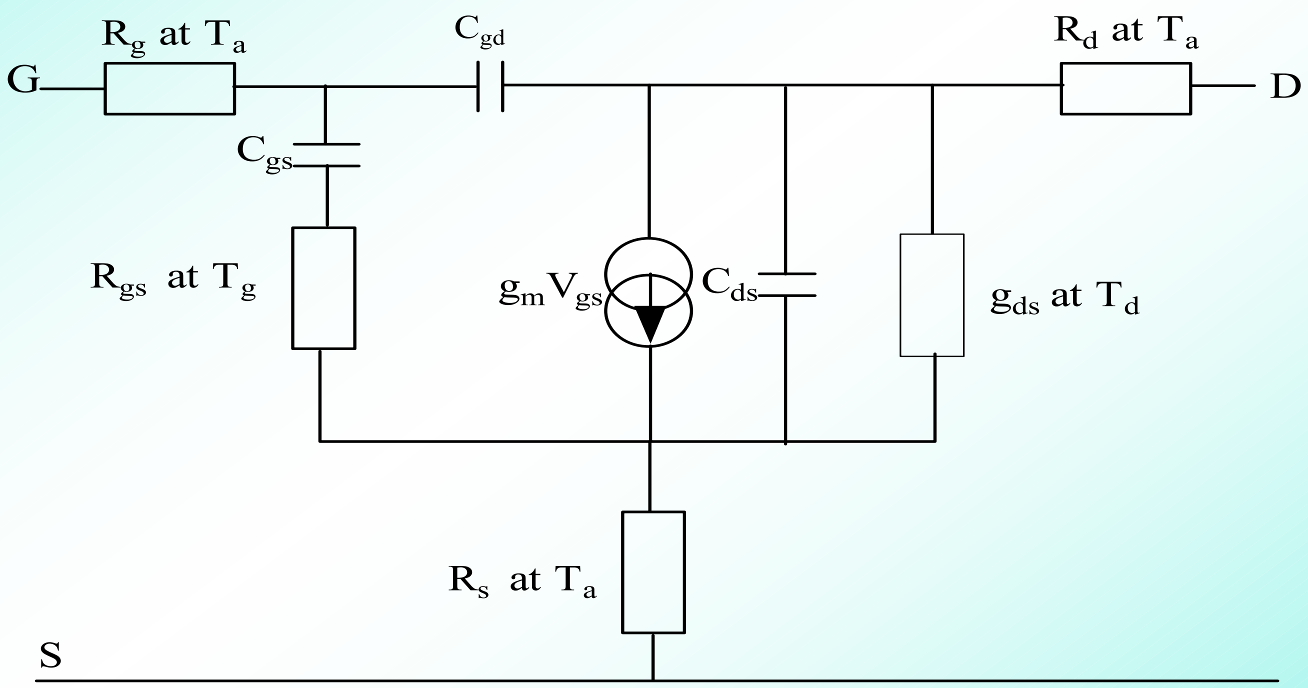
Noise Parameter of FET, Cont'd.

An extrinsic FET with parasitic resistances R_g and R_d can contribute thermal noise, and their influences can be calculated based on the ambient temperature, T_a. The noise properties of the intrinsic FET are treated by assigning an equivalent temperature T_g and T_d to R_{gs} and g_{ds}.



Noise Parameter of FET, Cont'd.

Assuming zero correlation between the noise sources represented by the equivalent temperature T_g and T_d, the modified noise parameters are expressed as:

$$T_{\min} = 2\frac{f}{f_T}\sqrt{R_{gs}g_{ds}T_gT_d + \left(\frac{f_T}{f}\right)^2 R_{gs}^2 g_{ds}^2 T_d^2} + 2\left(\frac{f_T}{f}\right)^2 R_{gs}g_{ds}T_d,$$

$$T_{\min} = (F_{\min} - 1)T_0$$

$$R_n = \frac{T_g}{T_o}R_{gs} + \frac{T_d}{T_0}\frac{g_{ds}}{g_m^2}(1 + w^2C_{gs}^2R_{gs}^2),$$

$$R_{opt} = \sqrt{\left(\frac{f_T}{f}\right)^2 \frac{R_{gs}}{R_{ds}}\frac{T_g}{T_d} + R_{gs}^2}$$

$$C_r = C\sqrt{R_n g_n} = \frac{T_d}{T_0}\frac{g_{ds}}{g_m^2}(w^2C_{gs}^2R_{gs} + jwC_{gs}),$$

$$Z_{opt} = R_{opt} + jX_{opt}$$

$$\frac{4NT_0}{T_{\min}} = \frac{2}{1 + \dfrac{R_{gs}}{R_{opt}}},$$

$$X_{opt} = \frac{1}{wC_{gs}}$$

$$f_T = \frac{g_m}{2\pi C_{gs}}$$

$$g_n = \left(\frac{f_T}{f}\right)^2 \frac{g_{ds}T_d}{T_0},$$

Noise Parameter of FET, Cont'd.

Approximation and Discussion

With some reasonable approximation, the expression of the noise parameters becomes much simpler. By introducing the following approximation, the obtained values from the calculation are typically vary less than 5% from exact one:

$$if\ \frac{f}{f_T} \le \sqrt{\frac{R_{gs}}{R_{ds}}\frac{T_g}{T_d}} \quad \textbf{and} \quad R_{opt} \ge R_{gs}\ \textbf{, then}$$

$$R_{opt} \cong \left(\frac{f_T}{f}\right)\sqrt{\frac{r_{gs}}{r_{ds}}\frac{T_g}{T_d}}, \qquad X_{opt} \cong \frac{1}{wC_{gs}}$$

$$T_{\min} \cong 2\frac{f}{f_T}\sqrt{r_{gs}g_{ds}T_gT_d}, \qquad g_n \cong \left(\frac{f_T}{f}\right)^2 \frac{g_{ds}T_d}{T_0}$$

$$f_T = \frac{g_m}{2\pi C_{gs}}, \qquad \frac{4NT_0}{T_{\min}} \cong 2$$

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Noise Parameter of FET, Cont'd.

Example : The temperature-dependent noise parameters for intrinsic FET are now calculated for two cases (room temperature), with the following intrinsic parameters are assumed:

R_{gs} = 2.5 Ohm	**C_{ds}= 0.067pF**
R_d =0 Ohm (absorbed in matching load)	**C_{gd}= 0.042pF**
r_{ds} = 400 Ohm	**g_m= 57mS**
C_{gs}= 0.28Pf	**f = 8.5GHz,**

1. Assume T_a =297 ^{0}K, T_g =304 ^{0}K, T_d =5514 ^{0}K, V_{ds}=2V, Ids=10mA.

$$f_T = \frac{g_m}{2\pi C_{gs}} = 32.39 GHz \qquad R_{opt} = \sqrt{\left(\frac{f_T}{f}\right)^2 \frac{r_{gs}}{g_{ds}} \frac{T_g}{T_d} + r_{gs}^2} = 28.42\Omega \qquad X_{opt} = \frac{1}{wC_{gs}} = 66.91\Omega$$

$$T_{\min} = 2\frac{f}{f_T}\sqrt{r_{gs} g_{ds} T_g T_d + \left(\frac{f_T}{f}\right)^2 r_{gs}^2 g_{ds}^2 T_d^2} + 2\left(\frac{f_T}{f}\right)^2 r_{gs} g_{ds} T_d = 58.74\ K$$

$$R_n = \frac{T_g r_{gs}}{T_0} + \frac{g_{ds} T_d}{T_0 g_m^2}(1 + w^2 r_{gs}^2 c_{gs}^2) = 17.27\ \Omega \qquad g_n = \left(\frac{f_T}{f}\right)^2 \frac{g_{ds} T_d}{T_0} = 3.27\ mS$$

$$F_{\min} = \frac{T_{\min}}{T_0} + 1 = \frac{58.7}{290} + 1 = 1.59\ dB$$

Noise Parameter of FET, Cont'd.

2. Assume $T_a = 12.5K$, $T_g = 14.5K$, $T_d = 1406K$, $V_{ds} = 2V$, Ids=5mA.(Cooled down to 14.5°K!)

$$f_T = \frac{g_m}{2\pi C_{gs}} = 32.39GHz \qquad F_{\min} = \frac{T_{\min}}{T_0} + 1 = \frac{7.4}{290} + 1 = 0.21dB$$

$$g_n = \left(\frac{f_T}{f}\right)^2 \frac{g_{ds}T_d}{T_0} = 0.87mS \qquad R_n = \frac{T_g r_{gs}}{T_0} + \frac{g_{ds}T_d}{T_0 g_m^2}(1 + w^2 r_{gs}^2 c_{gs}^2) = 3.86\Omega$$

$$T_{\min} = 2\frac{f}{f_T}\sqrt{r_{gs}g_{ds}T_gT_d + \left(\frac{f_T}{f}\right)^2 r_{gs}^2 g_{ds}^2 T_d^2} + 2\left(\frac{f_T}{f}\right)^2 r_{gs}g_{ds}T_d = 7.4K$$

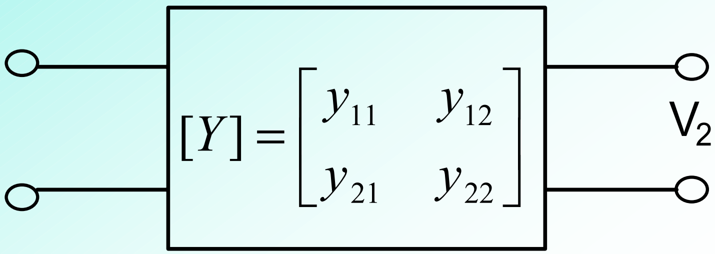
$$R_{opt} = \sqrt{\left(\frac{f_T}{f}\right)^2 \frac{r_{gs}}{g_{ds}}\frac{T_g}{T_d} + r_{gs}^2} = 12.34\Omega \qquad X_{opt} = \frac{1}{wC_{gs}} = 66.9\Omega$$

One final point for noise data is the inequalities: $1 \le \frac{4NT_0}{T_{\min}} < 2$

where the first equality occurs if the noise sources (at the input) are fully correlated, and the second inequality occurs if the noise sources are totally uncorrelated. This is a Valuable check on the data (or model) to insure the numbers are physically possible

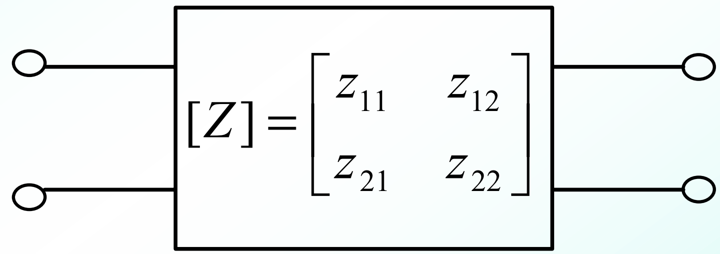
Stability Analysis of 2-Port Network

2-Port system (Y, Z, S, h parameters) :



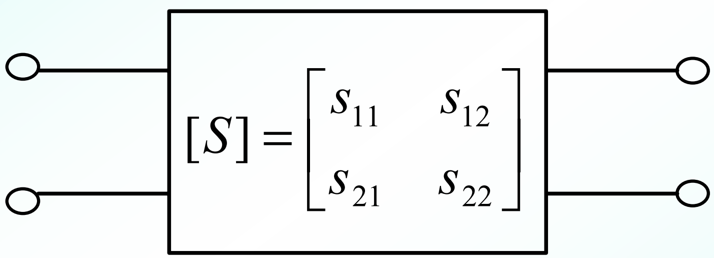
$$[Y] = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

Y-Parameter



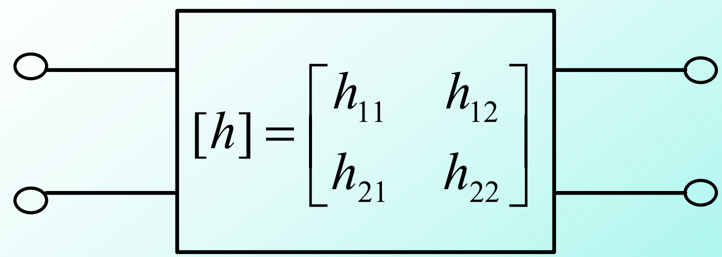
$$[Z] = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

Z-Parameter



$$[S] = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$$

S-Parameter



$$[h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

Hybrid-Parameter

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Stability Analysis of 2-Port Network, Cont'd.

Transformation of 2-port parameters (Y⇔S) :

S-Y Parameter

$$s_{11} = \frac{(1 - y_{11})(1 + y_{22}) + y_{12}y_{21}}{(1 + y_{11})(1 + y_{22}) - y_{12}y_{21}}$$

$$s_{12} = \frac{-2y_{12}}{(1 + y_{11})(1 + y_{22}) - y_{12}y_{21}}$$

$$s_{21} = \frac{-2y_{21}}{(1 + y_{11})(1 + y_{22}) - y_{12}y_{21}}$$

$$s_{22} = \frac{(1 + y_{11})(1 - y_{22}) - y_{12}y_{21}}{(1 + y_{11})(1 + y_{22}) - y_{12}y_{21}}$$

Y-S Parameter

$$y_{11} = \frac{(1 - s_{11})(1 + s_{22}) + s_{12}s_{21}}{(1 + s_{11})(1 + s_{22}) - s_{12}s_{21}}$$

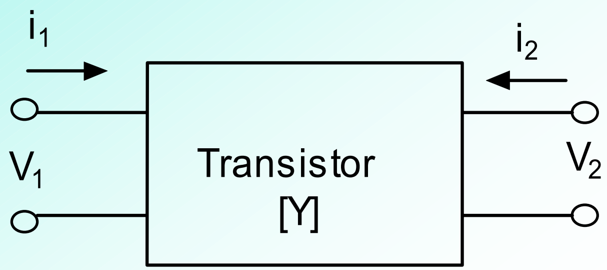
$$y_{12} = \frac{-2s_{12}}{(1 + s_{11})(1 + s_{22}) - s_{12}s_{21}}$$

$$y_{21} = \frac{-2s_{21}}{(1 + s_{11})(1 + s_{22}) - s_{12}s_{21}}$$

$$y_{22} = \frac{(1 + s_{11})(1 - s_{22}) - s_{12}s_{21}}{(1 + s_{11})(1 + s_{22}) - s_{12}s_{21}}$$

Stability Analysis of 2-Port Network, Cont'd.

Feedback from output to input through the internal capacitive coupling of a device is referred to as Miller effect. The Y-parameter of the 2-port network can be described by



$$\begin{bmatrix} i_1 \\ i \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$i_1 = y_{11}V_1 + y_{12}V_2$$
$$i_2 = y_{21}V_1 + y_{22}V_2$$

The stability factor of the transistor can be calculated from feedback (y_{12}) . The input and out put admittance is given by

$$Y_{in} = y_{11} - \frac{y_{12}y_{21}}{y_{22} + Y_L} \qquad Y_{out} = y_{22} - \frac{y_{12}y_{21}}{y_{11} + Y_s}$$

where y_L is load admittance and y_s is source admittance. If y_{12} is increased from a very small value, the input admittance can become zero, or the real part negative. If a tuned circuit were connected in parallel with y_{11} or Y_L were a tuned circuit, the system would become unstable and oscillate.

Stability Analysis of 2-Port Network, Cont'd.

(1) The system of two-port (amplifier) is unconditionally stable if the port admittances greater than zero for all passive load and source impedances :

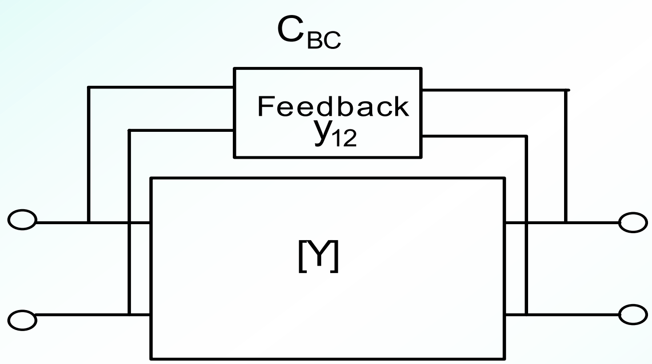
$$\mathrm{Re}[y_{11}] > 0$$
$$\mathrm{Re}[y_{22}] > 0$$

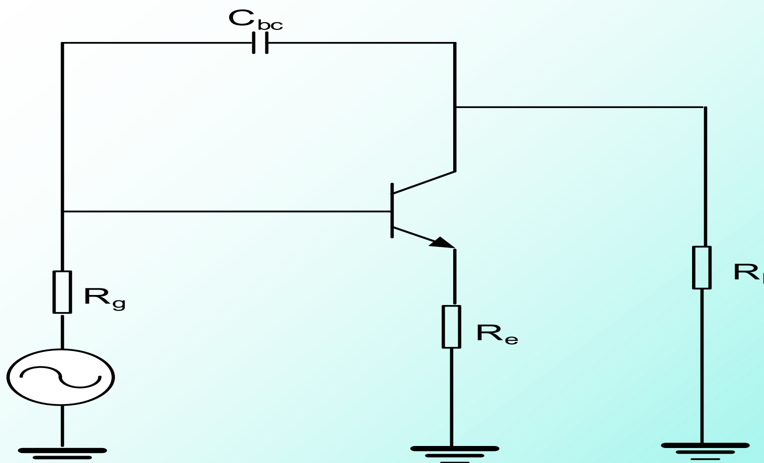
(2) The system of two-port (amplifier) is potentially unstable if port admittances less than or equal to zero for all passive load and source impedances; that is some passive load and source terminations can produce input and output impedances having a negative real part.

$$\mathrm{Re}[y_{11}] =< 0$$
$$\mathrm{Re}[y_{22}] =< 0$$

Stability Analysis of 2-Port Network, Cont'd.

The Figure shows the typical schematic diagram of common-emitter amplifier stage. Maximum gain is obtained if the collector impedance is raised to the maximum level at which the amplifier remains sable since the voltage gain is G_v=-$y_{21}R_L$. In this type of stage there is a polarity inversion between input and output. The current gain β decreases the by 3dB at the *β* cutoff frequency f_β. This in turn, reduces the input impedance and decreases the stage gain as the frequency increases further. In addition the collector-base feedback capacitance C_{CB} can further reduce the input impedance and can ultimately cause instability. The increase of input capacitance because of the voltage gain and feedback capacitance is called the Miller effect. The Miller effect limits the bandwidth of the amplifier.





If y_{12} is increased from a very small value, the input admittance can become zero, or the real part negative.

Noise in Oscillators

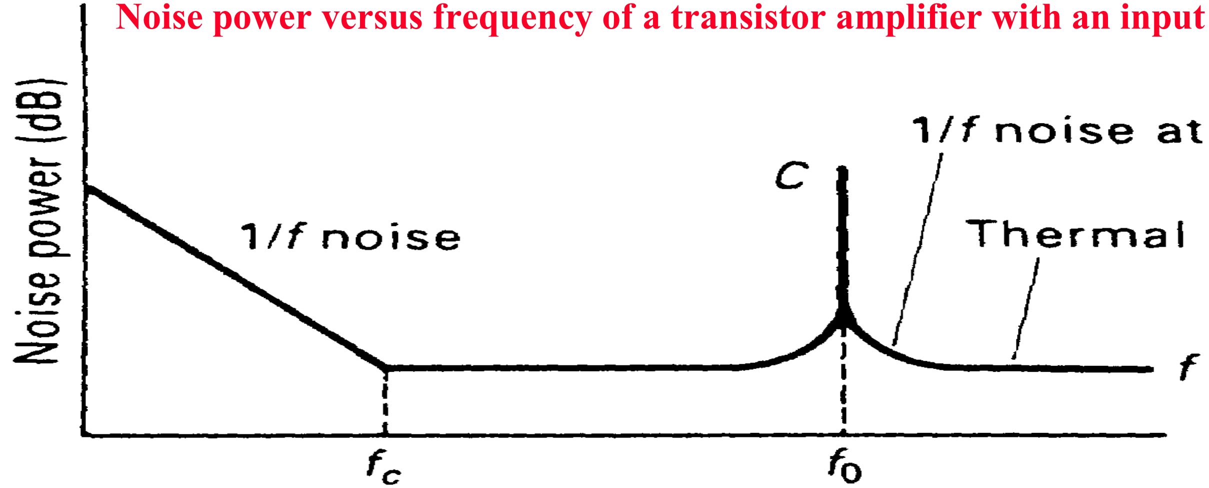
Linear Approach to the Calculation of Oscillator Phase Noise :

Since an oscillator can be viewed as an amplifier with feedback, it is helpful to examine the phase noise added to an amplifier that has a noise factor *F*. With *F* defined as

$$F = \frac{(S/N)_{in}}{(S/N)_{out}} = \frac{N_{out}}{N_{in}G} = \frac{N_{out}}{GkTB}, \qquad N_{out} = FGkTB, \qquad N_{in} = kTB$$

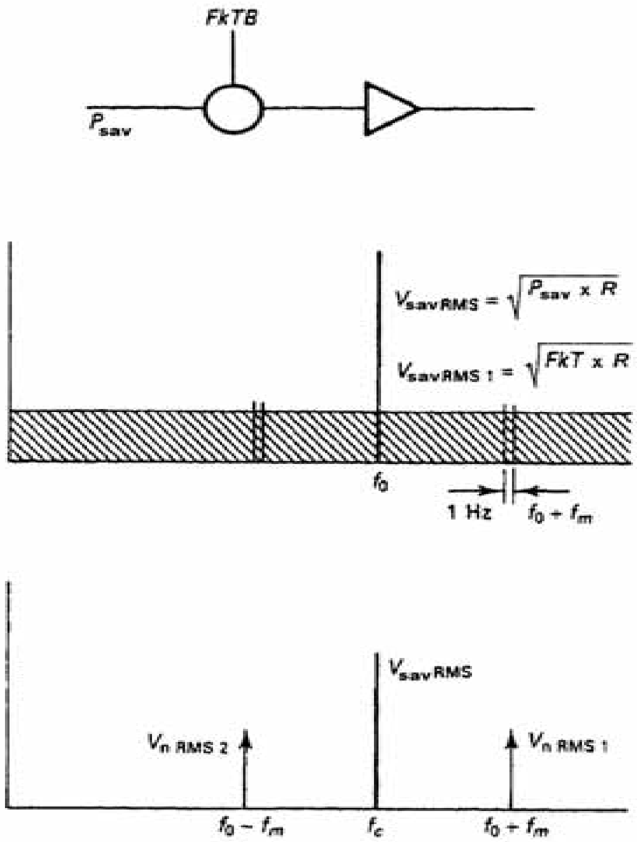
where N_{in} is the total input noise power to a noise-free amplifier.

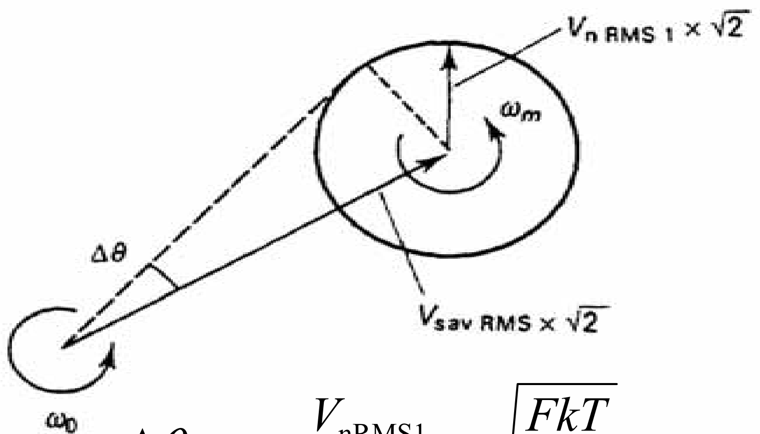
Noise power versus frequency of a transistor amplifier with an input signal applied



Noise in Oscillators, Cont'd.

The input phase noise in a 1 Hz bandwidth at any frequency $f_0 + f_m$ from the carrier produces a phase deviation as





$$\Delta\theta_{1\text{peak}} = \frac{V_{\text{nRMS1}}}{V_{\text{savRMS}}} = \sqrt{\frac{FkT}{P_{\text{sav}}}}$$

$$\Delta\theta_{\text{RMS}} = \frac{1}{\sqrt{2}}\sqrt{\frac{FkT}{P_{\text{sav}}}}$$

Phase noise added to the carrier

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Noise in Oscillators, Cont'd.

Since a correlated random phase noise relation exists at $f_0 - f_m$, the total phase deviation becomes

$$\Delta\theta_{\text{RMStotal}} = \sqrt{FkT / P_{sav}} \qquad (SSB)$$

The spectral density of phase noise becomes

$$S_\theta(f_m) = \Delta\theta^2_{\text{RMS}} = FkTB / P_{\text{sav}}$$

where B = 1 for a 1 Hz bandwidth.

Using $kTB = -174\,\text{dBm} \qquad (\text{B} = 1\text{Hz}, \quad \text{T} = 300\text{K})$

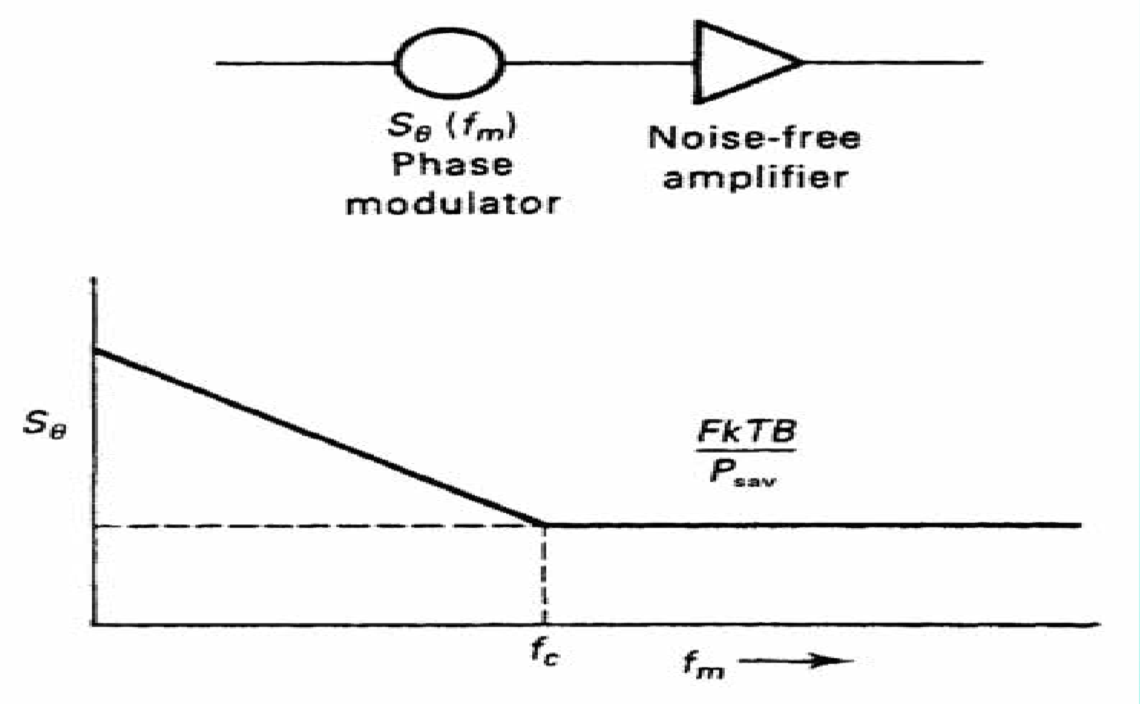
allows a calculation of the spectral density of phase noise that is far away from the carrier (that is, at large values of f_m). This noise is the theoretical noise floor of the amplifier. For example, an amplifier with +10 dBm power at the input and a noise figure of 6 dB gives

$$S_\theta(f_m > f_c) = -174\,\text{dBm} + 6\,\text{dB} - 10\,\text{dBm} = -178\,\text{dBm}$$

Only if P_{out} is > 0 dBm can we expect (signal-to-noise ratio) to be greater than 174 dBc/Hz (1 Hz bandwidth.)

Noise in Oscillators, Cont'd.

For a modulation frequency close to the carrier, $S_\theta(f_m)$ shows a flicker or $1/f$ component, which is empirically described by the corner frequency f_c. The phase noise can be modeled by a noise-free amplifier and a phase modulator at the input

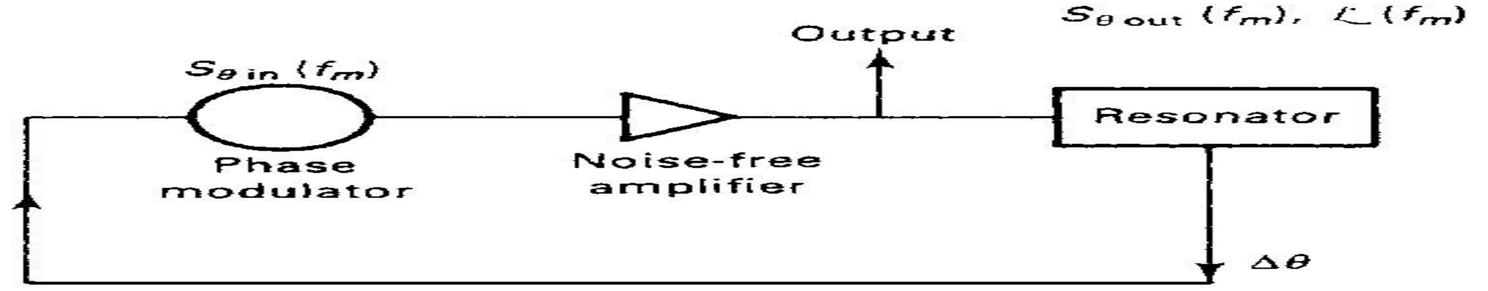


Noise in Oscillators, Cont'd.

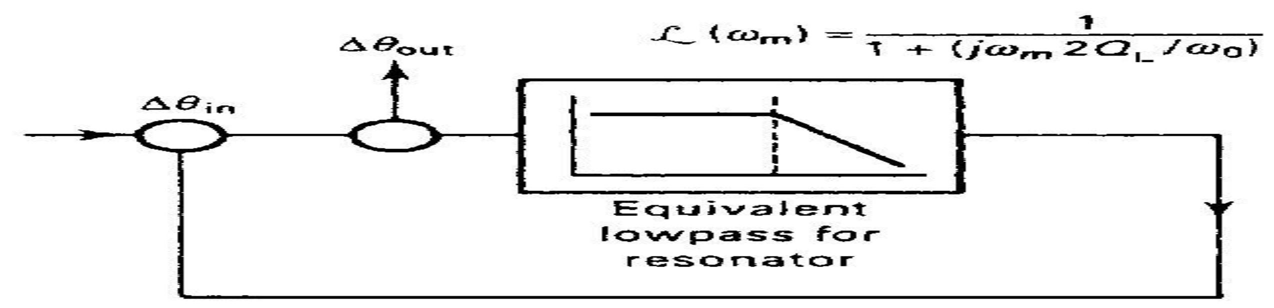
The purity of the signal is degraded by the flicker noise at frequencies close to the carrier. The phase noise can be described by

$$S_\theta(f_m) = \frac{FkTB}{P_{\text{sav}}}\left(1 + \frac{f_c}{f_m}\right) \qquad (B = 1)$$

Equivalent feedback models of oscillator phase noise



$$\omega_0 / 2Q_L = 2\pi\, B/2$$



Noise in Oscillators, Cont'd.

The closed loop response of the phase feedback loop is given by

$$\Delta\theta_{\text{out}}(f_m) = \left(1 + \frac{\omega_0}{j2Q_L\omega_m}\right)\Delta\theta_{\text{in}}(f_m)$$

The power transfer becomes the phase spectral density

$$S_{\theta\,\text{out}}(f_m) = \left[1 + \frac{1}{f_m^2}\left(\frac{f_0}{2Q_L}\right)^2\right]S_{\theta\,\text{in}}(f_m)$$

The equivalent expression of the single sideband (SSB) phase noise can be described by

$$£(\omega_m)_{SSB} = \frac{1}{2}\left[1 + \frac{1}{f_m^2}\left(\frac{f_0}{2Q_L}\right)^2\right]S_{\theta\,\text{in}}(f_m)$$

This equation describes the phase noise at the output of the amplifier (flicker corner frequency and AM-to-PM conversion are not considered). The phase perturbation $S_{\theta\text{in}}$ at the input of the amplifier is enhanced by the positive phase feedback within the half bandwidth of the resonator, $f_0/2Q_L$.

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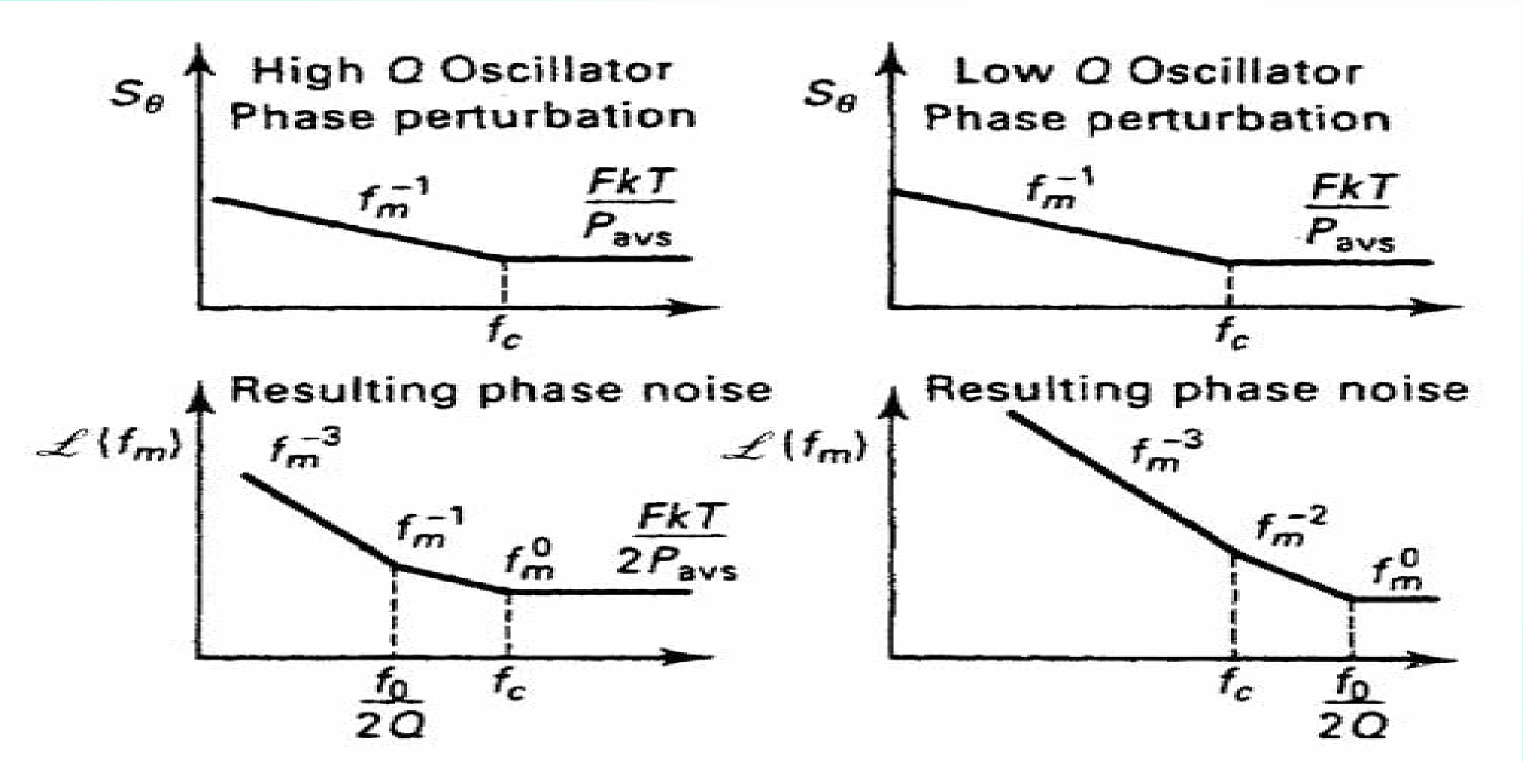
Noise in Oscillators, Cont'd.

Depending on the relation between f_c and $f_0/2Q_L$, there are two cases of interest. For the low Q case, the spectral phase noise is unaffected by the Q of the resonator, but the £ (f_m) spectral density will show a $1/f^3$ and $1/f^2$ dependence close to the carrier.

Case1 **Case2**



Equivalent feedback models of oscillator phase noise

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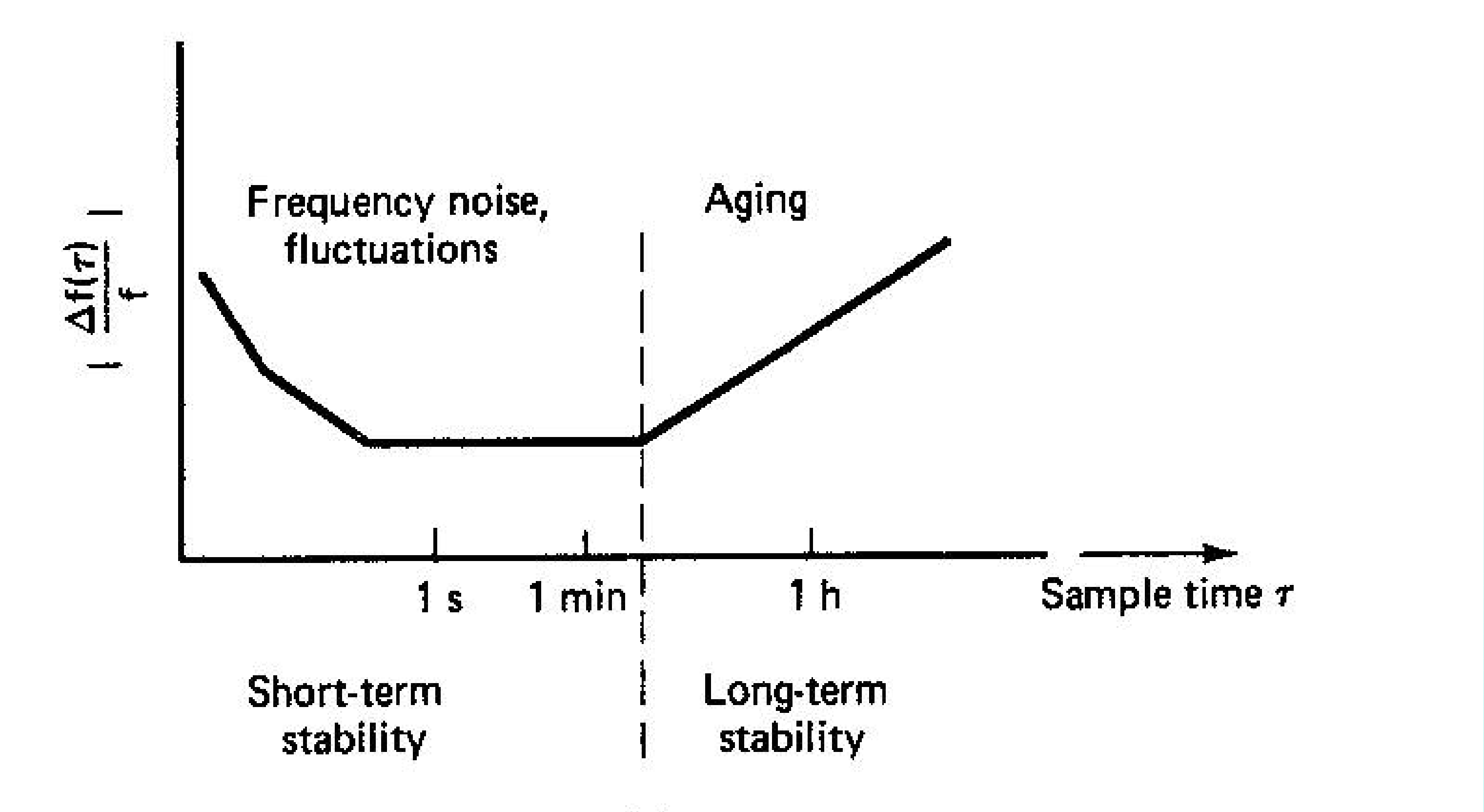
Noise in Oscillators, Cont'd.

Characterization of a noise sideband in the time and frequency domain and its contributions: time domain



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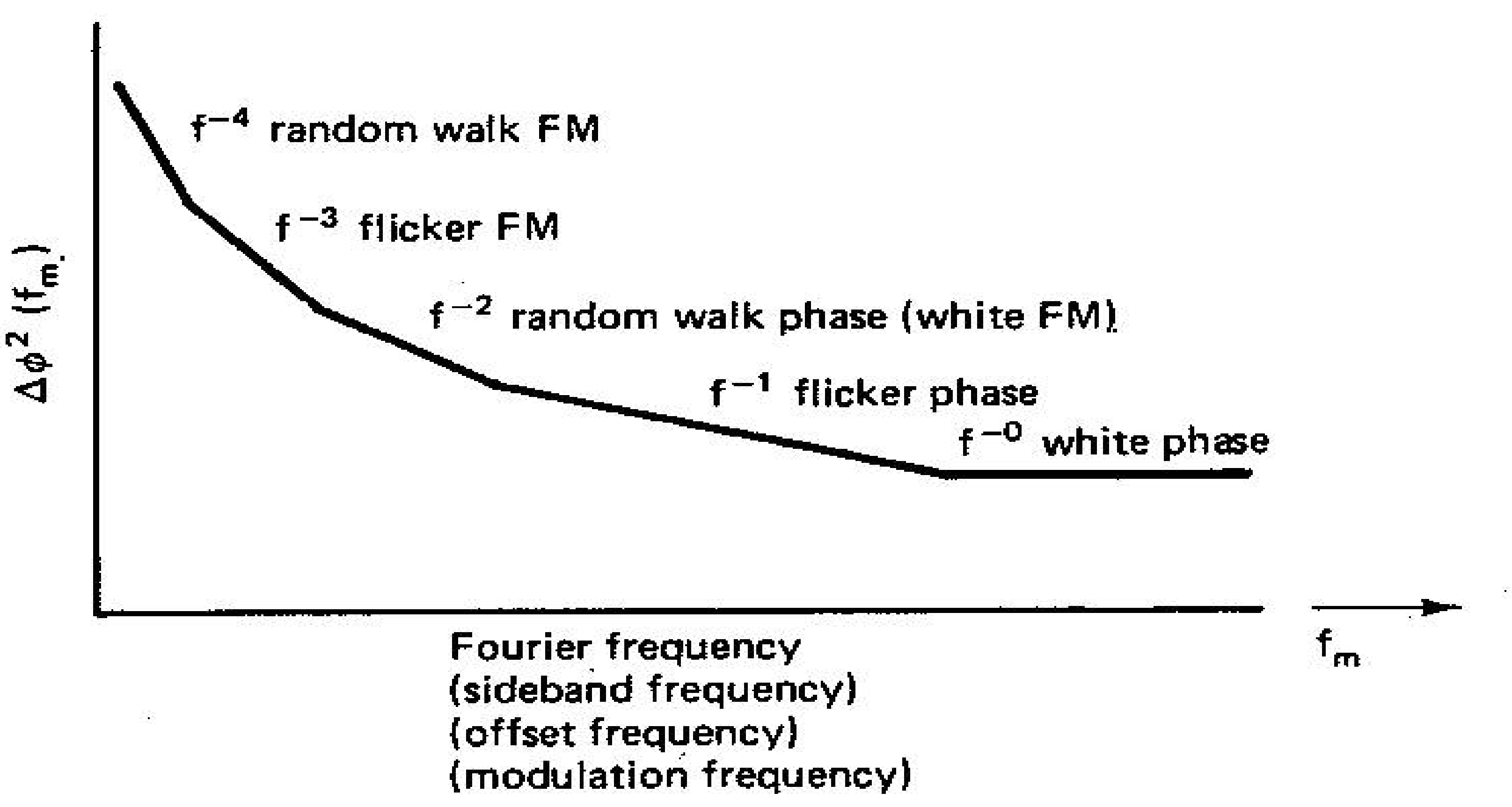
Noise in Oscillators, Cont'd.

Characterization of a noise sideband in the time and frequency domain and its contributions: frequency domain.



Noise in Oscillators, Cont'd.

Leeson phase noise model:

$$\pounds(f_m)=\frac{1}{2}\left[1+\frac{1}{f_m^{\ 2}}\left(\frac{f}{2Q_L}\right)^2\frac{FkT}{P_{sav}}\left(1+\frac{f_c}{f_m}\right)\right]==\frac{FkT}{2P_{sav}}\left[\frac{1}{f_m^{\ 3}}\frac{f^2 f_c}{4Q_L^2}+\frac{1}{f_m^{\ 2}}\left(\frac{f}{2Q_L}\right)^2+\left(1+\frac{f_c}{f_m}\right)\right]\quad dBc/Hz$$

The above Equation gives the four major causes of oscillator noise: the up-converted 1/*f* Noise or flicker FM noise, the thermal FM noise, the flicker phase noise, and the thermal noise floor, respectively.

$$\mathscr{L}(f_m)=\frac{1}{2}\left[1+\frac{\omega_o^2}{4\omega_m^2}\left(\frac{P_{\text{in}}}{\omega_o W_e}+\frac{1}{Q_{\text{unl}}}+\frac{P_{\text{sig}}}{\omega_o W_e}\right)^2\right]\left(1+\frac{\omega_c}{\omega_m}\right)\frac{FkT_o}{P_{\text{sav}}}$$

input power over reactive power

resonator Q

signal power over reactive power

flicker effect

phase perturbation

$$W_e=\tfrac{1}{2}CV^2$$

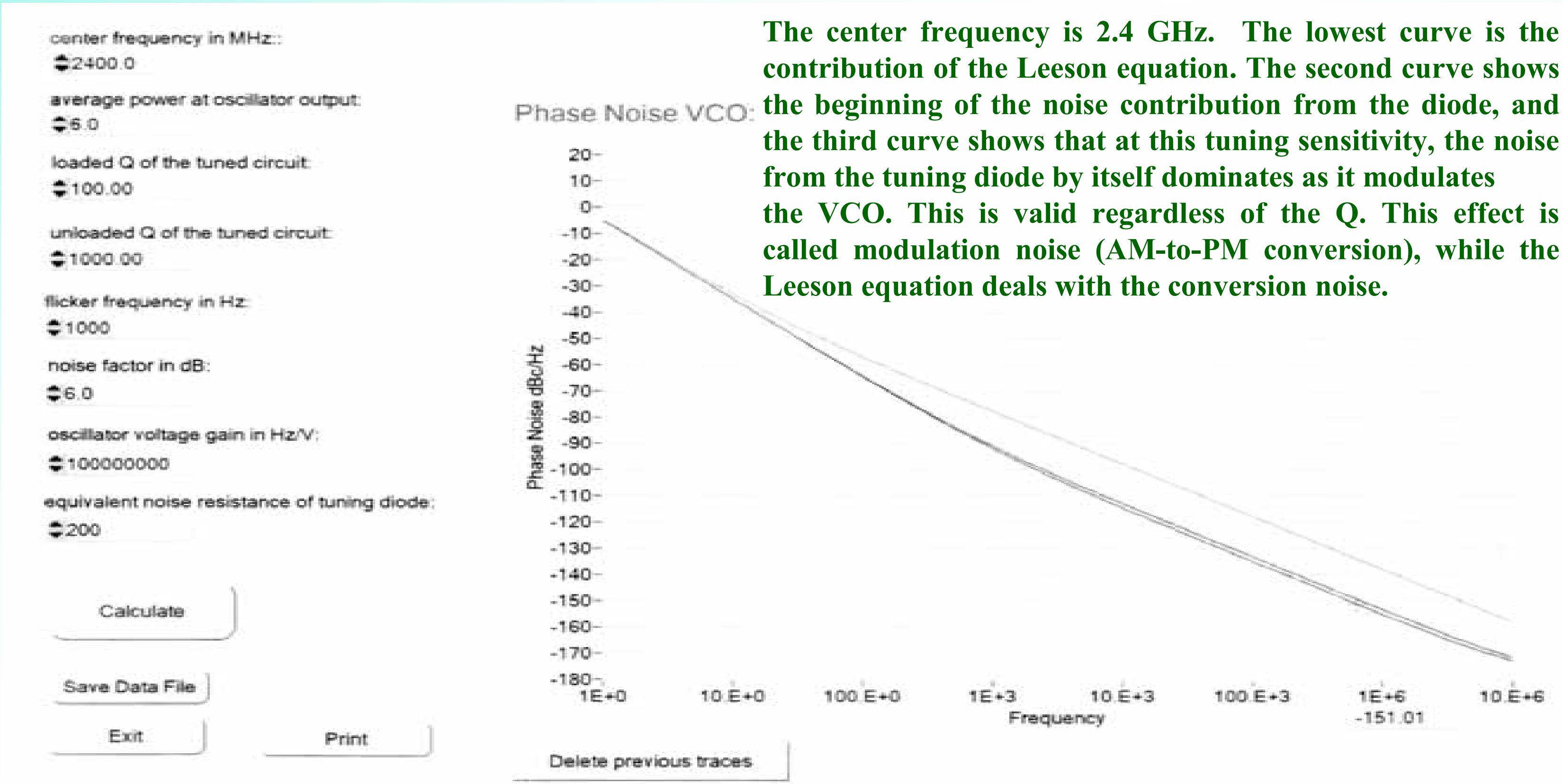
$$P_{\text{res}}=\frac{\omega_o W_e}{Q_{\text{unl}}}$$

$$Q_L=\frac{\omega_o W_e}{P_{\text{diss,total}}}=\frac{\omega_o W_e}{P_{\text{in}}+P_{\text{res}}+P_{\text{sig}}}=\frac{\text{reactive power}}{\text{total dissipated power}}$$

Noise in Oscillators, Cont'd.

Comments on the Leeson phase noise formulae:

The practical oscillator will experience a frequency shift due to the voltage and current dependent capacitances of the transistor and tuning diode.



The center frequency is 2.4 GHz. The lowest curve is the contribution of the Leeson equation. The second curve shows the beginning of the noise contribution from the diode, and the third curve shows that at this tuning sensitivity, the noise from the tuning diode by itself dominates as it modulates the VCO. This is valid regardless of the Q. This effect is called modulation noise (AM-to-PM conversion), while the Leeson equation deals with the conversion noise.

Noise in Oscillators, Cont'd.

Modified Leeson phase noise formulae (Rohde added the tuning diode noise contribution):

$$\pounds(f_m) = 10\log\left\{\left[1+\frac{f_0^{\,2}}{(2f_mQ_L)^2}\right]\left(1+\frac{f_c}{f_m}\right)\frac{FkT}{2P_{sav}}+\frac{2kTRK_0^{\,2}}{f_m^{\,2}}\right\}$$

where

$L\ (f_m)$ = ratio of sideband power in a 1 Hz bandwidth at f_m to total power in dB

f_m = frequency offset

f_0 = center frequency

f_c = flicker frequency

Q_L = loaded Q of the tuned circuit

F = noise factor

$kT = 4.1 \times 10^{-21}$ at 300 $^{K}{}_{0}$ (room temperature)

P_{sav} = average power at oscillator output

R = equivalent noise resistance of tuning diode (typically 50 Ω - 10 kΩ)

$^{K}{}_{o}$ = oscillator voltage gain

Noise in Oscillators, Cont'd.

Non-Linear Approach to the Calculation of Oscillator Phase Noise :

Noise Generation in Oscillators :

1. Noise components at low frequency deviations result in frequency modulation of the carrier through mean square frequency fluctuation proportional to the available noise power.

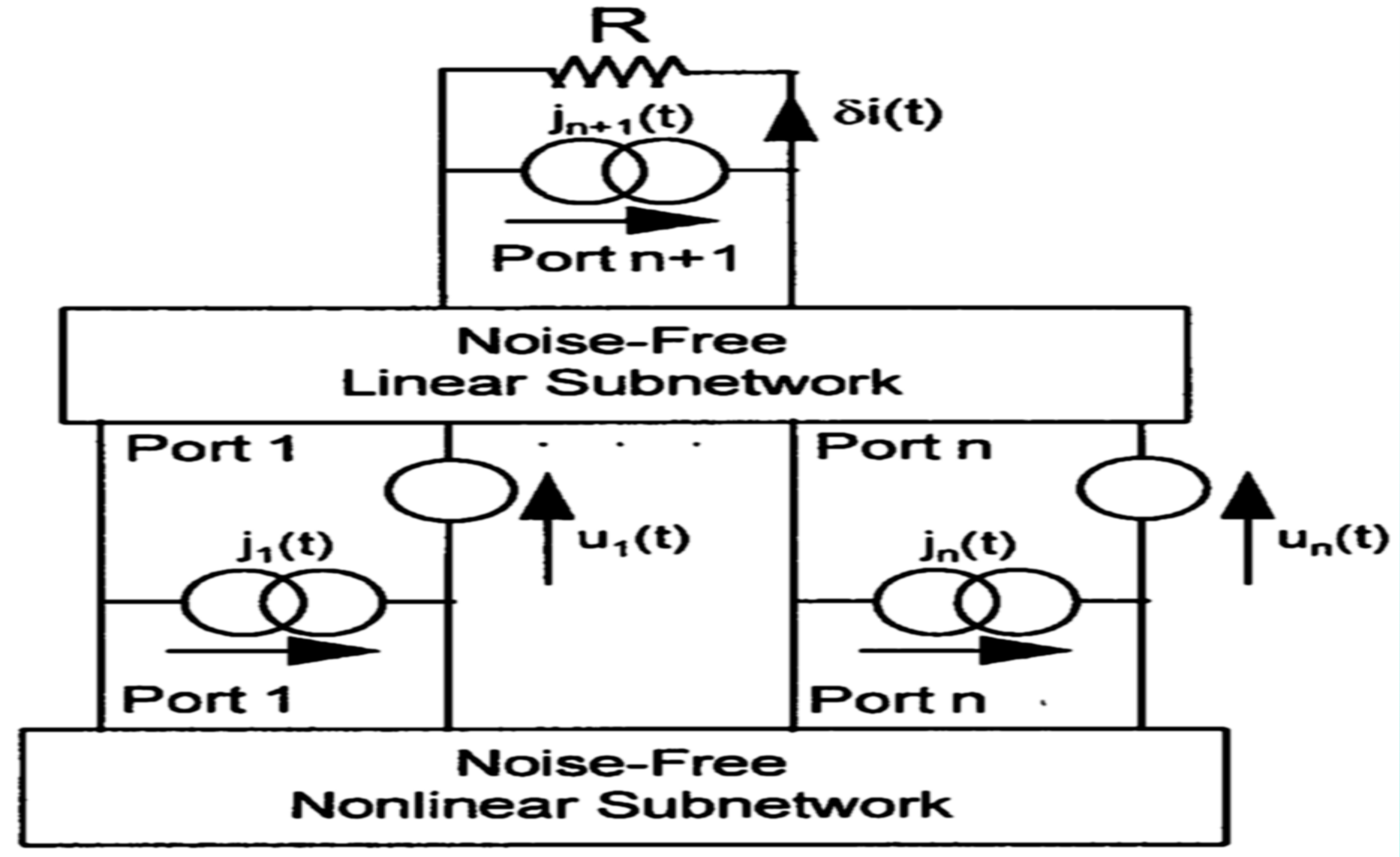
2. Noise components at high frequency deviations result in phase modulation of the carrier through mean square phase fluctuation proportional to the available noise power.

Equivalent Representation of a Noisy Nonlinear Circuit:

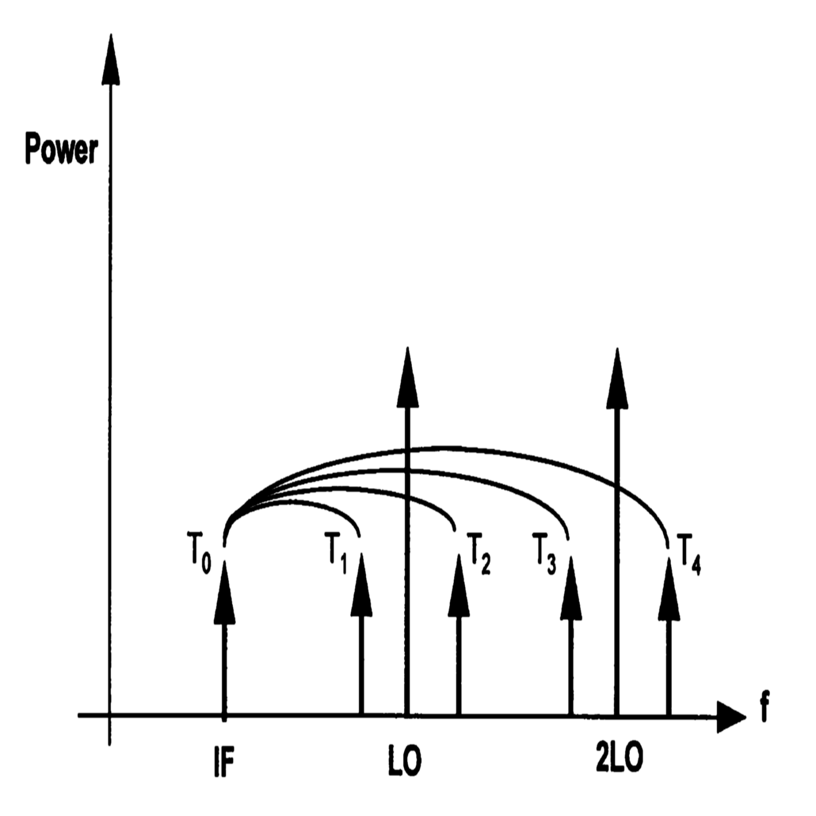
A general noisy nonlinear network can be described by dividing nonlinear circuit into linear and nonlinear sub-networks as noise-free multi n-ports.

Noise in Oscillators, Cont'd.

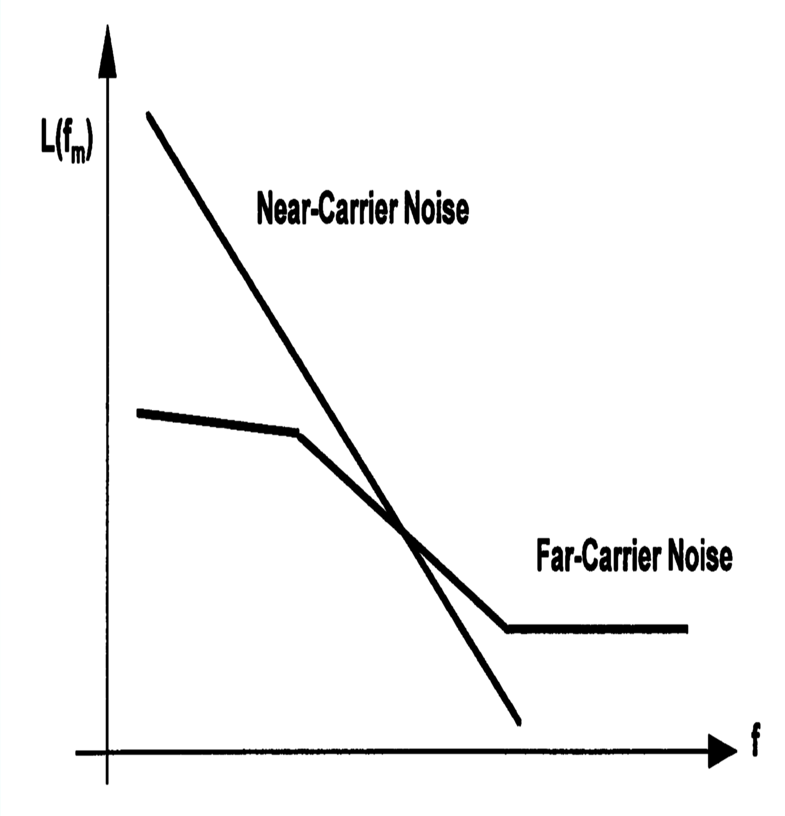
Equivalent circuit of a general noisy nonlinear network



Noise in Oscillators, Cont'd.



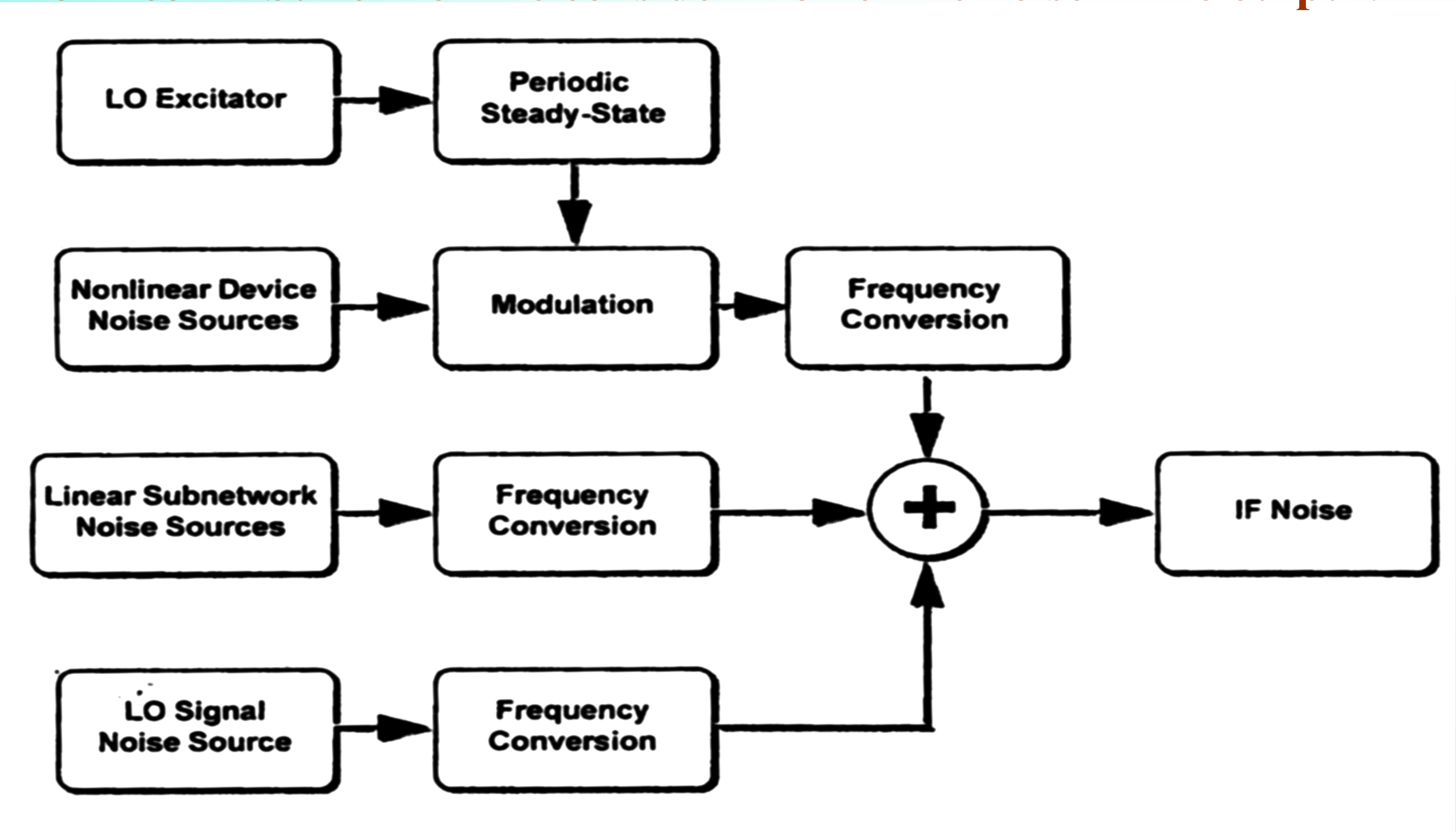
Noise sources where the noise at each sideband contributes to the output noise at the IF through conversion.



Oscillator noise components

Noise in Oscillators, Cont'd.

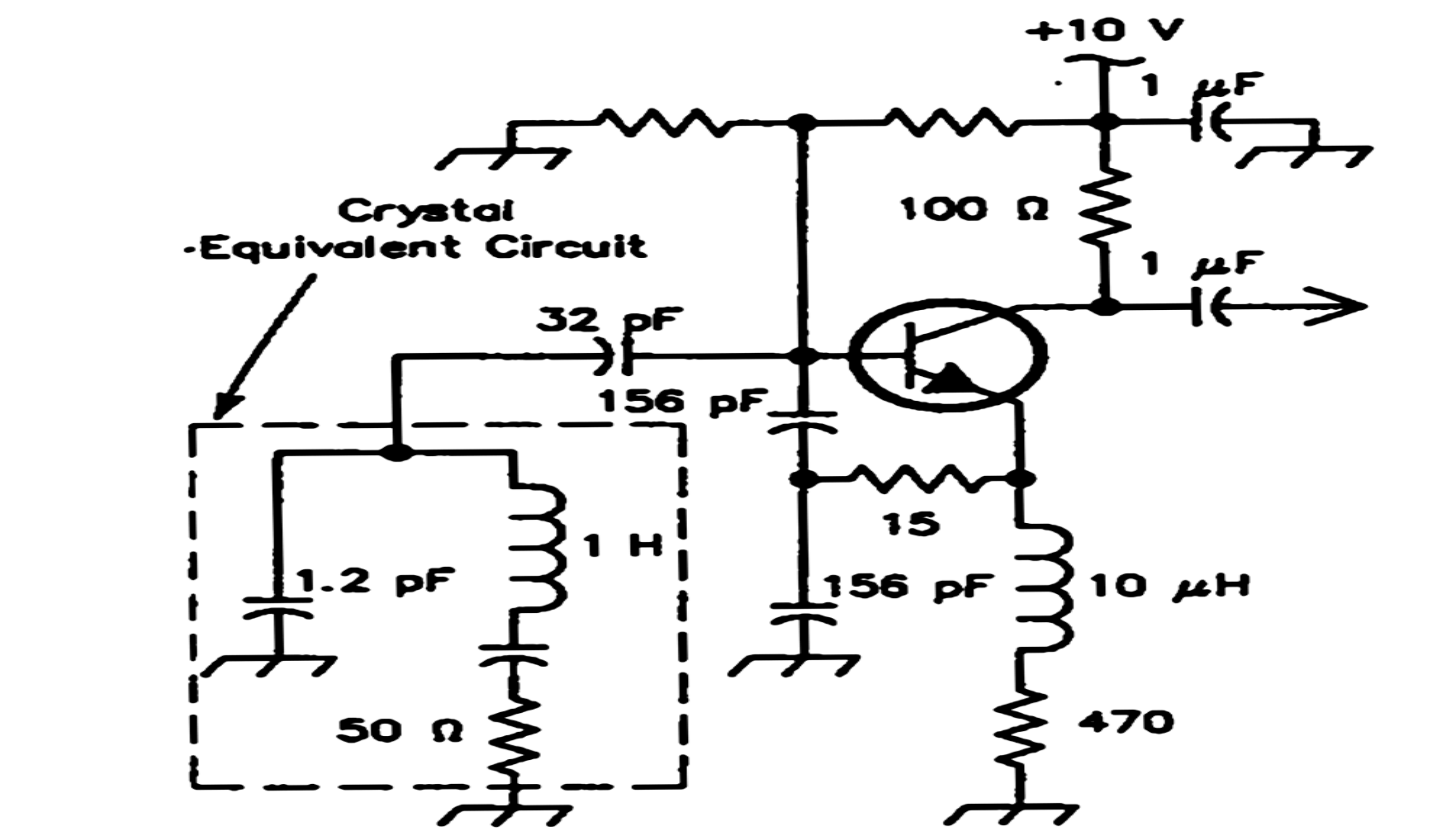
Total contribution for the consideration of the noise at the output:



Noise mechanism

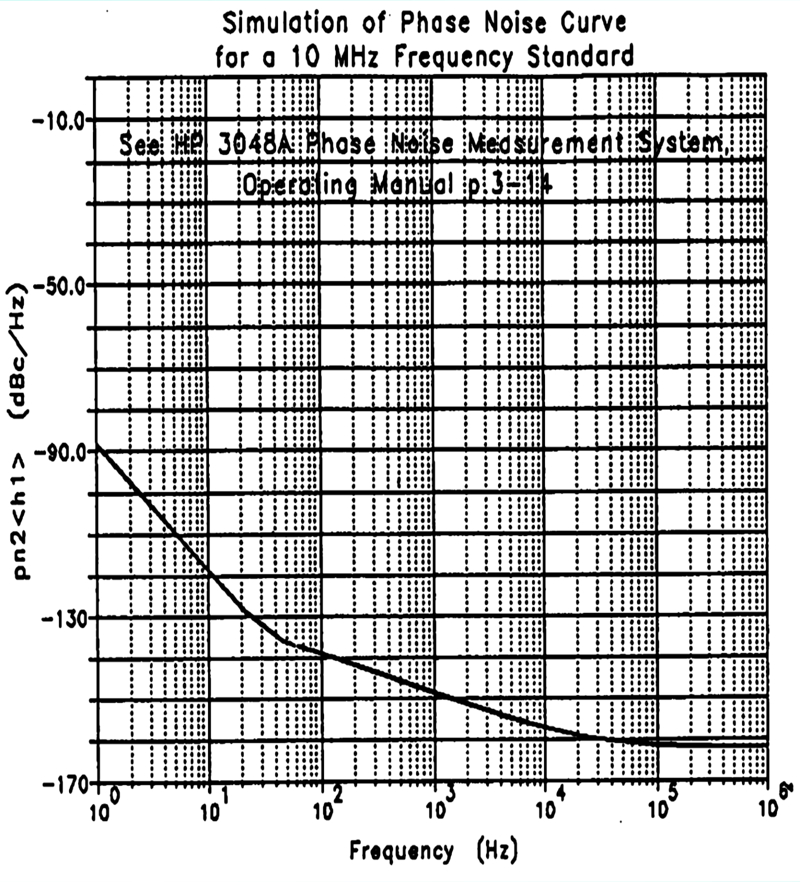
Oscillator Noise: Experimental Validation

10-MHz crystal oscillator circuit

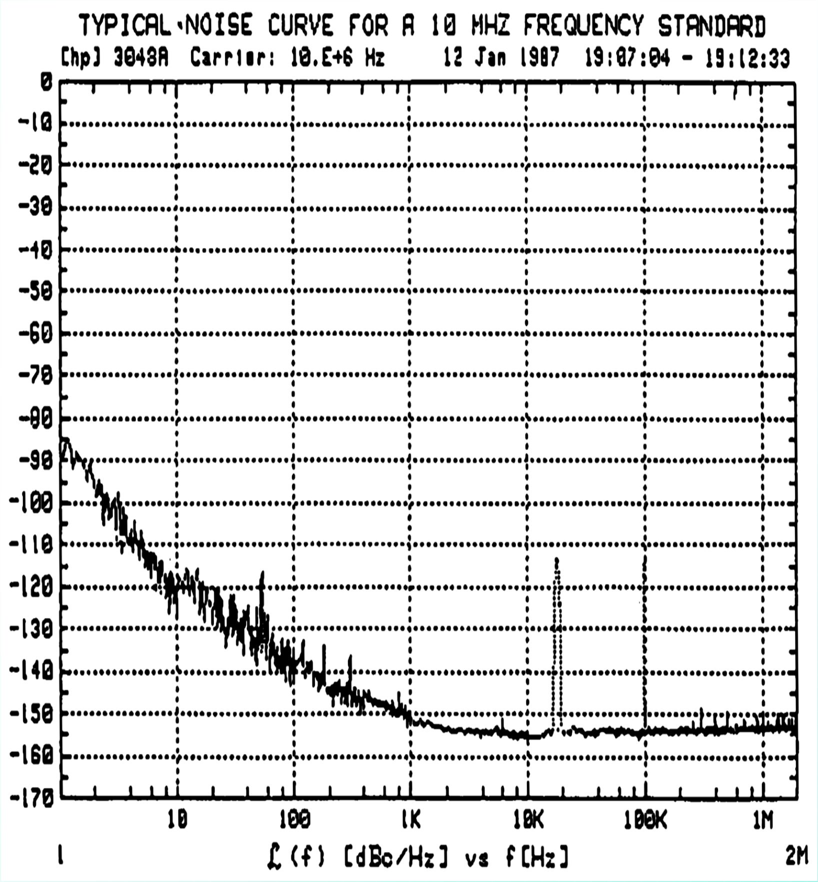


Oscillator Noise: Experimental Validation

Simulated phase noise



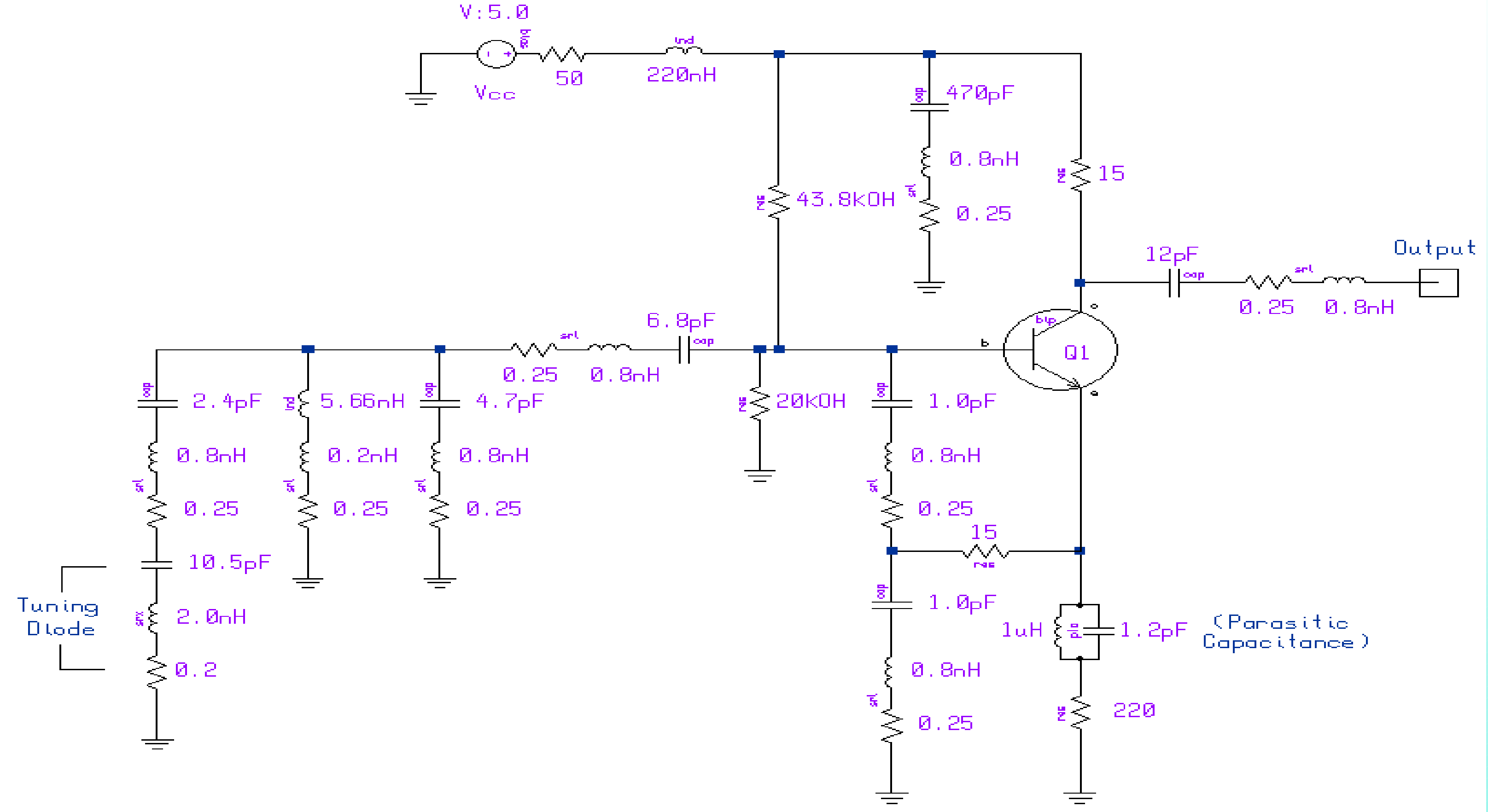
Measured phase noise



10-MHz crystal oscillator circuit

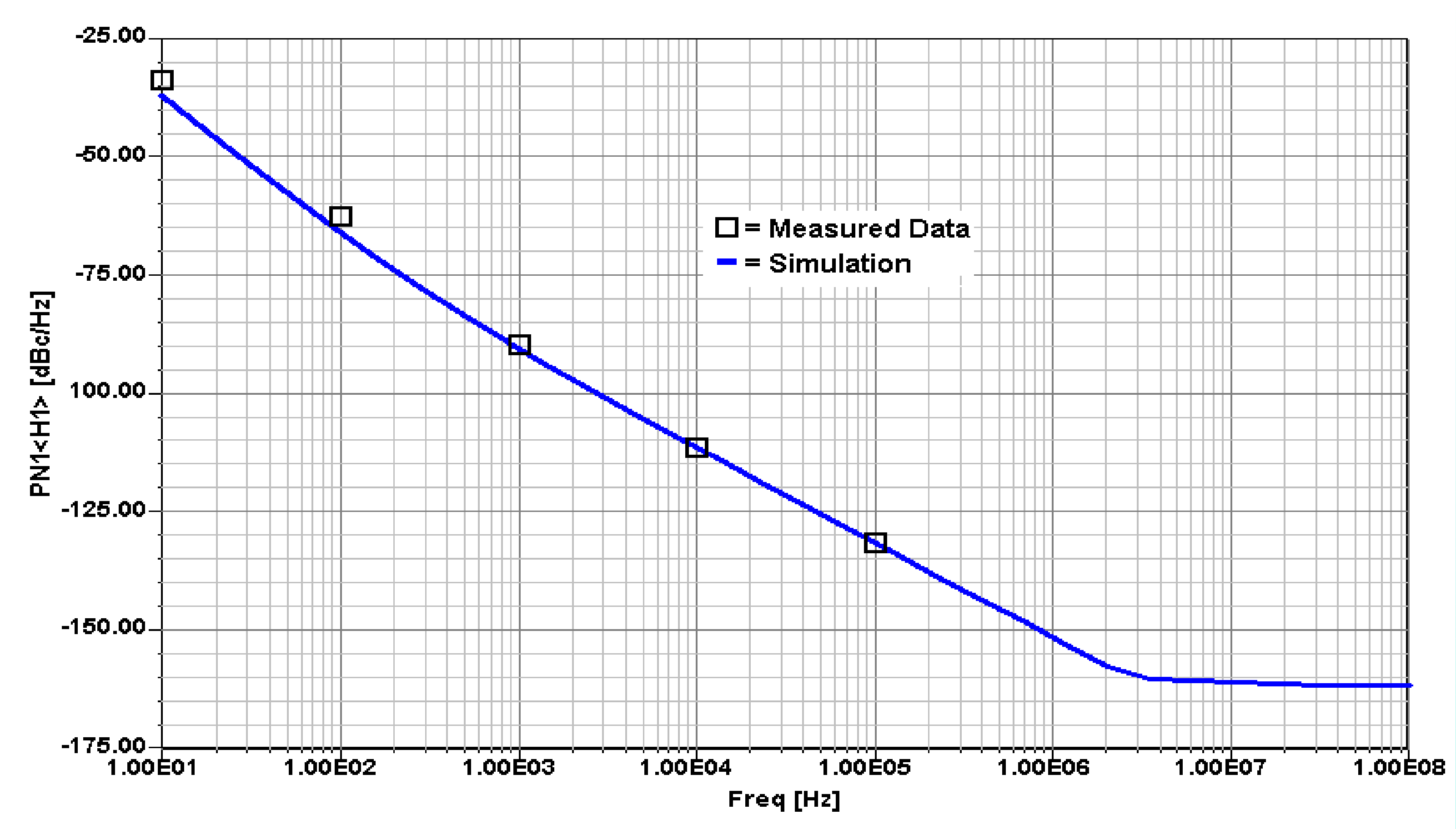
Oscillator Noise: Experimental Validation

800-MHz oscillator circuit (uses RF feedback in the form of a 15 ohm resistor between the emitter and the capacitive voltage divider)



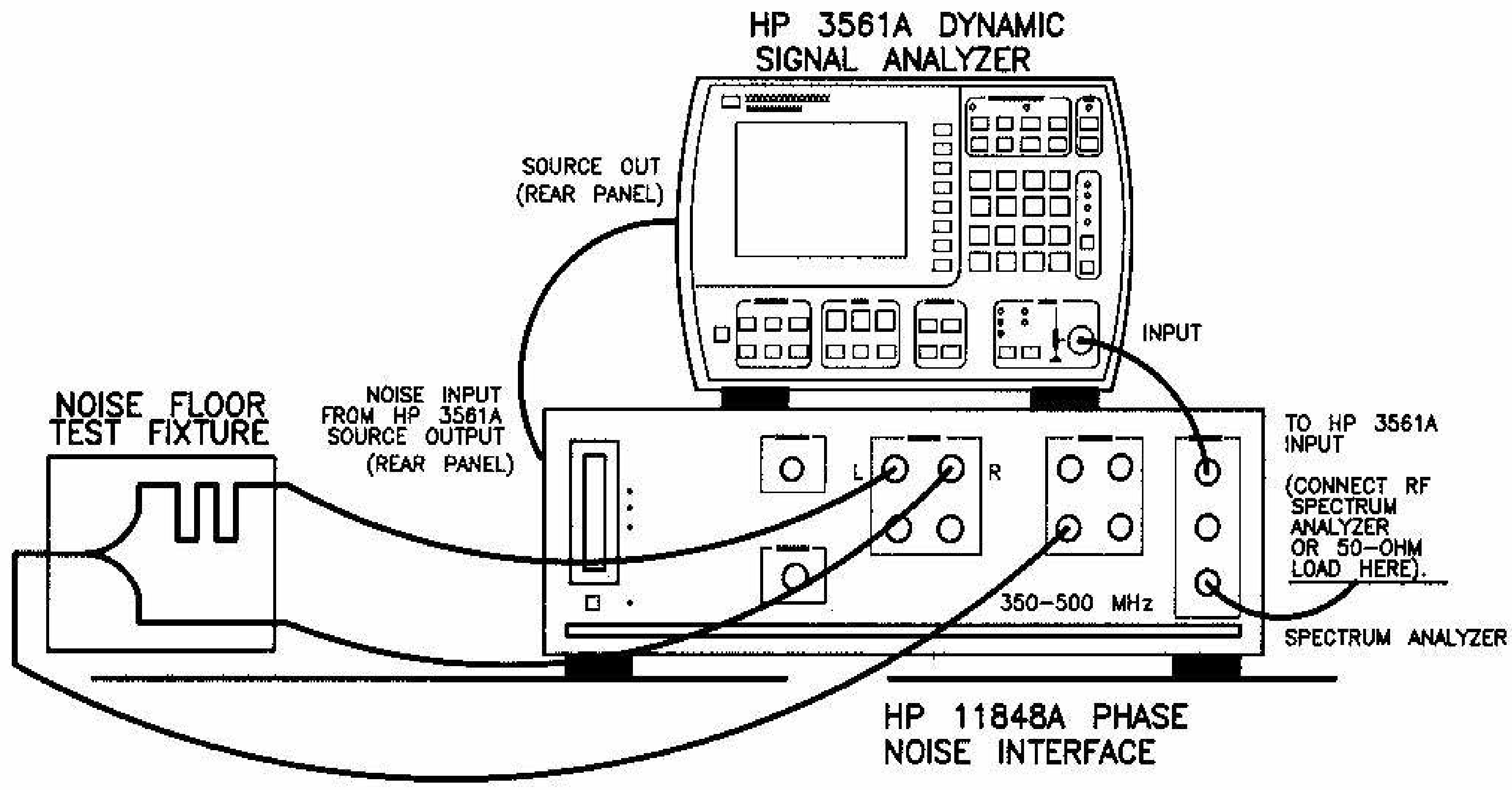
Oscillator Noise: Experimental Validation

800-MHZ VCO



Phase Noise Measurements

Model 3048A -based phase noise test setup



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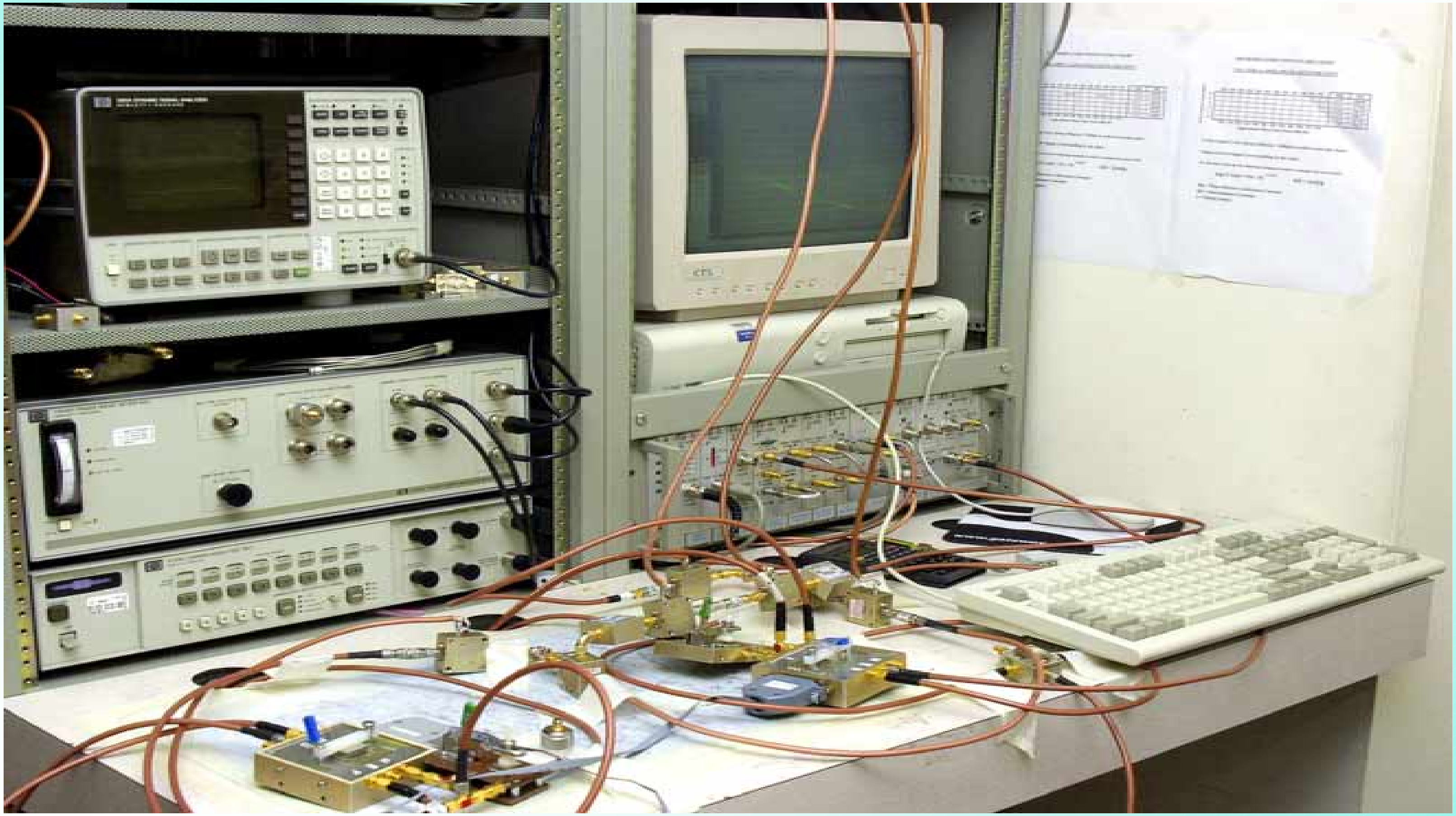
Phase Noise Measurements

Synergy Microwave Corp. in-house automated test system to measure oscillator phase noise.



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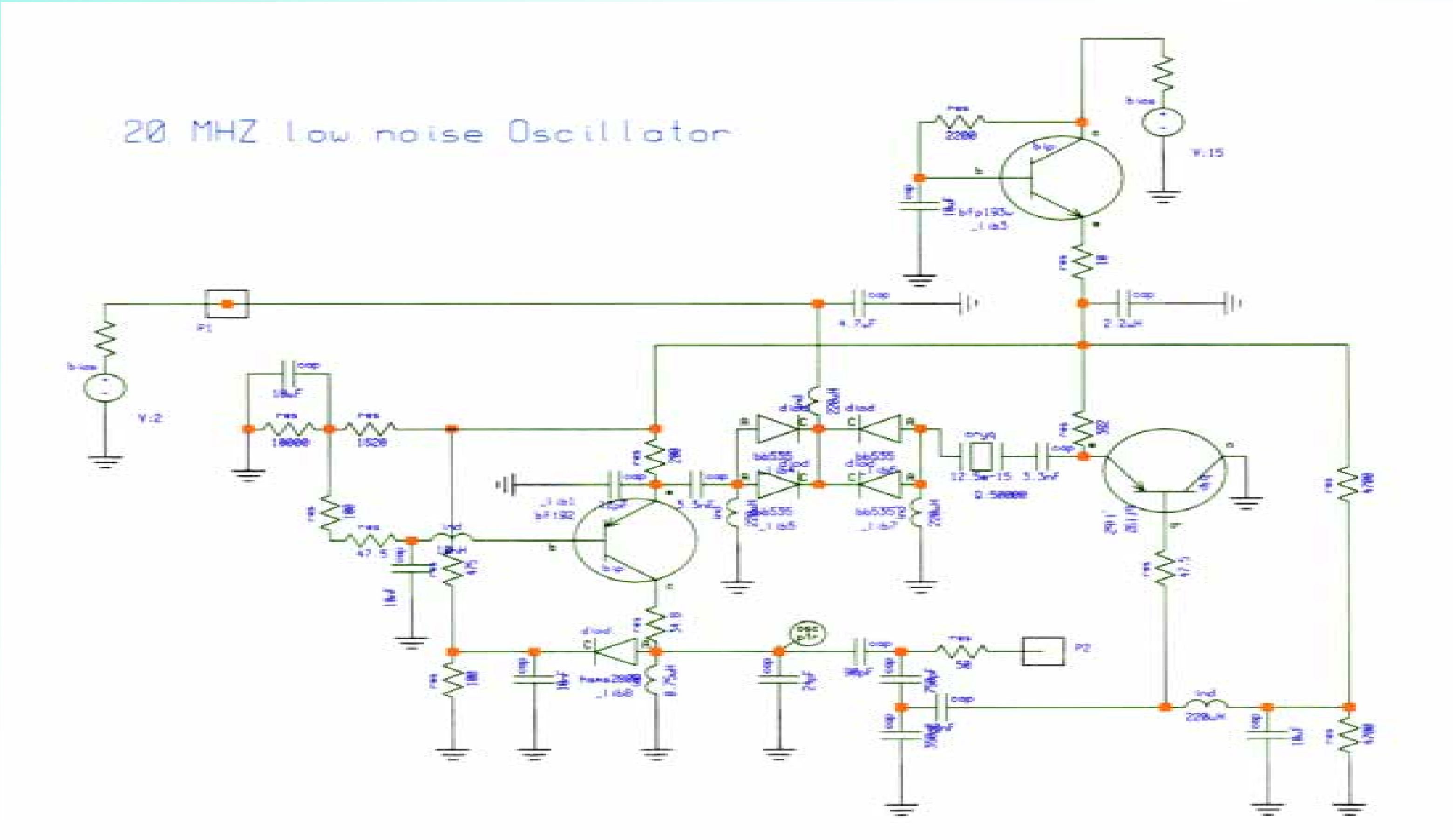
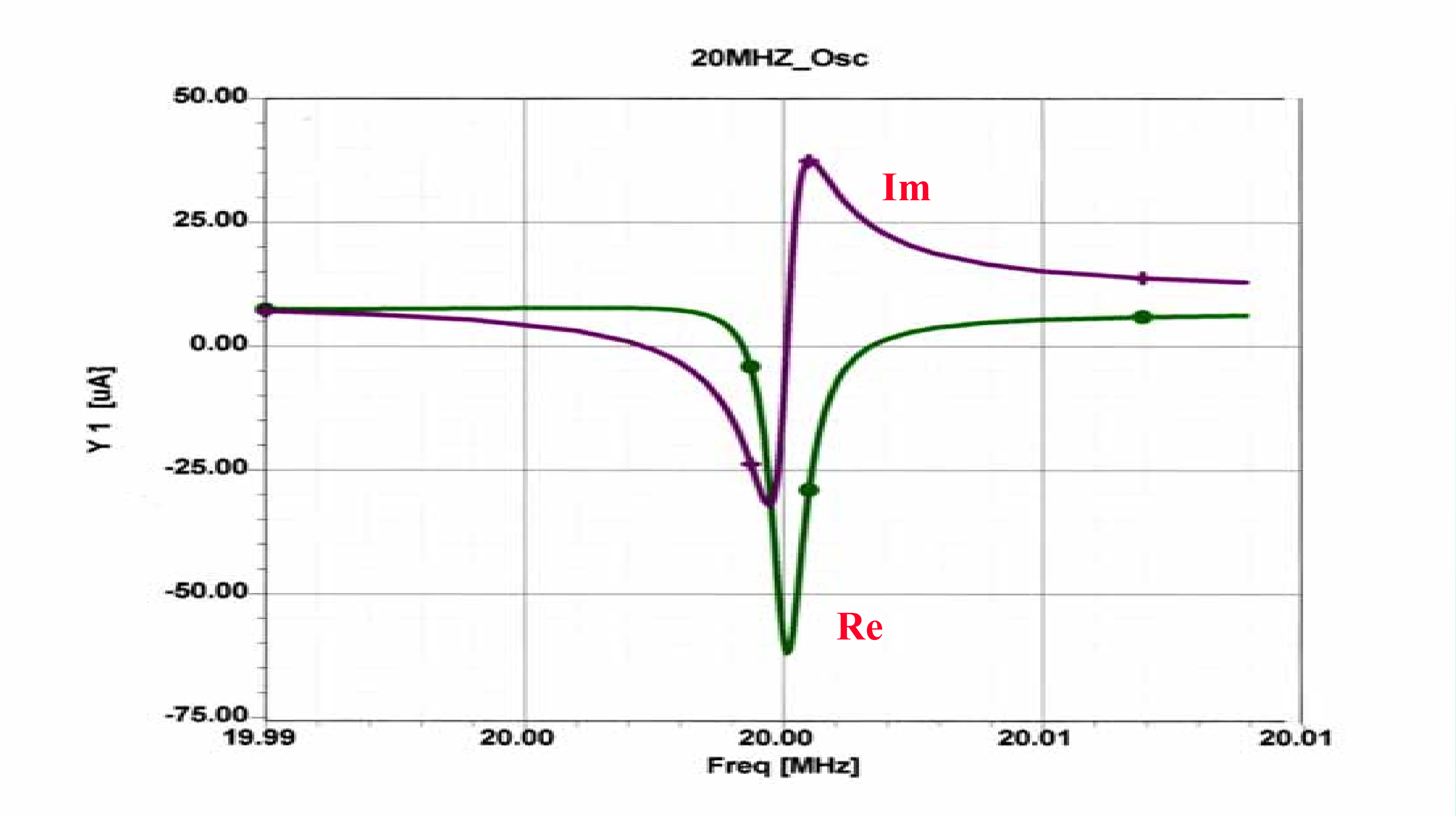
Low Phase Noise Oscillators

20 MHz Butler-type low phase noise oscillator



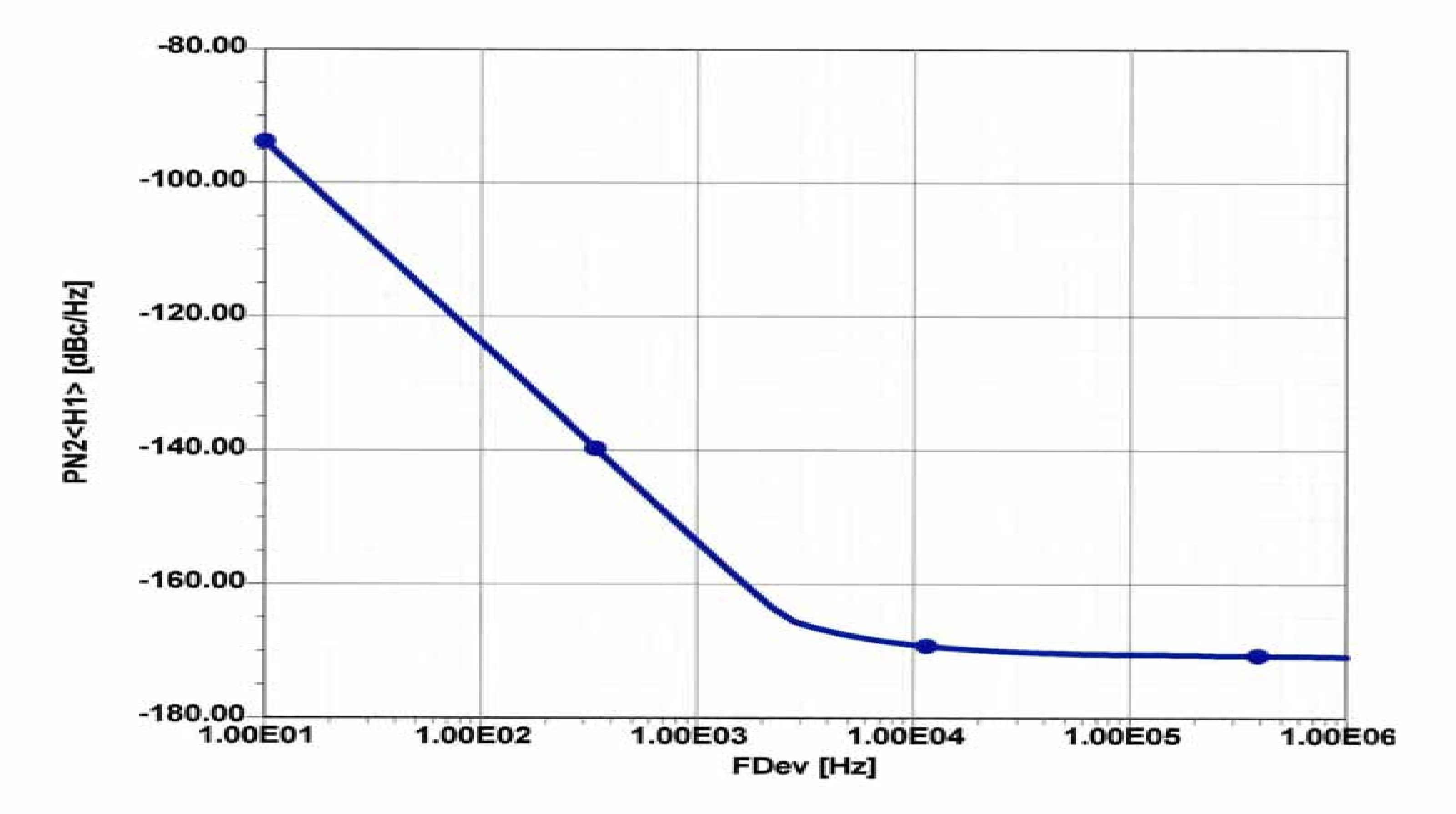
Low Phase Noise Oscillators

Resistive and reactive currents of the 20 MHz Butler-type low phase noise oscillator



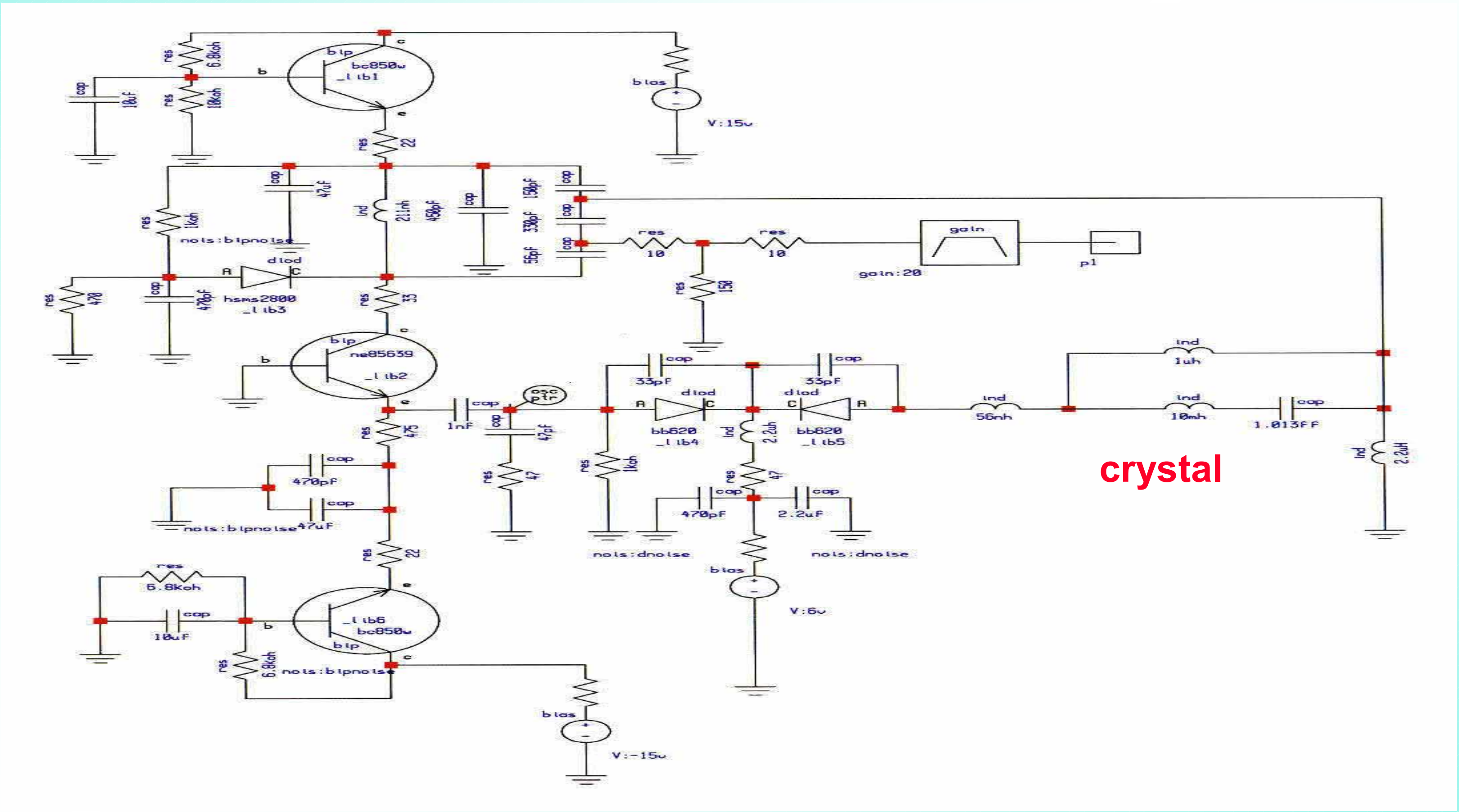
Low Phase Noise Oscillators

Predicted phase noise of the 20 MHz Butler-type low phase noise oscillator



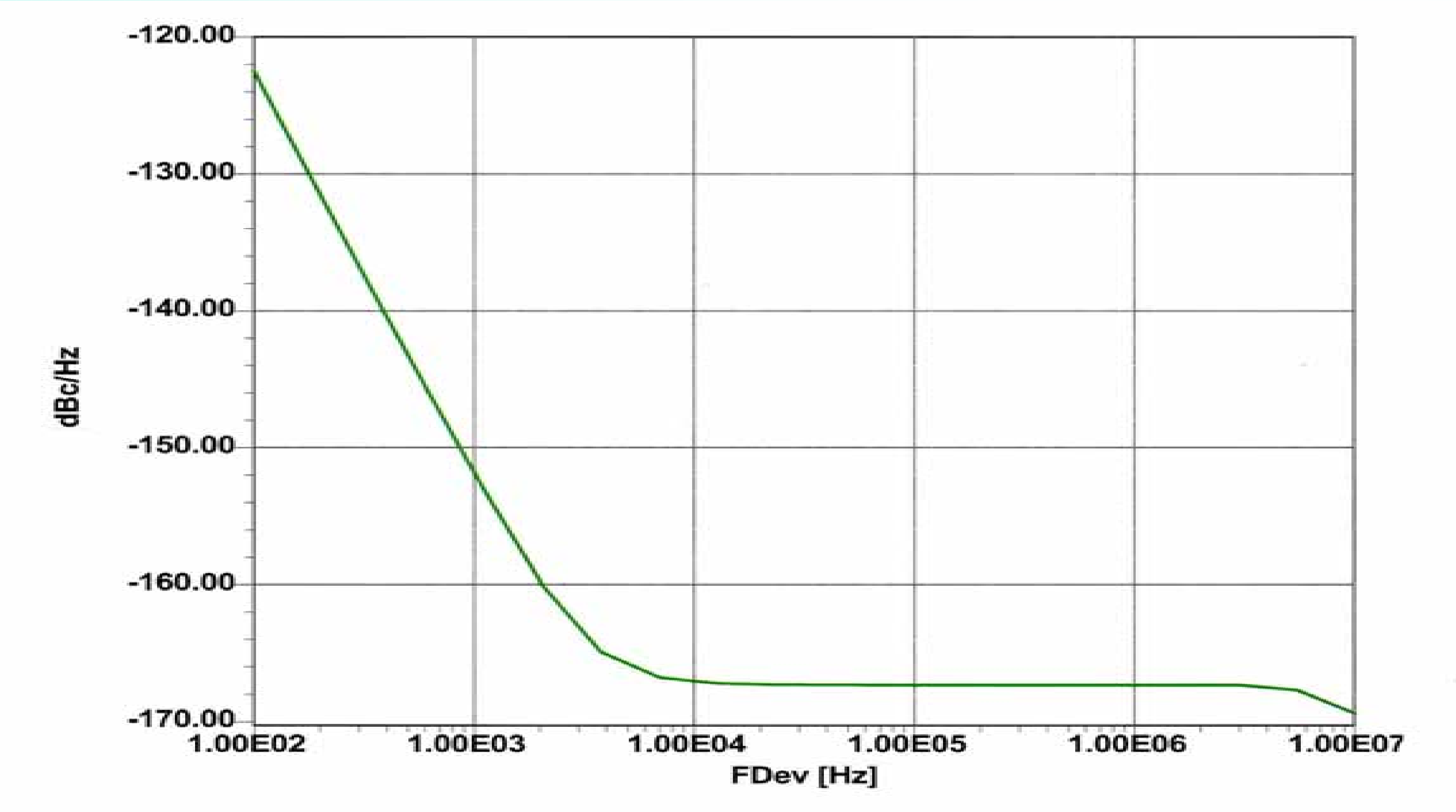
Low Phase Noise Oscillators

Schematic of 50 MHz crystal low phase noise oscillator



Low Phase Noise Oscillators

Predicted phase noise of the 50 MHz crystal type low phase noise oscillator



Mixer

Type of Mixer based on device :

- Passive mixer - Conversion loss, High dynamic range, Not suitable for integration, High LO power requirement etc
- Active mixer- Conversion gain, Low LO power requirement, High reverse isolation, suitable for integration etc.

Type of Mixer based on device :

- Single ended mixer
- Balanced Mixer

Single ended mixer: This is simplest and having no inherent isolation between LO and RF due to circuit geometry and shows poor performance.

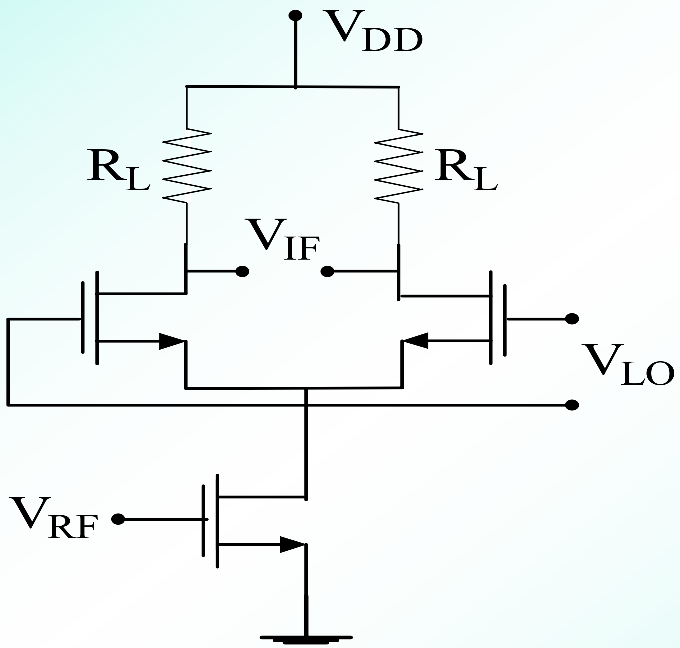
Balanced mixer: Double balanced mixer shows inherent isolation between LO and RF due to circuit geometry and shows better performance

Other type of mixers is image reject mixer, image recovery (enhanced) mixer and harmonic mixer.

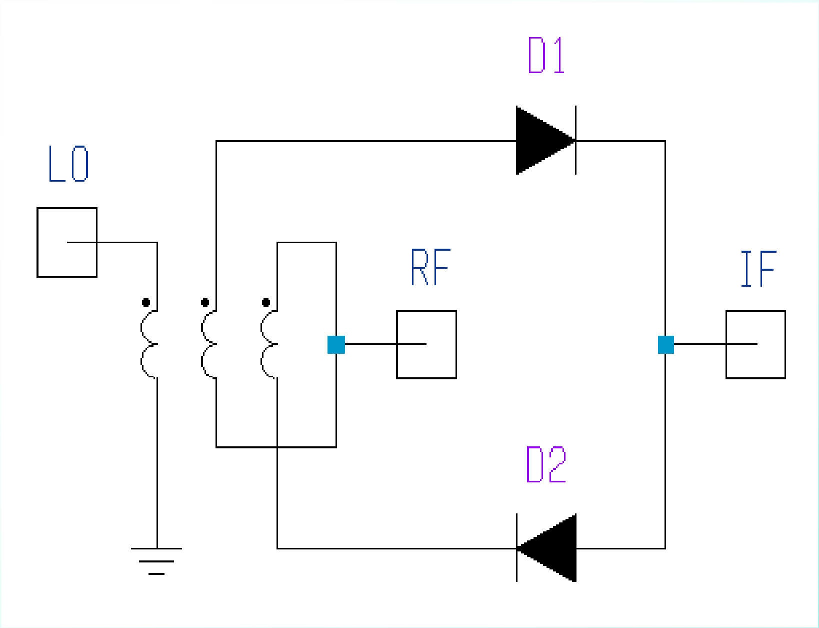
Mixer, Cont'd.

Single Balanced Mixer:

There is two configuration of the balanced mixer, single balanced and double balanced configuration. Figure below shows the schematic of passive and active single balanced mixer. Here two signals are mutually isolated due to the circuit geometry rather than the use of filters. Normally the LO and RF signals are mutually isolated and separated from the IF port by means of filter.



Active Single Balanced mixer

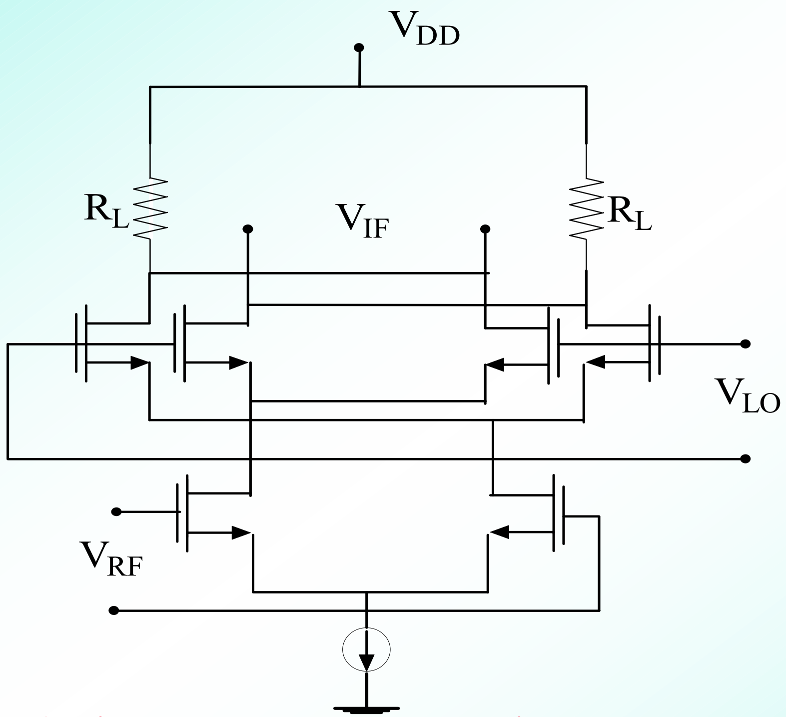


Passive Single Balanced mixer

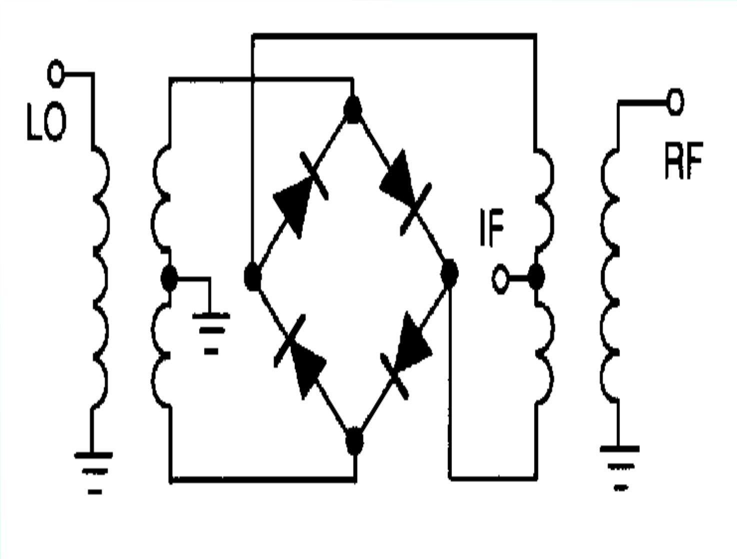
Mixer, Cont'd.

Double Balanced Mixer:

Figure below shows the schematic of passive (diode ring mixer) and active (Gilbert mixer) double balanced mixer. In this case three signals (LO, RF and IF) are mutually isolated due to the balanced nature of the circuit geometry.



Active Double Balanced mixer



Passive Double Balanced mixer

Mixer, Cont'd.

FET Mixer:

FETs, used in active and passive circuits, are a popular solution for low power integrated mixers. The linearity of the FETs is based on the fact that a FET follows a square law and therefore the first derivative, its transconductance, is supposed to be constant. and this is valid within a wide amplitude range. GaAs FET Mixers obtain up to +50dBm IP3 (Passive mixers).

The active FET mixer achieves gain at the expense of intercept point; the difference can be as much as 20dB. On the other hand, one can use any FET as a passive device similar to a diode mixer, in which the source-drain channel gets switched on and off. This impedance modulation is somewhat similar to a diode mixer, but the gate electrode is isolated from both source and drain. It nonetheless falls in the category of additive mixers because there is sufficient interaction between gate and source, although the impedance at the gate changes significantly less than in an additive diode mixer.

Implementation is a challenge in that building a high-performance passive FET mixer requires a pair or a quad of mixer cells that are sufficiently matched to suppress even-order IMD products. Dual-Gate MOSFET/GaAsFET significantly improves LO-RF isolation.

Mixer, Cont'd.

MOSFET Mixer:

The trend to go to smaller voltages has created some CMOS implementations of the Gilbert cell. While this may allow us to integrate reasonable mixers on the same chip, but they are frequently starved in operating voltage and current and rarely fare better than a single diode mixer with proper drive level. Its main advantage is the full integration in the front-end.

On the other hand, the symmetry reduces some of the unwanted spurious frequencies and because of the high impedance; the actual RF power level is much less than the diode would need. Because of the flicker corner frequency of MOS, the low frequency spot noise figure of the mixer will be high compared to other devices, but not as bad as its GaAsFET brothers.

In any case, to combine reasonably performance and simplicity, one need to have a preamplifier that reduces the noise figure as well as the third-order intercept point of the mixer by the amount of pre-amplification.

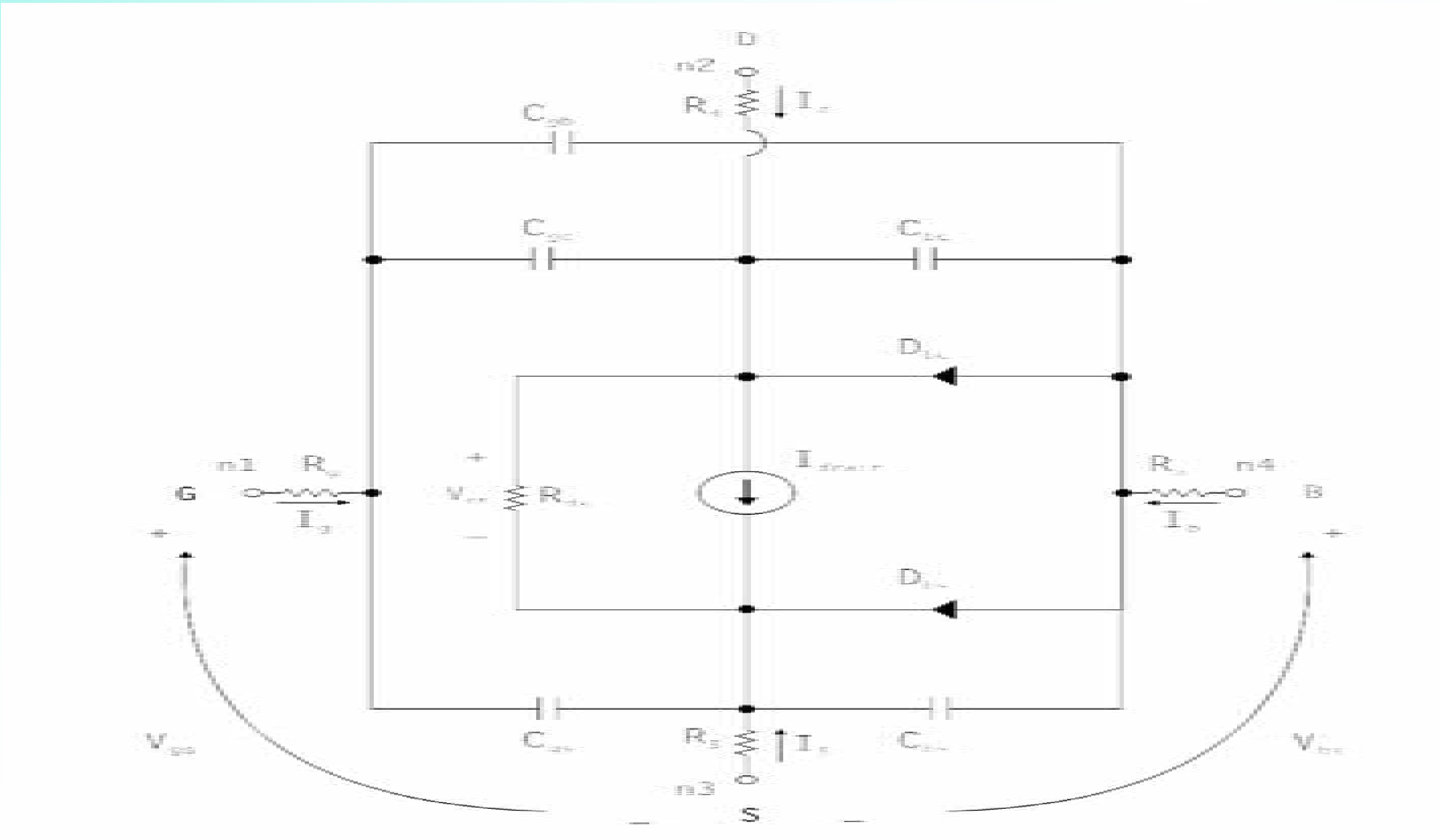
Mixer, Cont'd.

GaAsFET Mixer:

The cutoff frequency of the GaAs FET transistors today is still higher but its corner frequency f_c, also frequently referred to as flicker frequency, is somewhere between 10 and 100 MHz and therefore results in poor mixer performance from at low IF frequencies.

Mixer Noise Model

MOSFET : Intrinsic model for NMOS-MOSFET (Insulated Gate)



Typical small signal intrinsic model the MOSFET

Mixer Noise Model, Cont'd.

MOSFET Noise Model

Noise Model

Let Δf be the bandwidth (normalized to 1Hz). The noise generators introduced in the intrinsic device are shown below, and have mean-square values of:

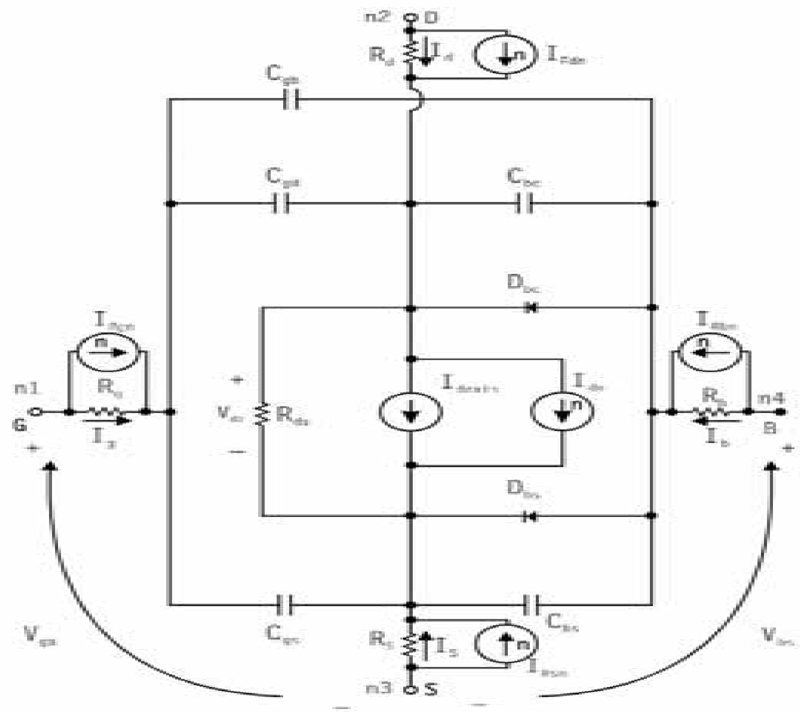
$$\langle i_{dn}^2 \rangle = \frac{8kTg_m}{3}\Delta f + \mathrm{KF}\,\frac{I_D^{\mathrm{AF}}}{f^{\mathrm{FCP}}}\Delta f$$

$$\langle i_{Rgn}^2 \rangle = 4\frac{kT}{R_g}\Delta f$$

$$\langle i_{Rdn}^2 \rangle = 4\frac{kT}{R_d}\Delta f$$

$$\langle i_{Rsn}^2 \rangle = 4\frac{kT}{R_s}\Delta f$$

$$\langle i_{Rbn}^2 \rangle = 4\frac{kT}{R_b}\Delta f$$



Noise of model the Intrinsic MOSFET

Mixer Noise Model, Cont'd.

MIXER Noise Analysis

Simple analytical equations are derived to estimate the low frequency noise (flicker) and high frequency noise (white noise) at the output of a switching mixer. The total mixer noise is the contribution from the (1) low frequency noise (1/f) and (2) high frequency noise (white noise). The noise model described here for FET and holds true for similar kind of devices (MOSFET/GaAsFET).

Low frequency noise associated with the mixer is due to the following:

(a) Transconductance noise
(b) Load-noise
(c) Switching noise (direct switch noise and indirect switch noise

High frequency noise associated with the mixer is due to the following:
(a) White noise in mixer switches
(b) Transconductor noise

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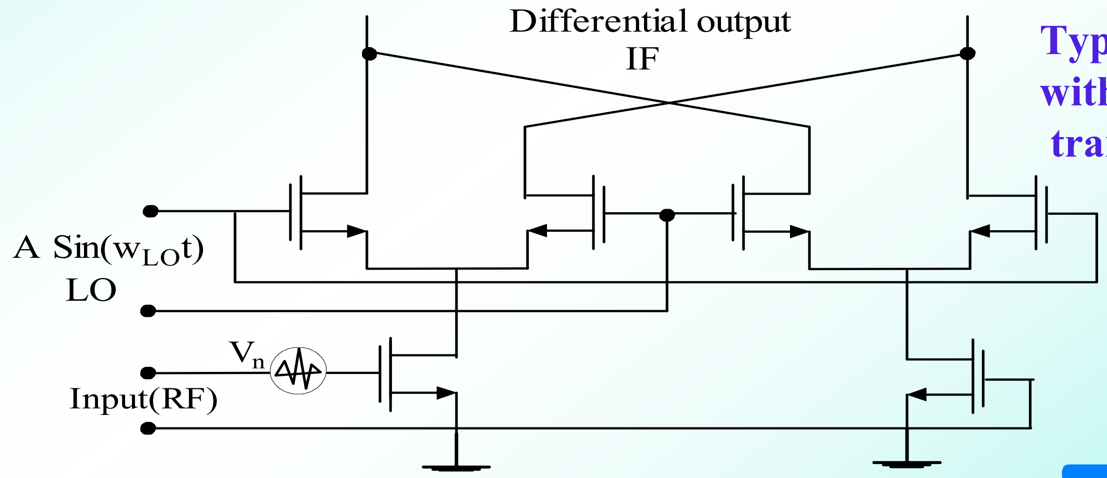
Mixer Noise Model, Cont'd.

(a) Transconductance noise:

Figure below shows the typical switching active mixer with noise source shown at the transconductor-input. The noise in the lower transconductance FET's accompanies the RF input signal, and is translated in frequency just like the signal is. Therefore, flicker noise in theses FETs is up-converted to ω_{LO} and to its odd harmonics while white noise at ω_{LO} (and to its odd harmonics) is translated to the DC. If the output of interest lies at low frequency or zero IF, then the transconductance FET's contribute white noise after frequency translation. The flicker corner frequency of theses devices is much lower than the IF.



Typical switching active mixer with noise source shown at the transconductor-input

Mixer Noise Model, Cont'd.

(a) Load noise:

In a zero-or low IF receiver, flicker noise in the loads of the down-conversion mixer competes with the signal.

P-MOSFET's show lower flicker noise as compared to N-MOSFET's of the same dimensions, therefore PMOS loads are preferred over NMOS in mixers (P-MOSFET's has low f_T)

The noise due to load resistance R_L is given by

$$[\hat{V}_{on}^{2}]_{noise-Load} = 4kTR_L + 4kTR_L = 8kTR_L$$

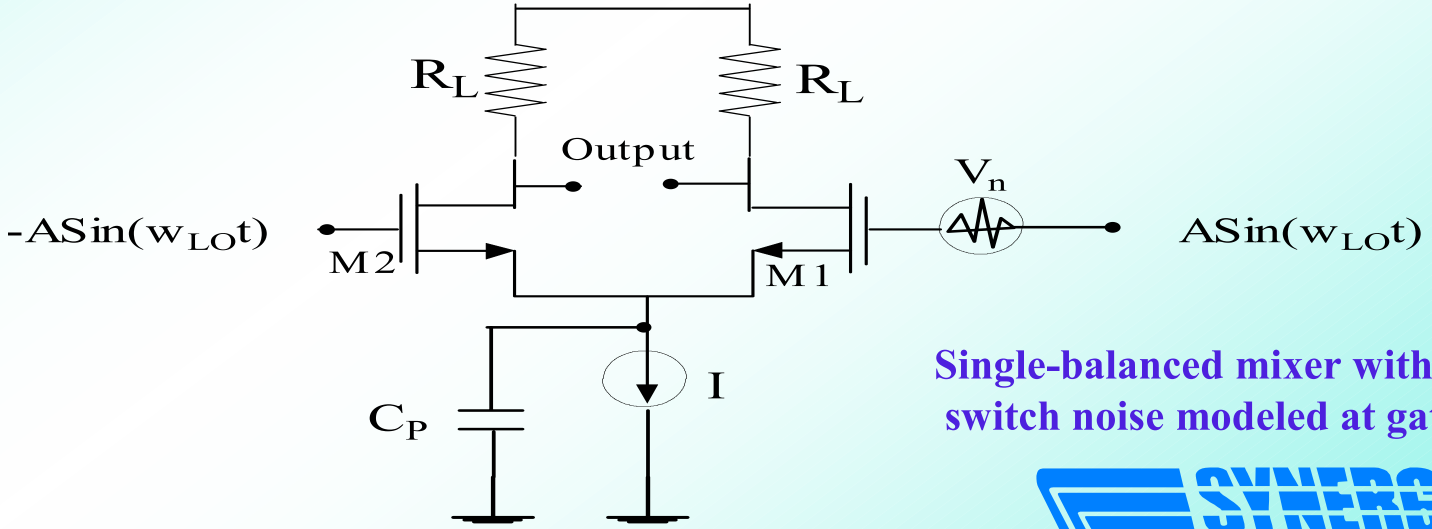
Mixer Noise Model, Cont'd.

(c) Switch Noise:

The mixer noise due to switching mechanism is characterized as direct switch noise and indirect switch noise.

Direct Switch Noise:

The Figure below shows the single-balanced mixer with switching noise modeled at the gate, where the bias current in the switch FET's M1 and M2 is periodic at a frequency ω_{LO}. Flicker noise arises from traps with much longer time constants than the typical period of oscillation at RF, and it may be assumed that the time-average inversion layer charge in the channel determines the root mean square (RMS) flicker fluctuations.



Single-balanced mixer with switch noise modeled at gate

Mixer Noise Model, Cont'd.

Direct Switch Noise:

For ease in analysis, it is assumed that the circuit switches sharply and a small differential voltage excursion causes the current to completely switch from one side of the differential pair to other side. The switching noise is characterized as direct and indirect switch noise. Considering the direct effect of the switch noise at the mixer output, the transconductance RF input stage is replaced by a current source "I" at the tail .

In the absence of noise, for positive value of LO voltage M1 switches ON and M2 switches OFF, and a current equal to "I " appears at the right branch and again in the next half period the current switches to the left branch, thereby generating output as a square wave at frequency ω_{LO} with zero DC value.

In the presence of the noise, the slowly varying noise voltage V_n modulates the time at which the pair M1, M2 switches and at every switching instant the skew in switching instant modulates the differential current waveform at the mixer output. The height of the square-wave signal at the output remains constant, however noise advances or retards the time of zero-crossing by $\Delta t = \frac{V_n(t)}{S}$ where S is the slope of the LO voltage at the switching time.

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Mixer Noise Model, Cont'd.

Direct Switching Noise:

The waveform at the mixer output consists of a square-wave of frequency ω_{LO} and the current amplitude I, representing the LO feed-through, superposed with a pulse train of random widths Δt and amplitude of 2I at a frequency of $2\omega_{LO}$, representing the noise. The average output current over one period is given by

$$i_{o,n}(t) = \frac{2}{T} \times 2I \times \Delta t = \frac{2}{T} \times 2I \times \frac{V_n}{S} = 4I\left[\frac{V_n}{ST}\right]$$

and the frequency spectrum of the base-band noise current at the output is given by

$$i_{o,n}(f) = 4I\left[\frac{V_n(f)}{ST}\right] = \left[\frac{1}{\pi}\right]\left[\frac{I}{A}\right]V_n(f)$$

Where T is the period of LO and S is the slope of the LO voltage at the switching time. Sampled images of this spectrum appear at integer multiples of $2\omega_{LO}$.

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Mixer Noise Model, Cont'd.

Direct Switching Noise:

The low frequency noise V_n at the gate switch appears at the output without frequency translation and corrupts a signal down-converted to zero IF. The zero-crossing modulation, Δt depends on the low-frequency noise V_n and the LO-voltage slope (S) at zero crossing normalized to LO frequency, $S \times T$. For a sine wave LO, $S \times T = 4\pi A$, where A is the amplitude and a factor of two accounts for the fact that V_n is compared to a differential LO signal with an amplitude of 2A. If the mixer is used for up conversion, the switches contribute no flicker noise to the output at ω_{LO}, although flicker noise in the transconductance stage is up-converted to this frequency. The signal to noise ratio of the mixer is given by

$$SNR = \left[\frac{S \times T}{2\pi(V_{GS} - V_t)}\right]\left[\frac{V_{in}}{V_n}\right] = \left[\frac{2A}{(V_{GS} - V_t)}\right]\left[\frac{V_{in}}{V_n}\right]$$

Where $(V_{GS}-V_T)$ is the transistor gate over-drive voltage is the period of LO and S is the slope of the LO voltage at the switching time. From the above expression SNR improves by raising the product of the slope of the LO waveform at zero-crossing and its period; by increasing the gate area of the switch FET's to lower flicker noise V_n; and by lowering the transconductance FET over-drive. However, increasing switch gate area (larger input capacitance) or lowering the transistor gate-over drive voltage will degrade the mixer bandwidth.

Mixer Noise Model, Cont'd.

Indirect Switching Noise:

The analysis so far suggest that flicker noise at the mixer output may be eliminated if the slope at zero-crossing is increased infinitely, however as the LO slope rises, the output flicker noise appears via another mechanism that depends on LO frequency and circuit capacitance, called the "indirect" mechanism. The output noise current is given by

$$i_{o,n} = \frac{2}{T}\int_0^{T/2} i_{Cp}(t)dt = \left[\frac{2}{T}C_p\right]V_n$$

Where i_{Cp} is the capacitive current, frequency equal to LO frequency, with zero DC value.

The conversion gain (CG) to flicker noise in V_n due to the indirect process is given as

$$[CG]_{indirect} = \left[\frac{2}{T}C_p\right]$$

Conversion gain due to the indirect mechanism $[CG]_{indirect}$ grows with LO frequency but usually smaller than the gain $[CG]_{direct}$ due to the direct mechanism .

In most practical case, flicker noise due to a sine wave LO is attributable to the direct mechanism, which is frequency independent. However, even a LO waveform with infinitely fast rise time and fall time does not eliminate flicker noise but pushes it down to a level determined by the tail capacitance. In general, LO waveforms with a large S×T product, which is a low frequency LO with sharp transitions, will have lower flicker noise.

Mixer Noise Model, Cont'd.

High Frequency Noise:

High frequency noise associated with the mixer is due to the following:

(a) White noise in mixer switches

(b) Transconductor noise

(a) White noise in mixer switches

The high frequency mixer output noise is white and cyclostationary and can be expressed as the product of a periodic and deterministic sampling function, and white and stationary switch input-refereed noise. The mixer output noise and sampling function is given by

$$i_{0,n} = p(\omega_{LO} t) V_n(t)$$

$$p(\omega_{LO} t) = \sum_n G_m (t - \frac{nT}{2})$$

Where $p(\omega_{LO} t)$ is a periodic and deterministic sampling function, $V_n(t)$ is the white and stationary input-referred noise, and G_m is periodic at twice the LO frequency (since there is two zero crossing over every cycle of the LO).

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Mixer Noise Model, Cont'd.

High Frequency Noise:

White noise in mixer switches: The switching noise V_n is transferred to the output only at zero crossing. Switches contribute noise to the mixer output when they are both ON and if one switch is OFF, it obviously contributes no noise, and neither does the other switch that is ON because it acts as a cascade transistor whose tail current is fixed to "I" by the RF input transconductance stage.

Starting with the direct mechanism, the noise current at the mixer output consists of train of pulses, with a rate of twice the LO frequency, with a height equal to 2I/S and a width which is randomly modulated by noise. The autocorrelation of the output noise is given by

$$R_{io,n}(t+\tau,t) = p(t).p(t+\tau).R_{vn}(\tau)$$

The autocorrelation of the white noise $R_{vn}(\tau)$, is a delta function and the autocorrelation of the output noise is a function of both t and τ, which indicates that the output noise is not stationary but periodic, white and cyclostationary. The input noise is white and stationary and its power spectral density is given by

$$[\hat{V}_n^2]_{noise-transconductance} = \left[\frac{4kT\gamma}{g_m}\right], \qquad [g_m]_{zero-crossing} = \frac{2I}{\Delta V}$$

Where γ is the channel noise factor and normally its value is 2/3 for long MOSFET's channel and g_m is the switch transconductance at the zero crossing.

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Mixer Noise Model, Cont'd.

High Frequency Noise:

The power spectral density of the output noise current is given by

$$[\hat{i}_{on}^{2}]_{output-noise} = \int_0^T p^2(t)dt.[\hat{V}_n^2] = \left[\frac{2}{T}\right]\left[\frac{2I}{S}\right]^2\left[\frac{1}{T_s}\right].[\hat{V}_n^2]$$

$$[\hat{i}_{on}^{2}]_{output-noise} = 4kT\gamma\left[\frac{4I}{ST}\right]$$

$$[\hat{i}_{on}^{2}]_{output-noise} = 4kT\gamma\left[\frac{4I}{\pi A}\right]$$

Where S is the slope of the LO waveform and for sine wave, S= 2Aω_{LO}.

From the above it shows that output noise power spectral density depends on LO magnitude (A) and bias current (I), and not the transistor size!

Mixer Noise Model, Cont'd.

High Frequency Noise:

Transconductor noise :

White noise originated in the transconductor is indistinguishable from the RF input signal, therefore mixer commutation is assumed square wave like; and the LO frequency and its odd harmonics down convert the respective components of the white noise to the IF and it is given by

$$[\hat{V}_{on}^2]_{noise-transconductance} = n\left[\frac{4kT\gamma}{g_m}\right]\left[\frac{2g_m R_L}{\pi}\right]^2$$

Any periodic LO waveform, sine-wave or otherwise (sufficient voltage), which switches the mixer results in square-wave commutation of the transconductance stage output current and the factor n is given as

$$n = 2\left[1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\right] = \frac{\pi^2}{4}$$

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Mixer Noise Model, Cont'd.

Total Mixer Output Noise:

The total mixer noise is the contribution from the low frequency noise (1/*f*) and) high frequency noise (white noise) and can be given by

$$[\hat{V}_{on}^{2}]_{total\ mixer-noise} = [\hat{V}_{on}^{2}]_{low-frequency\ (1/f)} + [\hat{V}_{on}^{2}]_{high-frequency\ (white)}$$

$$[\hat{V}_{on}^{2}]_{total\ mixer-noise} = 8kTR_L + 8kT\gamma\left[\frac{R_L^2 I}{\pi A}\right] + n\left[\frac{4kT\gamma}{g_m}\right]\left[\frac{2g_m R_L}{\pi}\right]$$

Where

K= 1.38E-23 (Joule/Kelvin)

R_L= Load resistor

g_m = Transconductance

γ = Channel noise factor

I= DC bias current

A= Amplitude of the LO signal.

n = $\pi^2/4$

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Mixer Noise Model, Cont'd.

Total Mixer Output Noise (MOSFET):

The simplified expression of the total mixer noise is given by

$$[\hat{V}_{on}^{2}]_{total\ mixer-noise} = 8kTR_L\left[1+\gamma\frac{R_L I}{\pi A}+\frac{\gamma\, g_m R_L}{2}\right]$$

Where the first term is due to the two-load resistor R_L, the second term is the output noise due to the two switches, and the third term is the noise of the transconductance stage transferred to the mixer output.

In the double-balanced mixer there are twice as many FET's in the transconductance stage and the switches, so the output noise is given as

$$[\hat{V}_{on}^{2}]_{total\ mixer-noise} = 8kTR_L\left[1+\gamma\frac{2R_L I}{\pi A}+\gamma\, g_m R_L\right]$$

Where I is the bias current in each side of the mixer.

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Mixer Noise Model, Cont'd.

Mixer Noise (MOSFET): Comments/Discussions

The mixer noise varies with different circuit parameters, such as LO amplitude (A), mixer DC bias current (I), load resistance (R_L) and transconductance g_m and allows the designer to design and optimize the mixer noise as per the desired specifications.

Comparing a scaled double-balanced mixer with the same total current as a single-balanced mixer (that is, the former is biased at half the current per branch but the same V_{GS}-V_t as the later), the output noise for double-balanced and single-balanced mixer is same. However, since the gain of the double-balanced mixer from the differential input is half, the input referred noise voltage is twice as large. Referred to a differential 100-ohm source, its noise figure is 3dB larger than that of a single-balanced mixer referred to a single-ended 50Ω-source resistance.

The main advantage of the double-balanced mixer is that it suppress LO feed-through, as well as noise or interferes superimposed on the LO waveform applied to the mixer but it cannot suppress the uncorrelated noise in the switches.

Mixer Noise Model, Cont'd.

Mixer Noise (GaAsFET):

$$[\hat{V}_{on}^{2}]_{total\ \ mixer-noise} = [\hat{V}_{on}^{2}]_{low-frequency\ (1/f)} + [\hat{V}_{on}^{2}]_{high-frequency\ (white)}$$

$$[\hat{V}_{on}^{2}]_{total\ \ mixer-noise} = 8kTR_L\left[1 + P\frac{R_L I}{\pi A} + P\frac{g_m R_L}{2}\right]$$

Where the first term is due to the two-load resistor R_L, the second term is the output noise due to the two switches, and the third term is the noise of the transconductance stage transferred to the mixer output.

In the double-balanced mixer there are twice as many GaAs FET's in the transconductance stage and the switches, so the output noise is given as

$$[\hat{V}_{on}^{2}]_{total\ \ mixer-noise} = 8kTR_L\left[1 + P\frac{2R_L I}{\pi A} + P\ g_m R_L\right]$$

Where
K= 1.38E-23 (Joule/Kelvin)
R_L= Load resistor
g_m = Transconductance
P = Channel noise factor
I= DC bias current
A= Amplitude of the LO signal

Mixer Noise Model, Cont'd.

Total Mixer Output Noise (MOSFET):

The simplified expression of the total mixer noise is given by

$$[\hat{V}_{on}^{2}]_{total\ mixer-noise} = 8kTR_L\left[1+\gamma\frac{R_L I}{\pi A}+\frac{\gamma\, g_m R_L}{2}\right]$$

Where the first term is due to the two-load resistor R_L, the second term is the output noise due to the two switches, and the third term is the noise of the transconductance stage transferred to the mixer output.

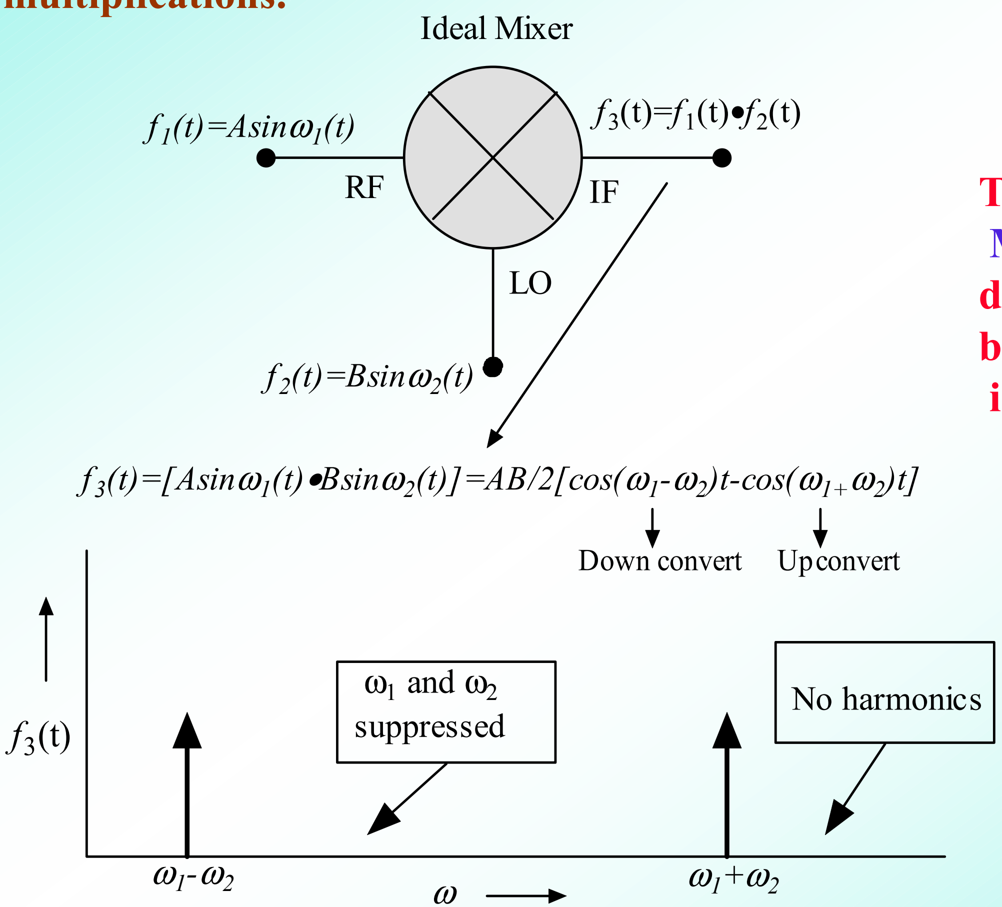
In the double-balanced mixer there are twice as many FET's in the transconductance stage and the switches, so the output noise is given as

$$[\hat{V}_{on}^{2}]_{total\ mixer-noise} = 8kTR_L\left[1+\gamma\frac{2R_L I}{\pi A}+\gamma\, g_m R_L\right]$$

Where I is the bias current in each side of the mixer.

Mixer Circuits

Mixer is a frequency translation device, as name appears mixing but it does not really 'mix' or 'sum' signals; it simply multiplies them. But in reality, practical mixer generates undesired output frequencies due to the nonlinear characteristic of the device used for multiplications.



The Figure shows the typical ideal Mixer; note that both sum and difference frequencies are obtained by the multiplication of the two input signals $f_1(t)$ and $f_2(t)$.

Where $\omega=2\pi f$

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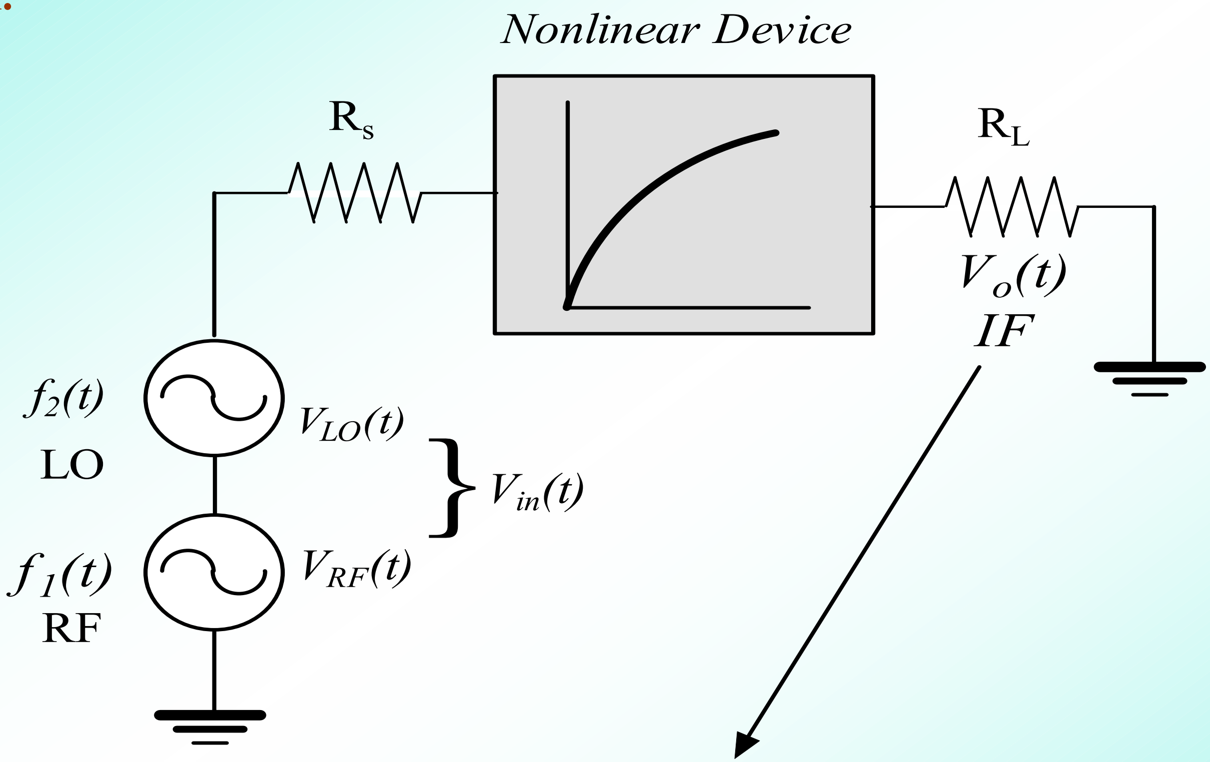
Mixer Circuits, Cont'd.

The Figure shows the typical simple unbalance nonlinear mixer operation in which any diode or transistor will exhibit nonlinearity characteristic at sufficiently high input signal level.



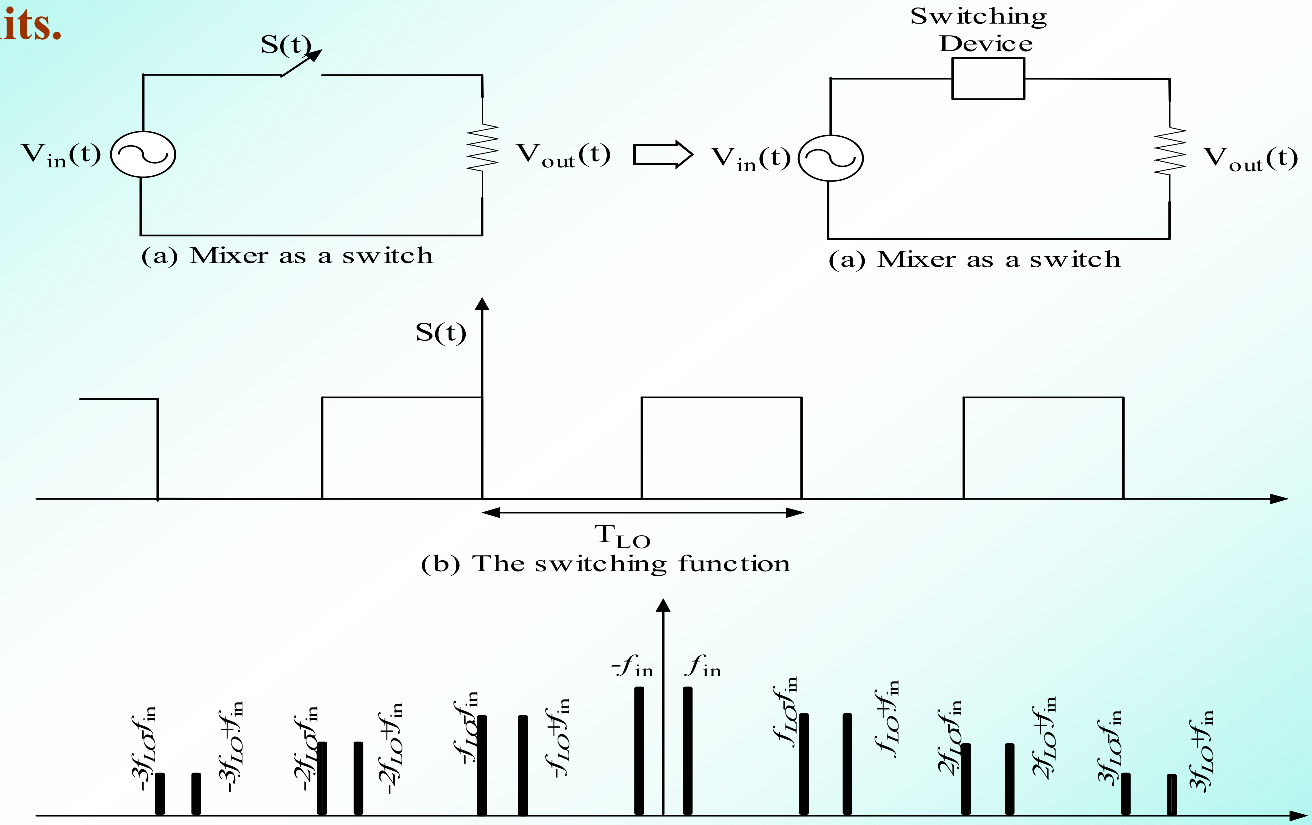
$$V_0(t) = a_0 + a_1 v_{in}(t) + a_2 v_{in}^2(t) + a_3 v_{in}^3(t) + \cdots a_{n-1} v_{in}^{n-1}(t) + a_n v_{in}^n(t)$$

Typical simplified nonlinear mixer circuit operation

Mixer Circuits, Cont'd.

Switching Mixer

The Figure shows the typical switching mechanism of the simplified ideal Mixer circuits.



(a) Mixer as a switch

(a) Mixer as a switch

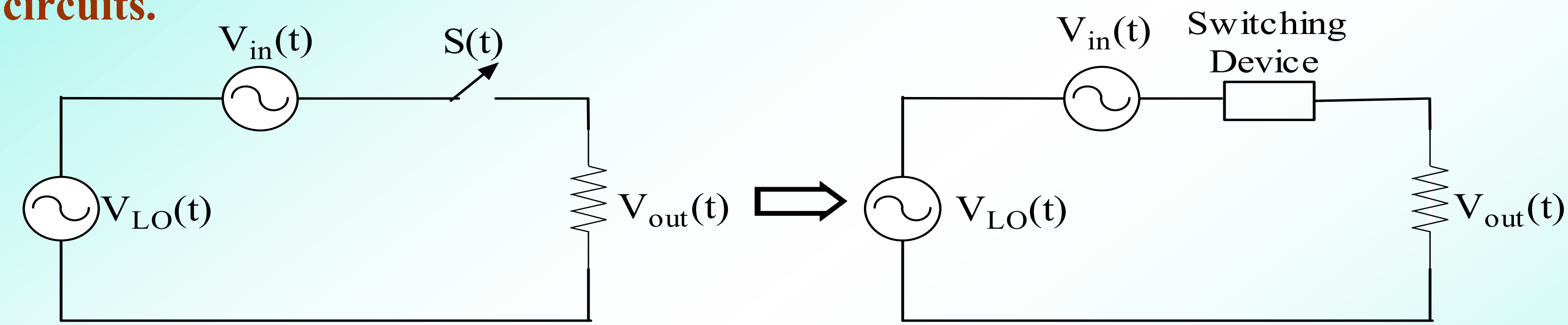
(b) The switching function

(c) The spectrum of the output voltage of an ideal mixer

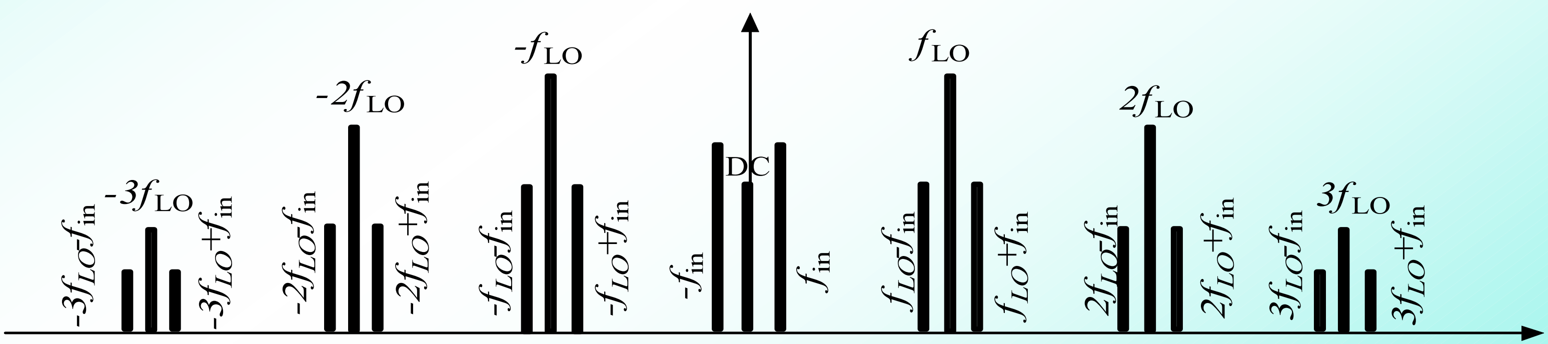
Mixer Circuits, Cont'd.

Switching Mixer

The Figure shows the typical switching mechanism of the simplified real Mixer circuits.



(a) Real Mixer as a switch　　　(a) Real Mixer as a switch

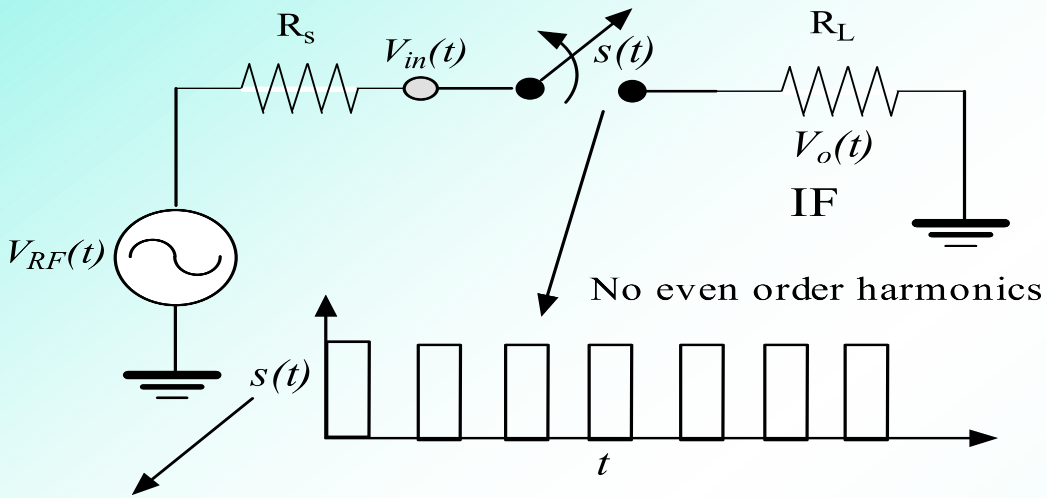


(b) The spectrum of the output voltage of real mixer

Typical simplified equivalent circuit of real mixer

Mixer Circuits, Cont'd.

The Figure shows the simplified version of typical switching mixer circuit.



where switch circuit s(t) is operated by the square wave LO signal with a 50% duty cycle .

$$s(t) = \frac{1}{2} + \frac{2}{\pi}\left[\sin(\omega_{LO}t) + \frac{1}{3}\sin(3\omega_{LO}t) + \frac{1}{5}\sin(5\omega_{LO}t) + \cdots\right]$$

$$V_0(t) = V_{RF}(t)s(t) = \left[\frac{V_R}{2}\cos(\omega_{RF}t)\right]_{RF-Feddthroug\quad h} +$$

$$\frac{2V_R}{\pi}\left[\cos(\omega_{RF}t)\sin(\omega_{LO}t)\right]_{2nd-order-product} +$$

$$\frac{2V_R}{\pi}\left[\frac{1}{3}\cos(\omega_{RF}t)\sin(3\omega_{LO}t)\right]_{4th-order-product} + \cdots$$

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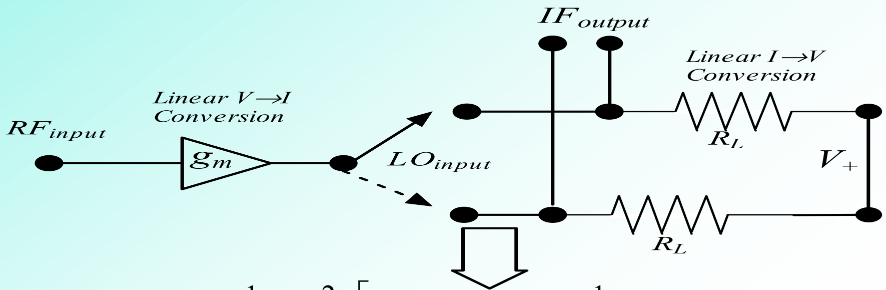
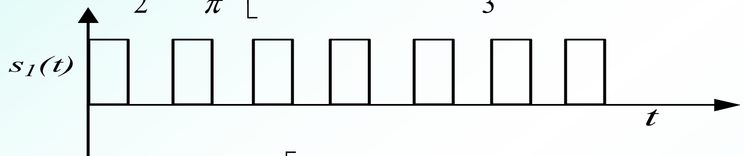
Mixer Circuits, Cont'd.

As depicted in mixer circuit in the Figure, by adding two switching functions $s_1(t)$ and $s_2(t)$, the DC terms (1/2 & -1/2) cancel in $s(t)$. From above, DC term was responsible for RF feedthrough in the unbalanced mixer due to multiplication of cos ($\omega_{RF}t$) term by $s_1(t)$.

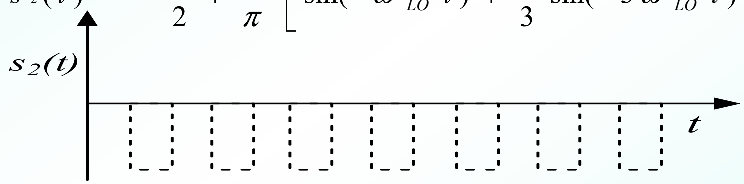


Typical LO switching function with reverse polarity

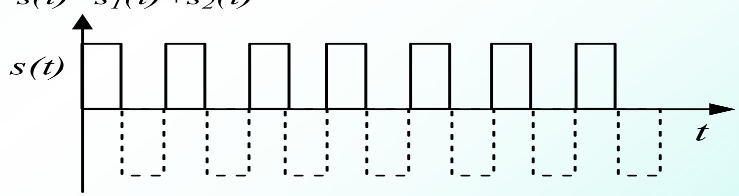
$$s_1(t) = \frac{1}{2} + \frac{2}{\pi}\left[\sin(\omega_{LO}t) + \frac{1}{3}\sin(3\omega_{LO}t) + \cdots\right]$$



$$s_2(t) = -\frac{1}{2} + \frac{2}{\pi}\left[\sin(\omega_{LO}t) + \frac{1}{3}\sin(3\omega_{LO}t) + \cdots\right]$$

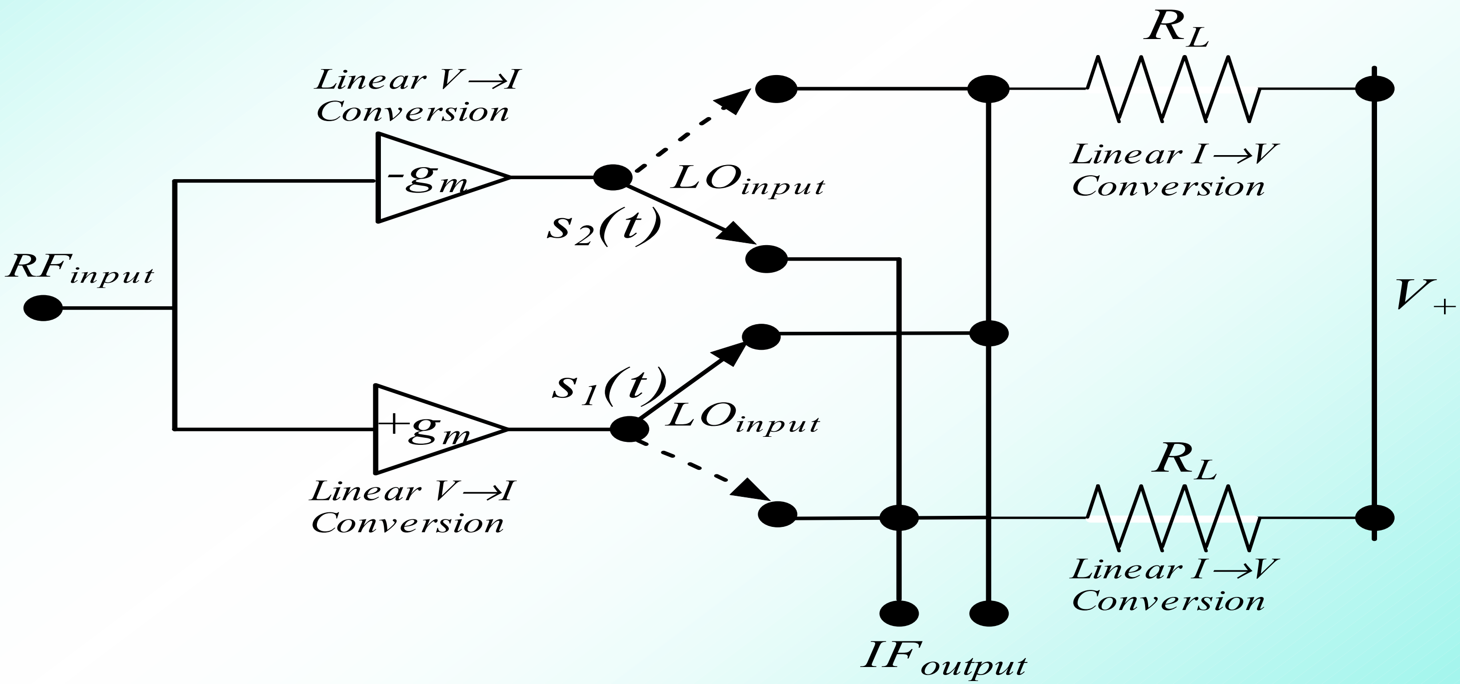


$s(t) = s_1(t) + s_2(t)$



Mixer Circuits, Cont'd.

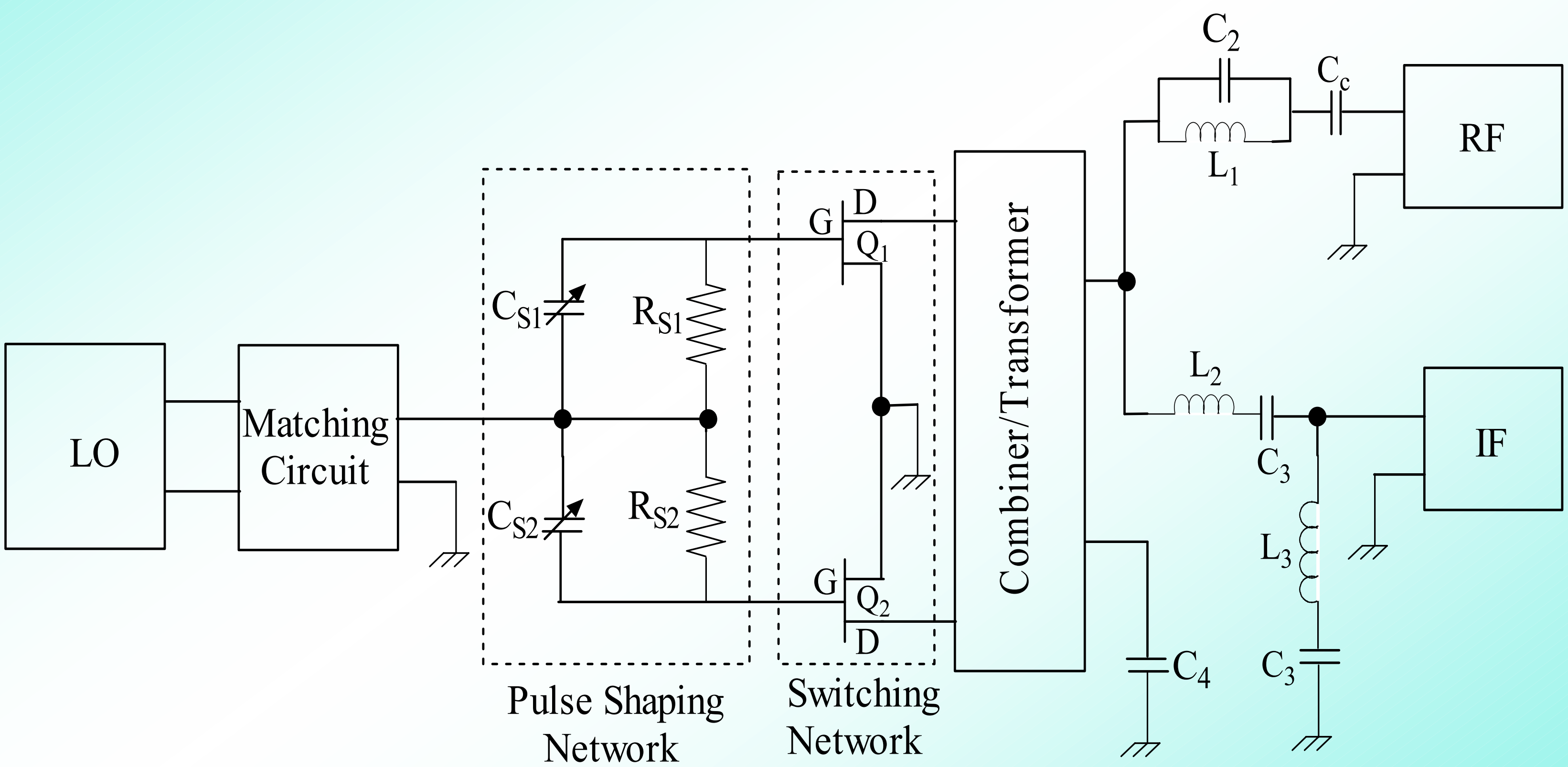
The Figure shows the typical ideal DBM (double balanced mixer), which consists of a switch driven by the LO that reserves the polarity of the RF input at the LO frequency and a differential transconductance amplifier stage. In this case, polarity reversing switch and differential IF cancels any output at the RF input frequency because the DC term cancels as was the case for the single balanced design. The double LO switch cancels out any LO frequency component, even with currents in the RF to IF path.



Typical ideal double balanced switching mixer circuit

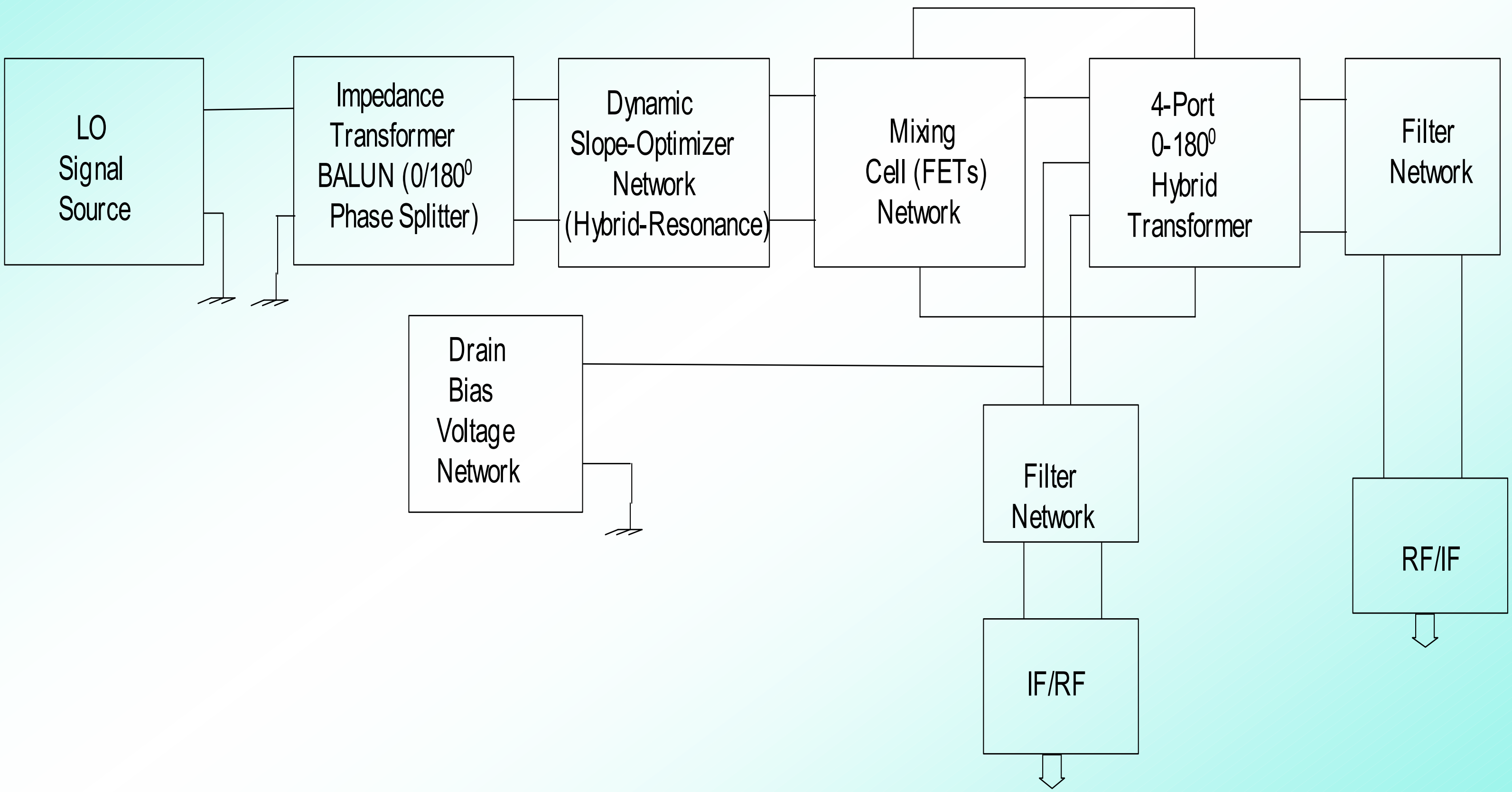
Mixer Circuits, Cont'd.

Typical circuit of passive reflection FET switching Mixer



Mixer Circuits, Cont'd.

Block diagram of high IP3 passive switching FETs Mixer (Patent pending)



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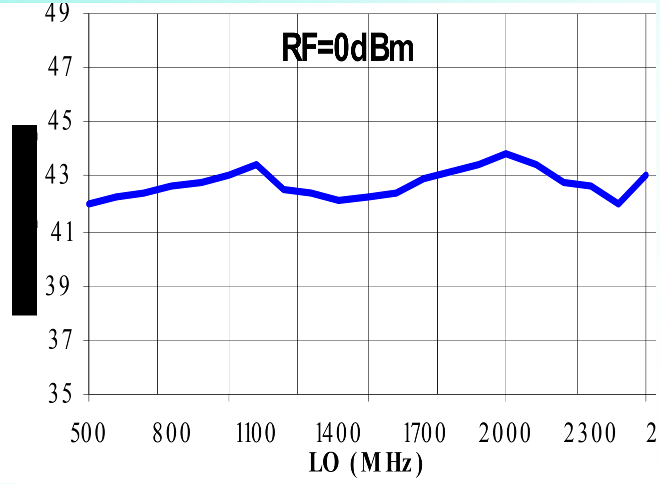
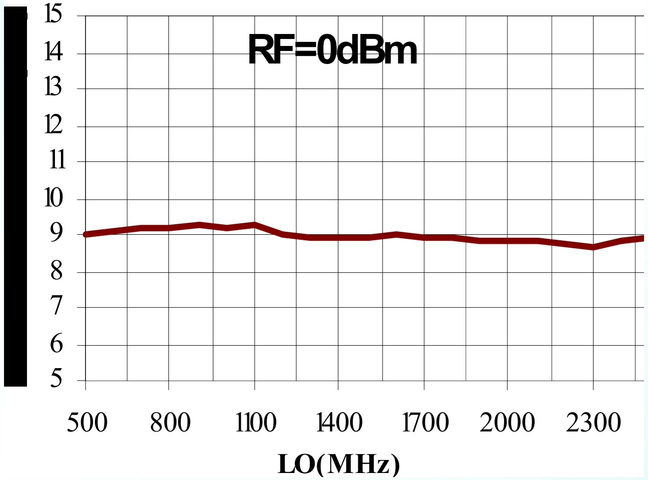
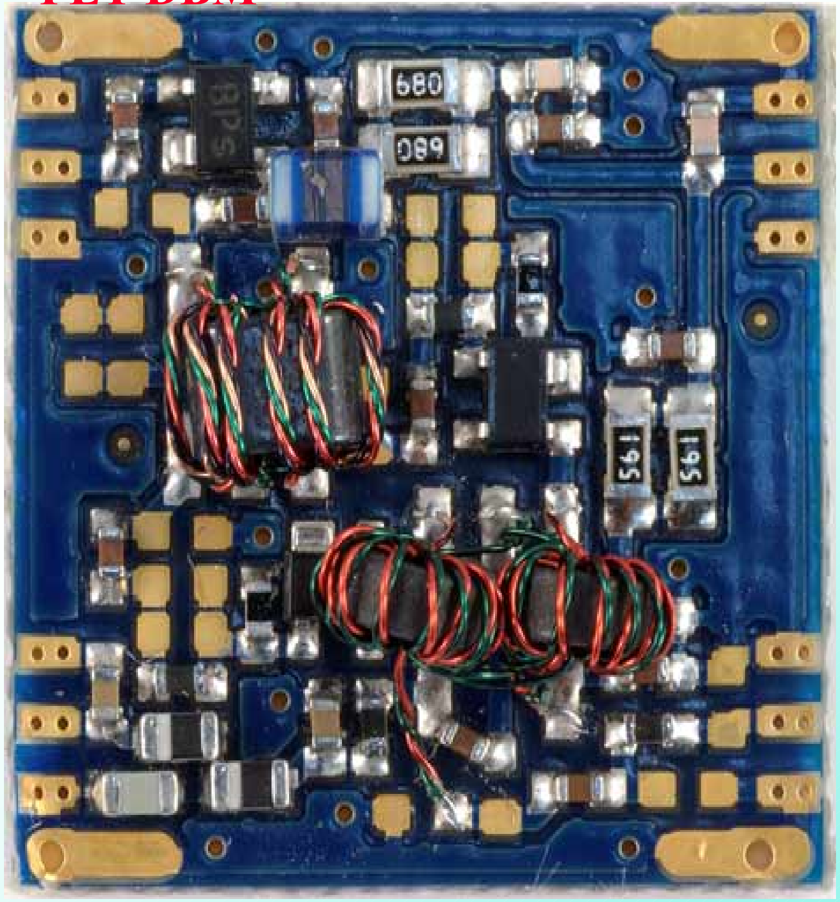
Passive Reflection FET Switching DBM

Experimental data of passive switching FET DBM





Typical Layout of Passive Switching FET DBM



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IMD (Intermodulation) Characterization Techniques

Nonlinear devices do not obey the law of the superposition theorem. This fundamental truth does not allow the use of any set of basis functions as a convenient means for describing their outputs to a general stimulus. So, the system's response to a certain input is as much useful as the input tested is closer to the excitation expected in real operation. But, since it is supposed that the system must handle information signals-which, by definition, are unpredictable-the input representation is a very difficult task.

Indeed, although electrical engineers are used to test their linear systems with sinusoids (a methodology determined by Fourier analysis), now their probing signals should typically approximate band-limited power spectral density (PSD).

The first and simpler approximation we will consider for this PSD is to concentrate all the power distributed in the channel's bandwidth (BW), into a single spectral line, and then to excite the system with that sinusoid. This corresponds to the 1-tone tests, in which fundamental output power and phase versus input power are measured, along with the output at a few of the first harmonics.

IMD (Intermodulation) Characterization Techniques, Cont'd.

Well-behaved nonlinear systems subject to a sinusoidal input excitations will produce Output signal spectral components that are harmonically related to the input frequency, and therefore, 1-tone test is inferior and as a characterization tool of those systems.

For example, no spectral regrowth can be observed in normal narrowband wireless telecommunication systems, and so, no interference can be measured either inside the tested spectral channel-cochannel interference-or in any other closely located channel-adjacent-channel interference.

To overcome the above mentioned difficulty, the 1-tone characterization was replaced by 2-tone test. In this case, the input PSD is represented by 2-tones of equal amplitude and located at the BW extremes, or somewhere in between. Now, although all even-order nonlinear components still constitute out-of-band distortion, there are a large number of odd-order combinations that produce in-band spectral regrowth. As we will explain later, this led to the definition of some of the most widely used nonlinear distortion standard as the intermodulation distortion ratio (IMR), or the third-order intercept point (IP_3).

IMD (Intermodulation) Characterization Techniques, Cont'd.

The main drawback associated with 2-tone test is their difficulty in evaluating cochannel distortion. Actually, since some of the odd-order mixing terms fall exactly at the same frequencies as the fundamentals, and the first-order, or linear, output components have much stronger amplitude than the distortion, there is no possibility of independently measuring cochannel distortion.

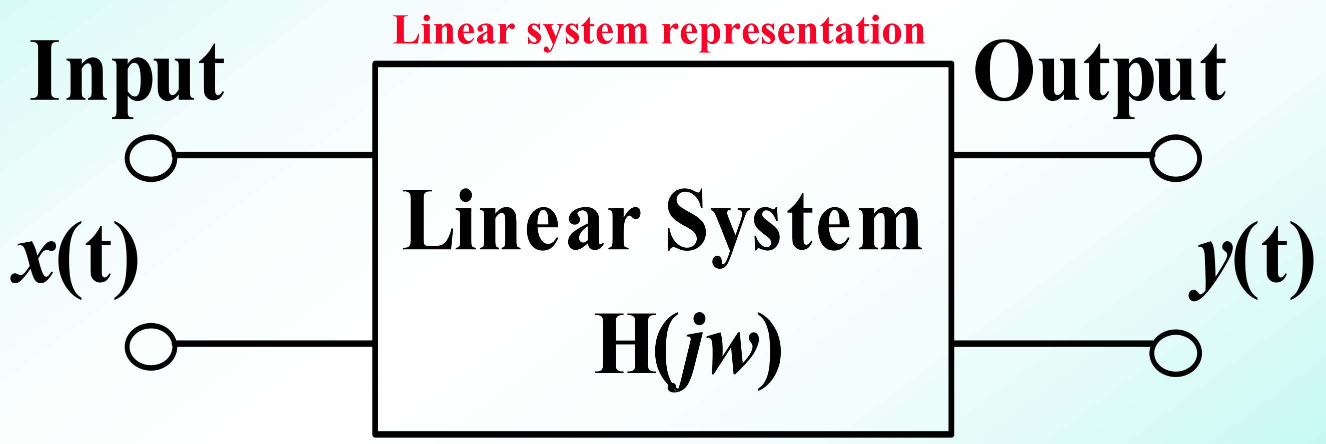
The one way to circumvent the drawbacks of 2-tone was to increase the resolution with which the input PSD is sampled.

Although a multi-channel input excitations stimulus approximation with a restricted number of tones is sometime adopted (as in cable TV system), nonlinear distortion tends to be specified from multi-tone or band-limited noise tests.

IMD (Intermodulation) Characterization Techniques, Cont’d.

Linear System: 1-Tone Characterization Tests

For linear system, a frequency sweep of the input excitations x(t) can only produce output changes in amplitude and phase, and the output can be described by



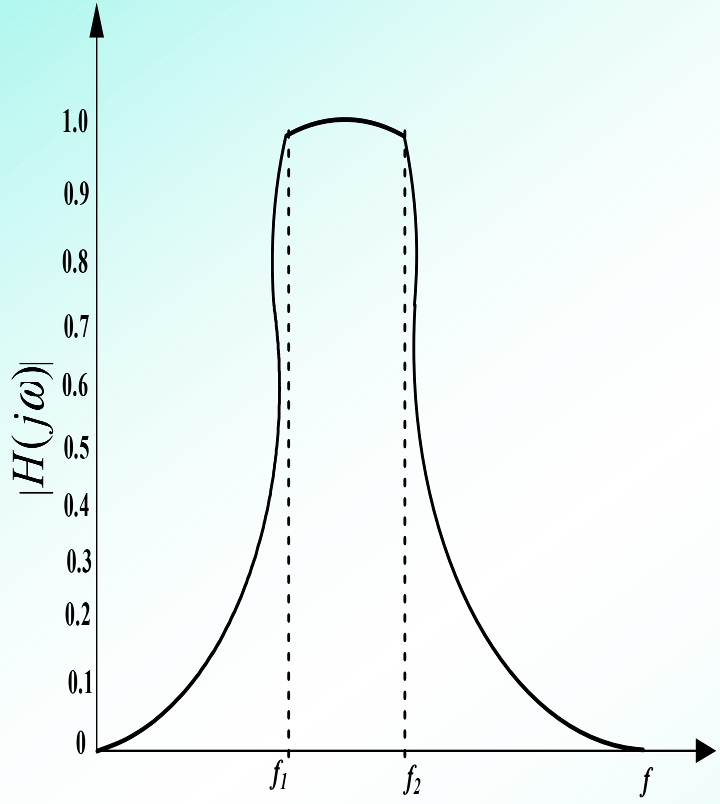
$$x(t) = A_n \cos(\omega t)$$

$$y(t) = A_0(\omega)\cos[\omega t + \varphi_0(\omega)]$$

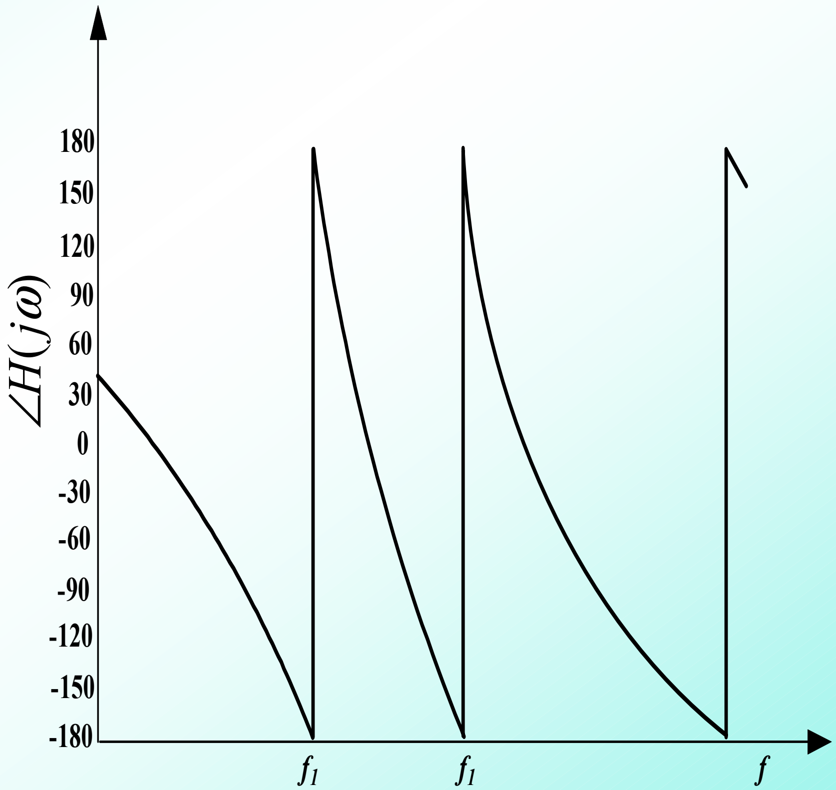
O/P response is at different amplitude and additional phase shift.

IMD (Intermodulation) Characterization Techniques, Cont'd.

Gain and phase characteristics of a linear system:



Gain (Frequency domain transfer characteristic)

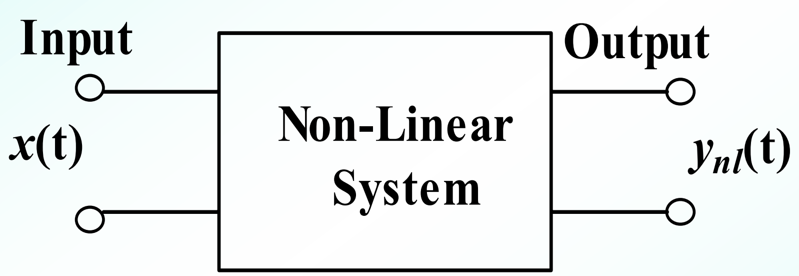


Phase Characteristic

IMD (Intermodulation) Characterization Techniques, Cont'd.

Non-Linear System: One-Tone Characterization Tests:

Although sinusoidal 1-tone excitation for linear system can be directly extended to non-linear system. In the case of a non-linear system, the output amplitude and phase will no longer be a scaled replica of the input level but they will vary nonlinearly with the input excitations. Furthermore, that DUT will also generate additional new frequency components located at the harmonics of the input excitations.

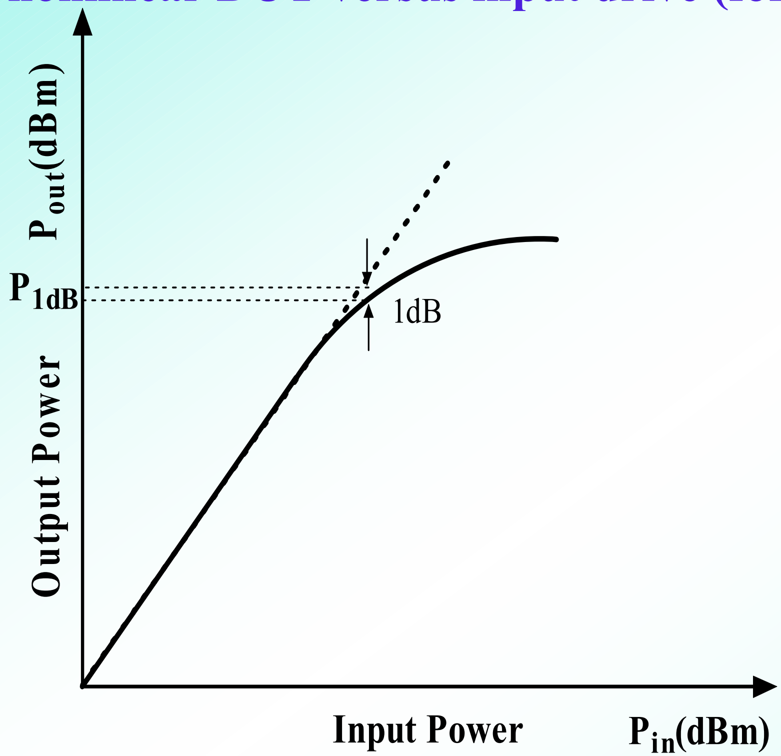


$$x(t) = A_n \cos(\omega t)$$

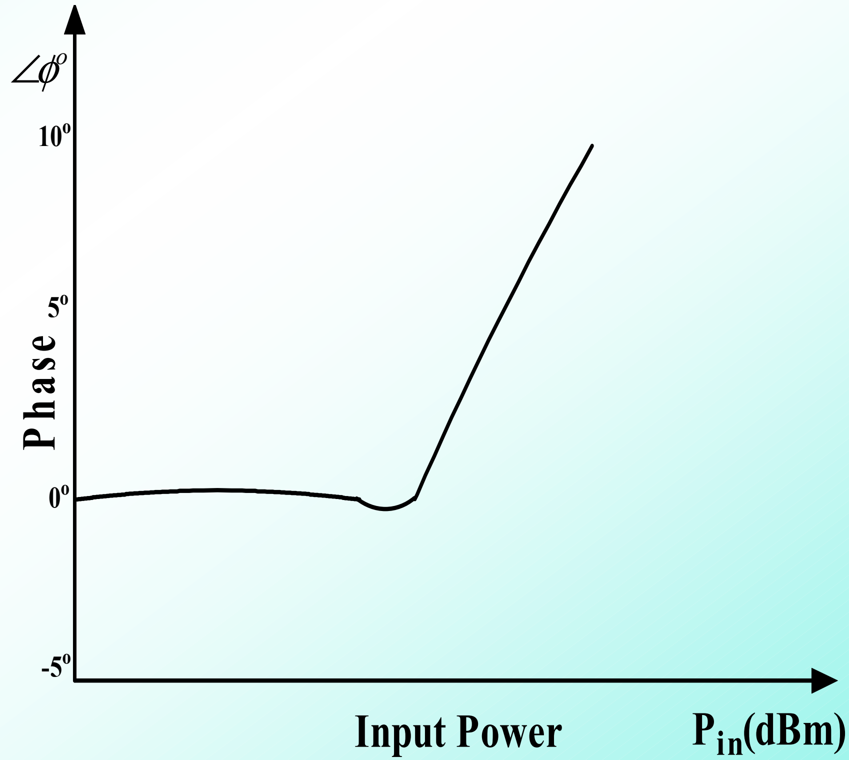
$$y_{nl}(t) = \sum_{m=0}^{\infty} A_{0_m}(\omega, A_n)\cos[m\omega t + \varphi_{0m}(\omega, A_n)]$$

IMD (Intermodulation) Characterization Techniques, Cont'd.

Non-Linear System: Typical amplitude and phase characteristics of a nonlinear DUT versus input drive (for constant frequency); 1-Tone test



Amplitude characteristics



Phase characteristics

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IMD (Intermodulation) Characterization Techniques, Cont'd.

Comments of 1-Tone test for a nonlinear systems characterizations:

The observed output amplitude and phase variation versus drive manifest themselves as if the nonlinear device could convert input amplitude variations into output amplitude and phase changes-or, in other words, as if it could transform possible amplitude modulation (AM) associated to its input, into output amplitude modulation (AM-AM conversion) or phase modulation (AM-PM conversion).

AM-AM conversion is particularly important in system based on amplitude modulation; while AM-PM has its major impact in modern telecommunication and wireless systems that relies on phase modulation formats.

The main application of this type of characterization (1-tone) is the extraction of behavioral models suitable to describe the nonlinear system performance at the excitation envelope. Nevertheless, since this is a static step-by-step characterization, the extracted behavioral models cannot present any memory to these envelopes.

IMD (Intermodulation) Characterization Techniques, Cont'd.

Comments of 1-Tone test for a nonlinear systems characterizations:

The DUT's capable for generating new harmonic components is characterized by the ratio of the integrated power of all the harmonics to the measured power at the fundamental, a figure of merit named total harmonic distortion (THD).

Following three figures of merits (1) AM-AM characterization, (2) AM-PM characterization, and (3) THD can be described based on n^{th} order power series as

$$y_{nl}(t) = \sum_{m=0}^{\infty} A_{0_m}(\omega, A_n)\cos[m\omega t + \varphi_{0m}(\omega, A_n)]$$

$$\Rightarrow y_{nl}(t) = k_1 x(t-\tau_1) + k_2 x(t-\tau_2)^2 + k_3 x(t-\tau_3)^3 + k_4 x(t-\tau_4)^4 + \cdots k_n x(t-\tau_n)^n + \cdots$$

IMD (Intermodulation) Characterization Techniques, Cont'd.

AM-AM characterizations:

AM-AM characterization describes the relation between the output amplitude of the fundamental frequency, m=1, with the input amplitude of a fixed input frequency.

$$y_{nl}(t) = \sum_{m=0}^{\infty} A_{0_3}(\omega, A_n)\cos[3\omega t + \varphi_{03}(\omega, A_n)]$$

AM-AM characterization is sometime expressed as a certain dB/dB deviation at a predetermined input power. It characterizes gain compression a nonlinear device versus input drive level.

AM-AM characterization enables the evaluation of 1-dB compression point, $P_{1\text{-}Db}$, which is defined as the output power level at which the signal output is compressed by 1dB, as compared to the output that would be obtained by simply extrapolating the linear system's small-signal characteristic.

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IMD (Intermodulation) Characterization Techniques, Cont'd.

AM-PM characterizations:

The vector addition of the output fundamental with distortion components in presence of varying input excitation signals leads to phase variation in the resultant output,which is defined as the AM-PM characteristics of non-linear system.

AM-PM characterization consists of studying the variation of the output signal phase, $\varphi_{01}(\omega, A_i)$, with input signal amplitude changes for a constant frequency, and may be expressed as a certain phase deviation, in degrees/dB, at a predetermined input power.

AM-AM and AM-PM characterizations are performed reading output signal components whose frequency is equal to the input excitation. Therefore, usual amplitude controlled sinusoidal-or continuous-wave (CW)-generator connected to a vector network analyzer are sufficient for these tasks. AM-AM behavior would be visible whether or not the system presented memory effects, But AM-PM is exclusive of dynamic systems.

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IMD (Intermodulation) Characterization Techniques, Cont'd.

Total Harmonic Distortion Characterization (THD): THD is defined as the ratio between the square roots of total harmonic output power and output power at the fundamental signal, and can be described by

$$THD = \frac{\sqrt{\frac{1}{T}\int_0^T \left[\sum_{m=2}^{\infty} A_{0m}(\omega, A_n)\cos[m\omega t + \varphi_{0m}(\omega, A_n)\right]^2 dt}}{\sqrt{\frac{1}{T}\int_0^T \left[A_{01}(\omega, A_n)\cos[\omega t + \varphi_{01}(\omega, A_n)\right]^2 dt}}$$

$$THD = \frac{\sqrt{\frac{1}{8}k_2^2 A_n^4 + \frac{1}{32}k_3^2 A_n^6 + \cdots}}{\sqrt{\frac{1}{2}k_1^2 A_n^2}} = \frac{1}{2}\frac{A_n}{k_1}\sqrt{k_2^2 + \frac{1}{4}k_3^2 A_n^2 + \cdots}$$

THD characterization can only be performed with a spectrum analyzer, as the measured output includes frequency components that are different from the input excitation.

IMD (Intermodulation) Characterization Techniques, Cont'd.

Non-Linear System: 2-Tone Characterization Tests:

A true representation of signal excitations is 2-tone stimulus than the pure sinusoid signals. Similarly to the 1-tone tests, this type of signal allows the characterization of generated harmonics-which, in band pass systems, are usually attenuated by the output matching networks-but it also enables the identification of new mixing component close to the fundamentals. These inband components play a dominant role in band pass systems, as they constitute the main sources of nonlinear distortion impairments. The output response of the 2-tone input excitations can be described by

$$x(t) = x_1(t) + x_2(t)$$

$$x(t) = A_{n1}(\omega)\cos(\omega_1 t) + A_{n2}(\omega)\cos(\omega_2 t)$$

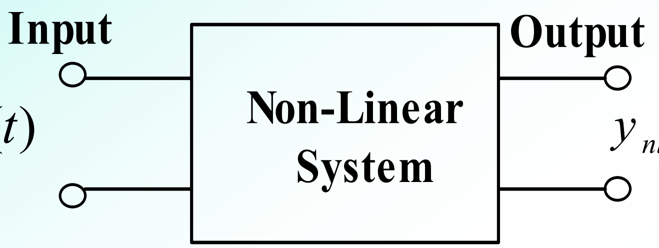
$$y_{nl}(t) = \sum_{m=1}^{\infty} A_{0m}\cos(\omega_m t + \varphi_{0m})$$

where $\omega_m = p\omega 1 + q\omega 2 \ and \ p, q \in Z$

IMD (Intermodulation) Characterization Techniques, Cont'd.

Non-Linear System: 2-Tone Characterization Tests:

$$x(t) = x_1(t) + x_2(t)$$



$$y_{nl}(t) = \sum_{m=1}^{\infty} A_{0m} \cos(\omega_m t + \varphi_{0m})$$

where $\omega_m = p\omega 1 + q\omega 2$ *and* $p, q \in Z$

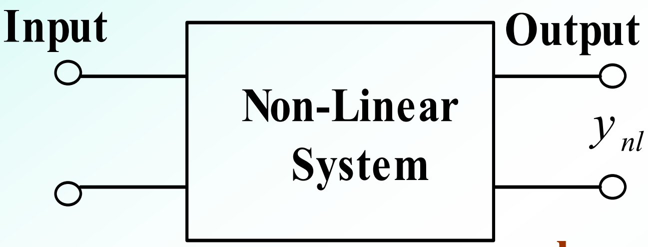
The output y_{nl}(t) would be composed of a very large number of mixing terms involving all possible combinations of $\pm\omega_1$ *and* $\pm\omega_2$. Referring to a usual narrowband RF subsystem, as the ones found in wireless transmission channels, two types of information can be expected from a 2-tone test: the so-called inband distortion measurements, in which p +q =1, and the out-of-band component's evaluation, where p +q ≠1.

Inband distortion products are the mixing components falling exactly over, or very close to, the output fundamental frequencies. Therefore, inband distortion frequencies will be those satisfying *p* +*q* =1.

IMD (Intermodulation) Characterization Techniques, Cont'd.

Non-Linear System: 2-Tone Characterization: Intermodulation Distortion (IMD)

$$x(t) = x_1(t) + x_2(t)$$



$$y_{nl}(t) = \sum_{m=1}^{\infty} A_{0m} \cos(\omega_m t + \varphi_{0m})$$

where $\omega_m = p\omega 1 + q\omega 2$ *and* $p, q \in Z$

Inband measurement performed at the fundamental frequencies: ω_1, ω_2; **third-order components** $(|p| + |q| = 3)$ **at:** $3\omega_1$-ω_2, $2\omega_2$-ω_1; **fifth-order components** $(|p| + |q| = 5)$ **at:** $3\omega_1$-$2\omega_2$, $3\omega_2$-$2\omega_1$; **seventh-order components** $(|p| + |q| = 7)$ **at:** $4\omega_1$-$3\omega_2$, $4\omega_2$-$3\omega_1$; **and so forth. These distortion products constitute a group of lower and upper sidebands, separated from the signals and from each other by the tone's frequency difference** ω_2-ω_1; **they are known as IMD.**

IMD (Intermodulation) Characterization Techniques, Cont'd.

Non-Linear System: 2-Tone Characterization: Intermodulation Ratio (IMR)

IMR is defined as the ratio between the fundamental and IMD output power as

$$IMR \equiv \frac{P_{fund}}{P_{IMD}} = \frac{P(\omega_1)}{P(2\omega_1 - \omega_2)} = \frac{P(\omega_2)}{P(2\omega_2 - \omega_1)}$$

Thus, third-order IMD output power at one of the sidebands (e.g., at $2\omega_1$-ω_2) will be given by

$$P_{IMD}(2\omega_1 - \omega_2) = \frac{1}{T_{(2\omega_1-\omega_2)}} \int_0^{T_{(2\omega_1-\omega_2)}} \left(\frac{3}{4} k_3 A_m^3 \cos[(2\omega_1 - \omega_2)t - \varphi_{32-1}] \right)^2 dt = \frac{9}{32} k_3^2 A_m^6$$

while the linear output power at ω_1 will be

$$P_{Linear}(\omega_1) = \frac{1}{T_{\omega_1}} \int_0^{T_{\omega_1}} (k_1 A_m \cos[\omega_1 t - \varphi_{110}])^2 dt = \frac{1}{2} k_1^2 A_m^2$$

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IMD (Intermodulation) Characterization Techniques, Cont'd.

Non-Linear System: 2-Tone Characterization: Intermodulation Ratio (IMR)

Now, applying the definition of IP_3, which is the extrapolated linear output power of one of the fundamentals that equals the extrapolated power of the considered third-order IMD sideband, we would get

$$\frac{1}{2}k_1^2 A_m^2 = \frac{9}{32}k_3^2 A_m^6 \Rightarrow \frac{4}{3}\frac{k_1^3}{k_3}$$

and thus, substituting this A_m^2 into $P(\omega_1)$,

$$IP_3 = P(\omega_1) = \frac{2}{3}\frac{k_m^2}{k_3}$$

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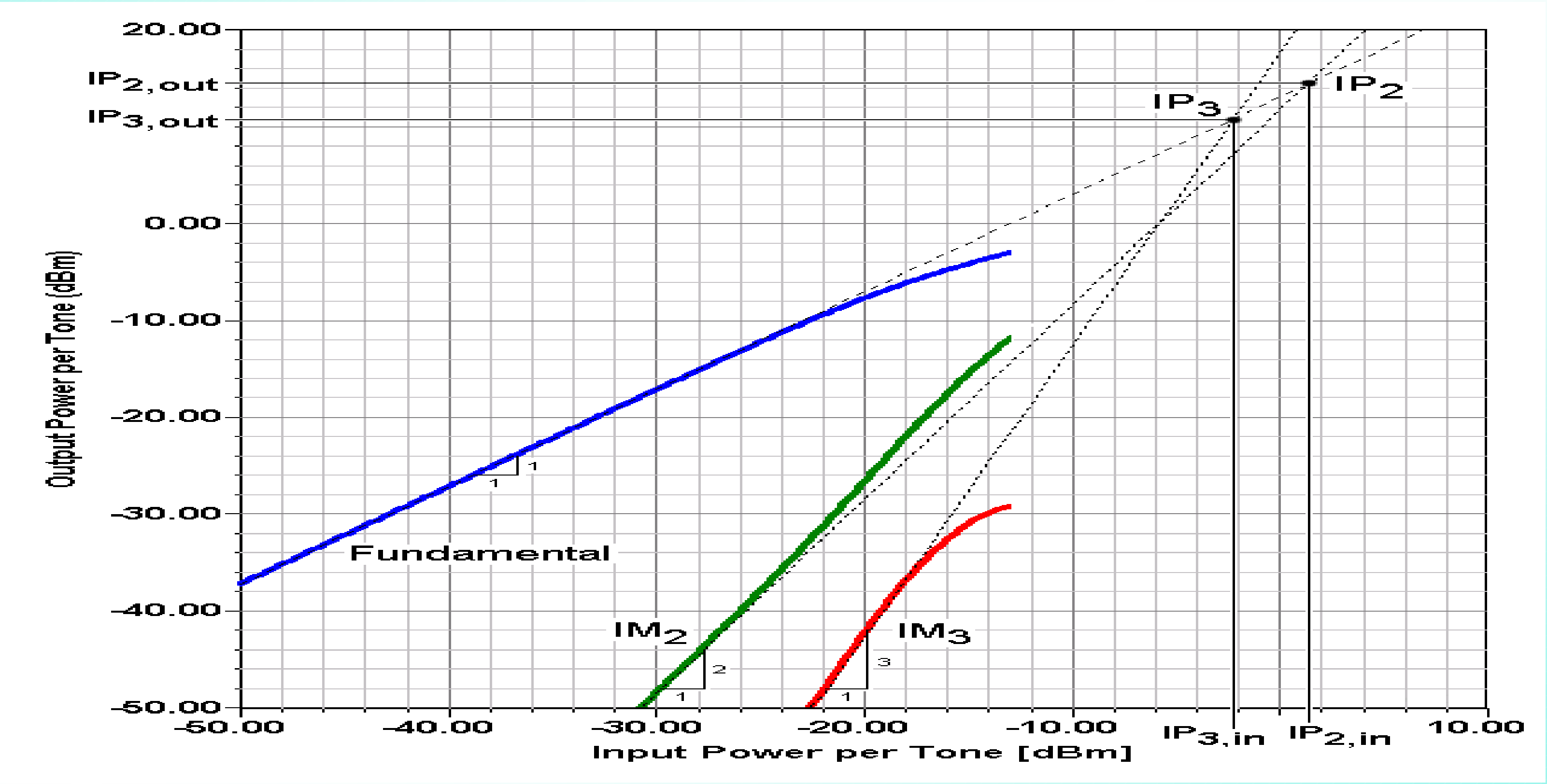
IMD (Intermodulation) Characterization Techniques, Cont'd.

Non-Linear System: 2-Tone Characterization: **Typical logarithmic plot of the fundamental output power at one of the fundamental signals and the IMD power measured in one of the distortion sidebands, versus input power.**



IMD (Intermodulation) Characterization Techniques, Cont'd.

Non-Linear System: 2-Tone Characterization:

At sufficiently small-levels, the fundamental output power increase 1dB for each decibel rise of input power, while a 3dB per decibel rate is noticed for IMD power. This is dictated by the dominance of polynomial power series system model's first and third-degree terms. However, at very large-signal levels, where the contribution of the higher order terms is no longer negligible, both curves tend to compare towards constant fundamental and IMD output power values. This behavior, common to the large majority of microwave and wireless systems, enables the definition of a very important figure of merit for characterizing the IMD in nonlinear devices: the third-order intercept point IP_3.

IP_3 is a fictitious point that is obtained when the extrapolated 3-dB/dB slope line of IMD power. Since IP_3 is determined by the system's third-order distortion behavior, it cannot be used for IMD characterization unless it is guaranteed that no large signal effects are involved. In other words, and contrary to a loose practice seen in various product specifications and sometimes even in scientific publications, IP_3 can only be extrapolated from the small-signal zone where IMD presents a distinct and constant 3-dB/dB slope.

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IMD (Intermodulation) Characterization Techniques, Cont'd.

Non-Linear System: 2-Tone Characterization: Out-of-Band Distortion Characterization

The out-of-band components are the mixing products that obey $p+q \neq 1$ for harmonics of each of the fundamentals, like in the one-tone case, but also new mixing products at $p\omega_1+q\omega_2$ that fall, either near dc (p + q=0), or close to the various harmonics (p + q =2, 3, 4,..).

$$y_{nl}(t) = \sum_{m=1}^{\infty} A_{0m} \cos(\omega_m t + \varphi_{0m})$$ where $\omega_m = p\omega 1 + q\omega 2$ *and* $p, q \in Z$

Table illustrates out-of-band products generated by a third-order nonlinearity subject to a two-tone excitation.

Mixing product order	dc	$\omega_2-\omega_1$	$2\omega_1$	$\omega_2+\omega_1$	$2\omega_2$	$3\omega_1$	$2\omega_1+\omega_2$	$2\omega_2+\omega_1$	$3\omega_2$
	2nd	2nd	2nd	2nd	2nd	3rd	3rd	3rd	3rd

IMD (Intermodulation) Characterization Techniques, Cont'd.

Non-Linear System: 2-Tone Characterization: Out-of-Band Distortion Characterization

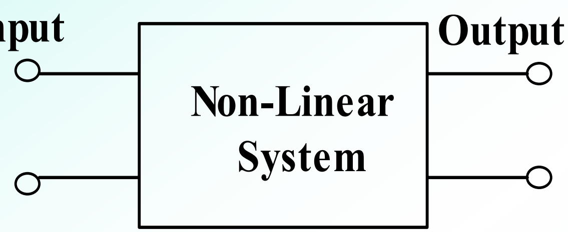
Comments:

The product located at dc describes the bias point shift from the quiescent point, when input driving level increases. Then, the one at ω_2-ω_1 is usually called the base band. The reason for this designation comes from the fact that if the two-tone excitation were considered as a carrier at $(\omega_1+\omega_2)/2$, amplitude modulated in double-sideband format (suppressed carrier) by a baseband modulating signal. According to what was defined for the inband distortion components, these out-of-band components can be also described by corresponding intercept points. As their name indicates, out-of-band components appear at zones of the output spectrum quite far from the fundamental signals. So, rigorously speaking, they are only out-of-band in narrowband systems, but not in multi-octave ones. Furthermore, because they become relatively simple to be filtered out in narrowband systems, their importance, as transmission quality impairment, is only evident on ultrawideband applications.

IMD (Intermodulation) Characterization Techniques, Cont'd.

Non-Linear System: Multi-Tone Characterization

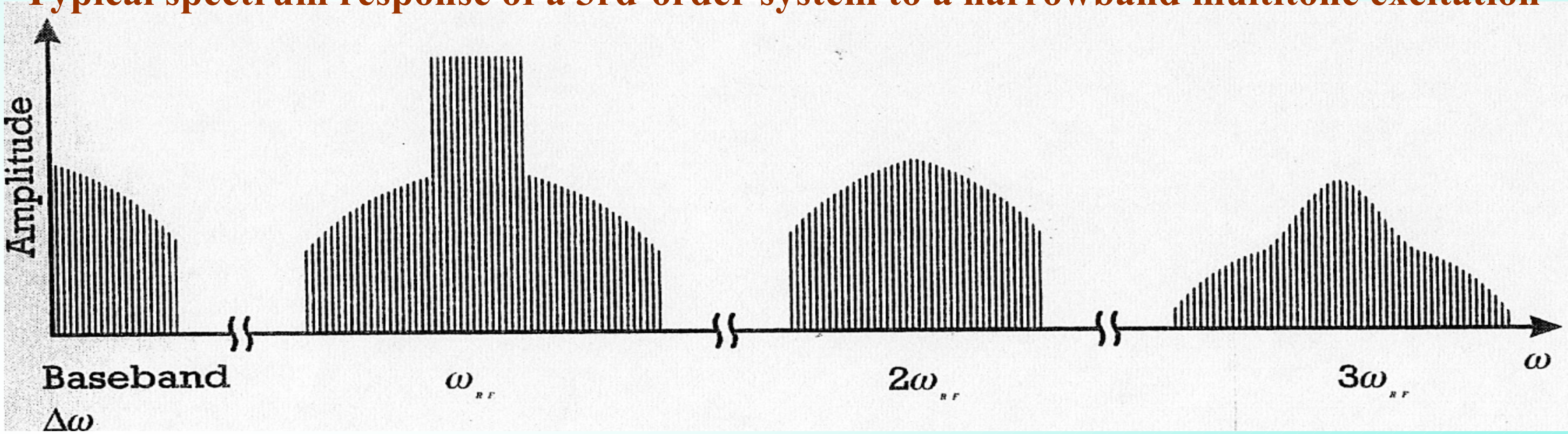
$x(t) = x_1(t) + x_2(t) + ...$



$y_{nl}(t) = \sum_{m=1}^{M} A_m \cos(\omega_m t + \varphi_m)$

where $\omega_m = p_0\omega_0 + \ldots p_q\omega_q + p_{Q-1}\omega_{Q-1}$, $|p_0| + \ldots |p_q| + \ldots |p_{Q-1}| \leq R$, $\omega_{0,\ldots}\ \omega_{q,\ldots}\ \omega_{Q-1}$, are the input frequencies and R is the maximum order of the mixing product under consideration.

Typical spectrum response of a 3rd-order system to a narrowband multitone excitation



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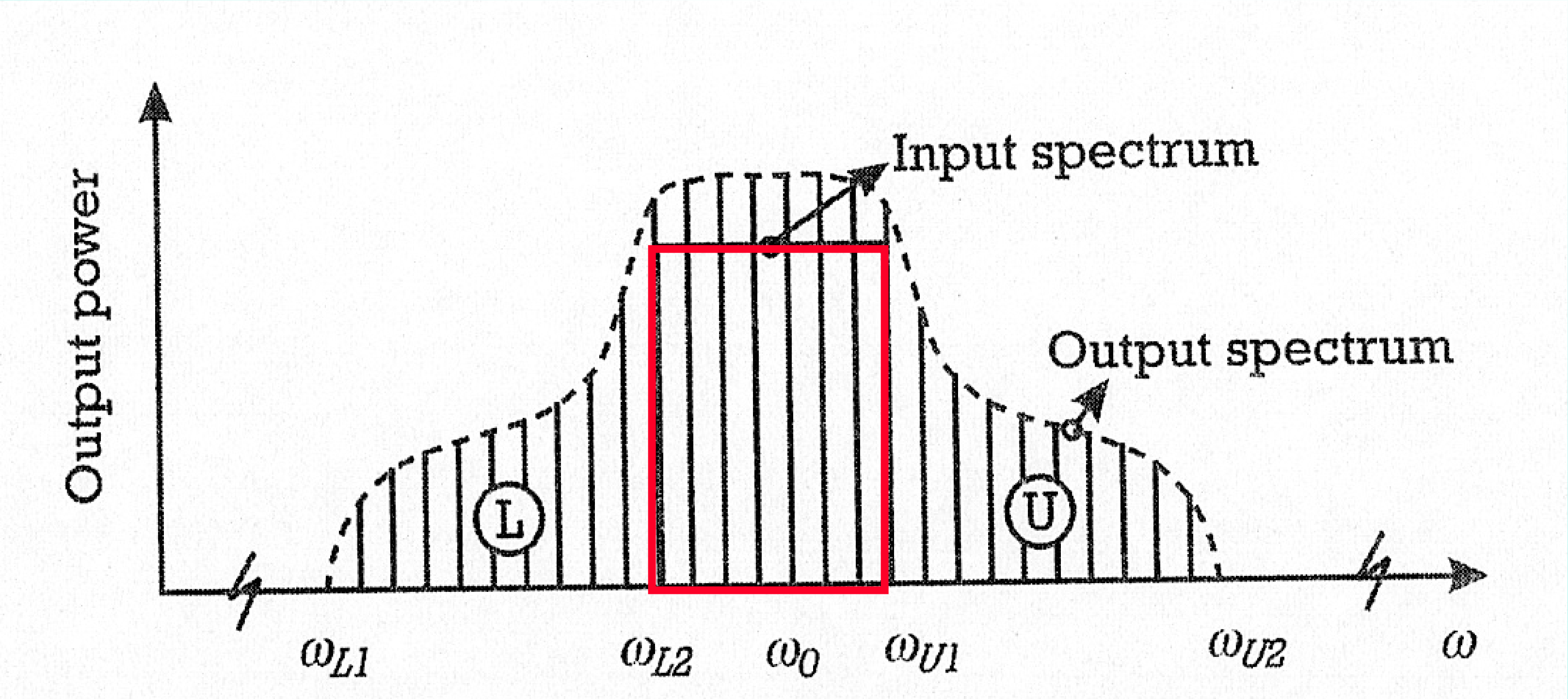
IMD (Intermodulation) Characterization Techniques, Cont'd.

Non-Linear System: Multi-Tone Characterization

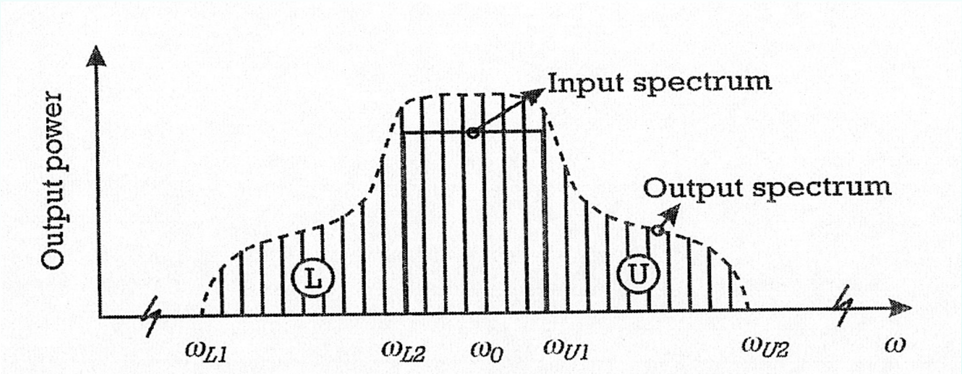
Typical input and inband output spectra observed in a system excited by a narrowband multitone input excitations



IMD (Intermodulation) Characterization Techniques, Cont'd.

Non-Linear System: Multi-Tone Characterization; Comments:

Comparing the input and output spectra shown in Figure, it is clear that the output contains many more frequency components. These are usually named spectral regrowth, because they are a consequence of the property of nonlinear systems in generating, or "growing," new frequency lines. In general, whenever the input tones are not evenly spaced this spectral regrowth includes not only the components adjacent to the signal, as seen in above Figure, but also new mixing products located among the fundamentals, but not coincident with them. The first type of spectral regrowth components is the adjacent-channel distortion [or alternate-channel distortion, in case the mixing product is located at distance greater than (Q-1) $\Delta\omega$ and lesser than 2(Q-1) $\Delta\omega$, from the input tone of highest or lowest frequency], whereas the second type constitutes cochannel distortion.



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Noise and Gain in Circuits/Systems

System Specifications and Their Relationship to Circuit Design:

Wireless communication involves a large range of signal powers—from levels on the order of 10^{-18} watt (at the receiver input) to 10^{2} watts (at a base-station transmitter output). A receiver must be able to demodulate signals that have been attenuated billions of time through propagation; a transmitter must be able to produce a properly modulated signal at a frequency suitable for propagation, at a level high enough to overcome worst-case propagation losses and provide a useful signal at the receiver.

Gain is therefore an essential attribute of wireless systems. Because no single active device can provide all the gain required for transmission or reception, we must distribute the gain among multiple stages, designing each for optimum performance across the power span it bridges.

Two inescapable realities impose limits on the gain and absolute power output we may achieve with a given circuit: All real electrical and electronic networks generate noise to some degree, and all real electrical and electronic networks distort the signals applied to them to some degree. A signal weaker than a circuit's inherent noise cannot be amplified by that circuit because it remains indistinguishable from the noise. A signal that exceeds the power-handling capability of the circuit to which it is applied may be degraded, even rendered useless, by the resulting distortion.

Noise and Gain in Circuits/Systems, Cont'd.

System Noise and Noise Floor :

Assuming that a system's gain is sufficient, the weakest signal that may be processed satisfactorily, a figure of merit referred to as noise floor or (in receivers) minimum detectable signal (MDS), is limited by thermal noise, assumed to be equal to the noise Power available from a resistor at 290 K (about 17 °C or 62 °F), an arbitrary reference value near standard room temperature.

The noise power is given by $P_n = kTB$ where P_n is the noise power, k is Boltzmann's constant (1.38×10^{-23} watts per kelvin), T is the temperature in kelvins, and B is the bandwidth (in Hertz) in which the noise appears. For T = 290, P_n is therefore 4.00×10^{-21} watts, or –174 dBm in a 1-Hz bandwidth.

Increasing B to a value suitable for digital communications, such as 160 kHz for a GSM system, admits more noise to the network, raising the minimum noise power against which an incoming signal must compete to –122 dBm. If the noise figure and bandwidth are known, the system noise floor can be calculated using the equation: Noise floor = –174 dBm + NF + 10 log B . The trouble with this equation is that the "integrated" bandwidth depends so much on the selectivity shape factor, which is not always known.

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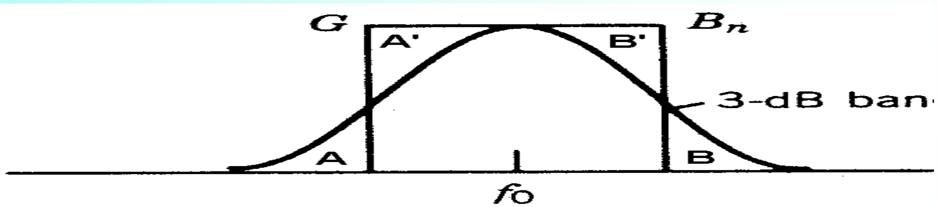
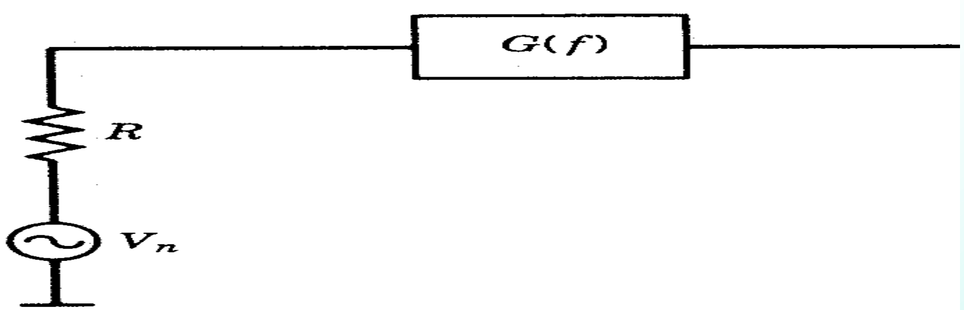
Noise and Gain in Circuits/Systems, Cont'd.

Figure shows the translation of the bandwidth of a single tuned circuit with its Gaussian shape into its rectangular equivalent. The transformation is done by sizing the rectangle such that A′ = A and B′ = B; when this is true, the area of the rectangular equals the area under the curve.



$$G(f) = \left| \frac{V_O(f)}{V_n(f)} \right|^2$$



$$V_o^2 = \int_0^\infty 4kT_O R G(f)\, dF$$
$$= 4kT_O R \int_0^\infty G(f)\, dF$$
$$B_n = \frac{1}{G} \int_0^\infty G(f)\, dF$$
B_n = noise bandwidth

Graphical and mathematical explanation of the noise bandwidth from a comparison of the Gaussian-shaped integrated bandwidth to the rectangular filter response

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Noise and Gain in Circuits/Systems, Cont'd.

Signal-to-Noise Ratio (S/N, SNR) and Sensitivity:

Successful radio communication depends on the achievement of a specified minimum ratio of signal power to noise power, expressed in decibels, at the output of the receiver. The input voltage, expressed in absolute units or decibels relative to a microvolt (dBμV), necessary to achieve a particular signal-to-noise ratio in a particular bandwidth may be specified as a figure of merit called *sensitivity*. Because techniques used to measure S/N actually measure the ratio of signal-*plus-noise* to noise, specifications may refer to (or imply) S+N/N or (S+N)/N rather than S/N. The difference between (S+N)/N and S/N becomes negligible at high ratios of signal to noise; even at an SNR of 10dB—*a common value—the difference is only 0.46 dB*.

Most receivers are designed to operate optimally when connected to an antenna system of a specified impedance (commonly 50 Ω), but relatively few receivers exhibit this design load impedance at their input terminals; that is, they are not designed for a conjugate input match. It is therefore customary to specify sensitivity in terms of "open circuit" voltage—the signal voltage that, with the receiver's antenna input terminated in its design antenna impedance, results in the desired ratio of signal to noise.

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Noise and Gain in Circuits/Systems, Cont'd.

Signal-to-Noise Ratio (S/N, SNR) and Sensitivity:

The input voltage for a given SNR is determined using instrumentation calibrated in terms of *closed*-circuit voltage--that is, in terms of voltage across a load resistance equal to the instrument's source resistance—the voltage indicated will be 1/2 the open-circuit value for the SNR specified.

By convention, the open-circuit measurement condition is indicated by a sensitivity specification in volts of electromotive force (EMF).

Specifying sensitivity in terms of available signal power (usually decibels relative to 1 mW, or dBm) eliminates this open/closed-circuit confusion.

Noise and Gain in Circuits/Systems, Cont'd.

Signal-Plus-Noise-and-Distortion (SINAD):

Extending the measurement of signal-plus-noise to noise to include distortion results in a figure of merit called *SINAD* (signal-plus-noise-and-distortion), commonly applied to FM receivers:

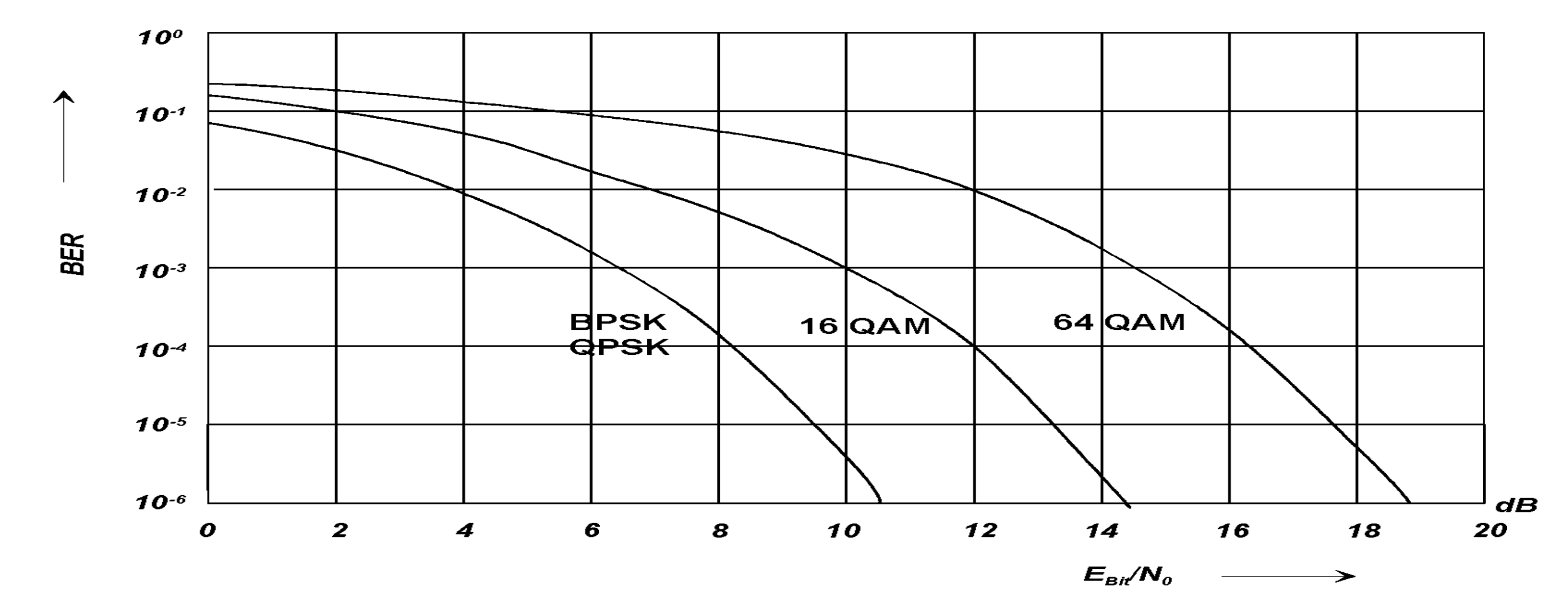
$$\mathrm{SINAD} = 10\ \log_{10} \frac{S+N+D}{N+D}$$

where SINAD is in decibels, *S* is signal power, *N* is noise power, and *D* is distortion power. At a SINAD ratio of 12 dB—a common specification—the noise-and-distortion power is 25% that of the desired signal. As is true of (S+N)/N, SINAD closely approximates S/N at high ratios of signal to noise.

Noise and Gain in Circuits/Systems, Cont'd.

Bit Error Rate and Noise:

For digital systems, signal-to-noise ratio and bit error rate are related. As introduced earlier, depending on the waveform, coding, and filtering, different BERs are related to particular SNRs. Bit error rate versus E_{bit}/N_0 for BPSK/QPSK, 16-QAM and 64-QAM, showing the significantly greater SNR necessary for a given BER as the number of signal states is increased.



Noise and Gain in Circuits/Systems, Cont'd.

Noise Factor and Noise Figure:

The degree to which a network's noise contribution degrades the noise floor is evaluated by its *noise factor (F)*, which is expressed as the ratio

$$F = \frac{N_{in} + N_{added}}{N_{in}}$$

where F is noise factor, N_{in} is the noise power available from the source and N_{added} is the noise power added by the network, with both powers determined in the same bandwidth.

Expressing this ratio in decibels ($10 \log_{10} F$), returns *noise figure (NF)*, a bandwidth independent figure of merit of great value in evaluating the noise performance of networks and communication systems. We can express NF as the ratio of the network's input SNR to its output SNR:

$$NF = 10 \, \log_{10}\left[\frac{(S_{\text{in}} / N_{\text{in}})}{(S_{\text{out}} / N_{\text{out}})}\right]$$

where NF is noise figure in decibels, S is signal power and N is noise power, with the input and output values of these quantities signified by the subscripts and all powers determined in the same bandwidth. The noise figure of an ideal noiseless network is 0 dB; for all real, noisy networks, NF is positive. The NF of a lossy passive device is equal to its insertion loss.

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Noise and Gain in Circuits/Systems, Cont'd.

Antenna's Noise Figure:

Noise figure for antenna systems can be described by

$$NF_{ant} = 10 \log_{10} \frac{N_t + N_{ant}}{N_t}$$

where NF_{ant} is the antenna system noise figure in decibels, N_t is the antenna's system's thermal noise power, and N_{ant} is the total noise power picked up by the antenna system.

From the lower end of the radio spectrum, and decreasingly up to approximately 400 MHz, noise intercepted by an antenna system from atmospheric, man-made and galactic sources will dominate NF_{ant} , and N_t can be considered as equivalent to the noise power of a resistor at 290 K.

Atmospheric noise subsides above 40 MHz; from this region to perhaps 700 MHz, N_t is till largely negligible, with noise from man-made and/or sky sources largely determining an antenna's noise figure.

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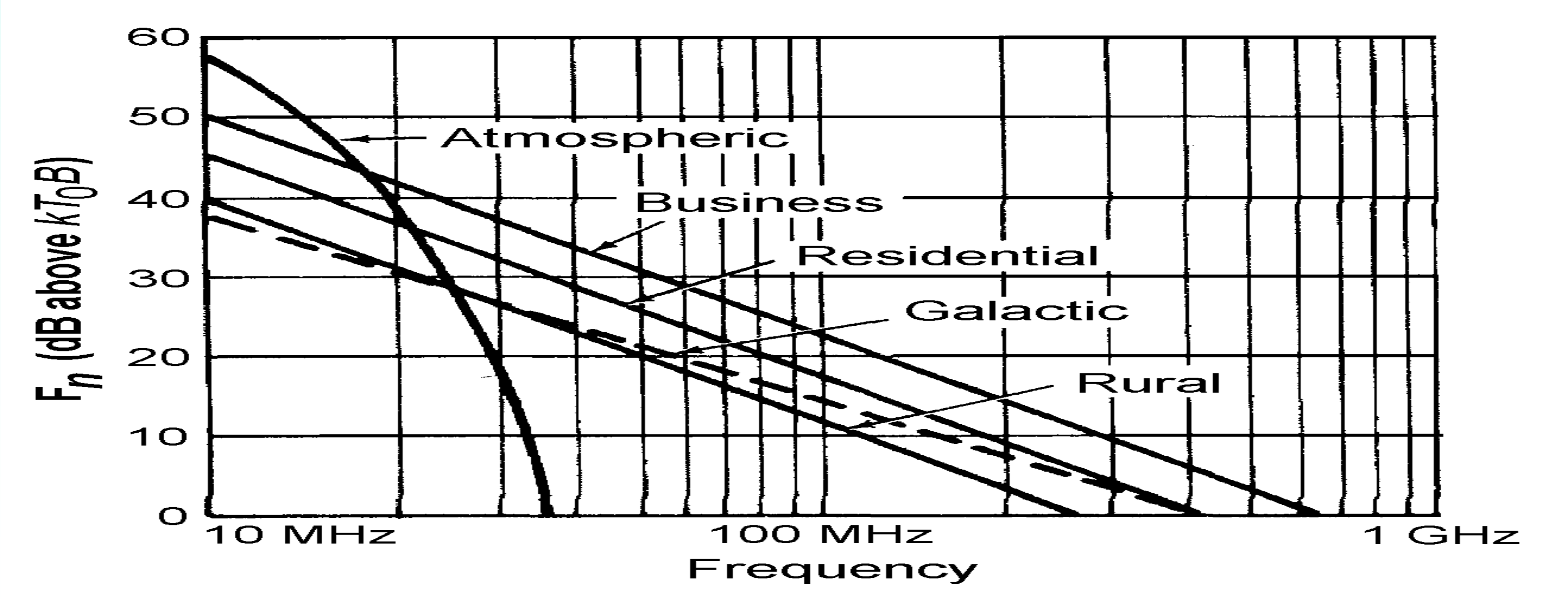
Noise and Gain in Circuits/Systems, Cont'd.

Antenna's Noise Figure:

Atmospheric noise subsides above 40 MHz; from this region to perhaps 700 MHz, N_t is still largely negligible, with noise from man-made and/or sky sources largely determining an antenna's noise figure.

The higher the frequency, the quieter the RF environment becomes, although the noise profile of specific sources may contradict this general rule.



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Noise and Gain in Circuits/Systems, Cont'd.

Noise Figure of Cascaded Networks:

The noise figure of two networks in cascade may be determined from

$$NF_{total} = 10\log_{10}\left(F_1 + \frac{F_2 - 1}{G_1}\right)$$

where NF is noise figure in dB, F_1 is the noise factor of the first network, F_2 is the noise factor of the second network, and G_1 is the gain (as a numerical ratio, *not* in dB).

The noise figure of a system with more than two stages can be evaluated through repeated iterations.

Note that above equation assumes two conditions: (1) that F_1 and F_2 are determined in the same bandwidth, and (2) that the networks' input and output terminations are resistive--a condition that is commonly *not* true of RF amplifiers optimized for lowest noise.

Noise and Gain in Circuits/Systems, Cont'd.

System Amplitude and Phase Behavior

If we could build electronic systems that were absolutely amplitude-and phase-linear, radiocommunication system design would be greatly simplified. An amplifier designed for a power gain of 10 dB, for example, would merely increase the magnitude of all signals at its input by a factor of 10, regardless of their frequencies, while perfectly maintaining their relative phases. But all real electrical and electronic networks, even those designed (or supposed) to be amplitude- and phase-linear, exhibit amplitude and phase nonlinearity to some degree, just as they all generate noise to some degree.

The effects of amplitude nonlinearity, generically referred to as *nonlinear distortion*, include the generation, through *harmonic distortion* and/or *intermodulation distortion* (IMD), of output signals at frequencies not present at a system's input. Nonlinear distortion also results in *gain compression*--changes in system gain with changes in input-signal level. By convention, when workers in electronics refer to or consider a network's "linearity," they usually mean its *amplitude* linearity; likewise, by "distortion" they usually mean *nonlinear* distortion.

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Noise and Gain in Circuits/Systems, Cont'd.

System Amplitude and Phase Behavior

The effects of phase or frequency nonlinearity are generically referred to as *linear distortion* because they occur independently of signal amplitude and polarity. We often intentionally apply linear distortion through *filtering*, which modifies the amplitude relationships among existing spectral components of a signal without creating any new frequencies.

Another linear-distortion effect, *phase* or *delay distortion (group delay)*, results in the delay of signals of differing frequencies by differing amounts of time. In a system where signal phase conveys information, as is true of most wireless links, phase distortion can seriously degrade communication.

The fact that all real (high Q, band-limiting, ripple) networks are amplitude-and angle-nonlinear to some degree means that all real networks modify the amplitude and angle characteristics of the signals they handle. What is perhaps less obvious is that subjecting a signal to amplitude and angle nonlinearities causes "crosstalk" between its amplitude and angle characteristics. For example, through a nonlinear distortion effect called AM-to-PM conversion, changes in a signal's amplitude result in changes in its phase.

Noise and Gain in Circuits/Systems, Cont'd.

Gain Compression:

Gain compression occurs when a network cannot increase its output amplitude in linear proportion to an amplitude increase at its input; gain *saturation* occurs when a network's output amplitude stops increasing (in practice, it may actually decrease) with increases in input amplitude. The output response of the typical nonlinear circuit can be described by

$$y = k_1 f(x) + k_2[f(x)]^2 + k_3[f(x)]^3 + \text{higher-order terms}$$

where y represents the output, the coefficients k_n represent complex quantities whose values can be determined by an analysis of the output waveforms, and $f(x)$ represents the input. For many practical purposes, the first three terms adequately describe such a network's nonlinearity

$$y = k_1 f(x) + k_2\left[f(x)\right]^2 + k_3\left[f(x)\right]^3$$

Noise and Gain in Circuits/Systems, Cont'd.

Gain Compression:

The system output response 'y' presence of input excitations $f(x) = A_1 \cos\omega_1 t + A_2 \cos\omega_2 t$ **as**

$$y = k_1(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t) + k_2(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^2 + k_3(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^3$$

$$= k_1(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)$$

$$+ k_2\left[A_1^2 \frac{1+\cos 2\omega_1 t}{2} + A_2^2 \frac{1+\cos 2\omega_2 t}{2} + A_1 A_2 \frac{\cos(\omega_1+\omega_2)t + \cos(\omega_1-\omega_2)t}{2}\right]$$

$$+ k_3\left\{\begin{array}{l}\left[A_1^3\left(\frac{\cos\omega_1 t}{2} + \frac{\cos\omega_1 t}{4} + \frac{\cos 3\omega_1 t}{4}\right) + A_2^3\left(\frac{3\cos\omega_2 t}{4} + \frac{\cos 3\omega_2 t}{4}\right)\right] \\ + A_1^2 A_2\left[\frac{3}{2}\cos\omega_2 t + \frac{3}{4}\cos(2\omega_1-\omega_2)t + \frac{3}{4}\cos(2\omega_1-\omega_2)t\right] \\ + A_2^2 A_1\left[\frac{3}{2}\cos\omega_1 t + \frac{3}{4}\cos(2\omega_2+\omega_1)t + \frac{3}{4}\cos(2\omega_2-\omega_1)t\right]\end{array}\right\}$$

The second- and third-order terms represent the effects of harmonic distortion and intermodulation distortion. Second-order effects include second-harmonic distortion (the production of new signals at $2\omega_1$ **and** $2\omega_2$ **) and IMD (the production of new signals at** $\omega_1 + \omega_2$ **and** $\omega_1 - \omega_2$ **). Third-order effects include gain compression, third-harmonic distortion (the production of new signals at** $3\omega_1$ **and** $3\omega_2$ **), and IMD (the production of new signals at** $2\omega_1 \pm \omega_2$ **and** $2\omega_2 \pm \omega_1$ **).**

Noise and Gain in Circuits/Systems, Cont'd.

Gain Compression, Blocking or Desensitization:

When the amplitude of the $\cos\omega_1 t$ signal become $A_1' = k_1 A_1 + k_3\left(\frac{3}{4} A_1^3 + \frac{3}{2} A_1 A_2^2\right)$ gain compression occurs. Because k_3 will normally be negative, a large signal can effectively mask a smaller signal $A_1 \cos\omega_1 t$ by reducing the network's gain.

This third-order effect, known as *blocking* or *desensitization* when it occurs in a receiver, is a special case of gain compression. The presence of additional signals results a greater reduction in gain; the gain reduction for each signal is a function of the relative levels of all signals present.

A receiver's blocking behavior may be characterized in terms of the level of off-channel signal necessary to reduce the strength of an in-passband signal by a specified value, typically 1 dB; alternatively, the decibel ratio of the off-channel signal's power to the receiver's noise-floor power may be cited as *blocking dynamic range*. Desensitization may be also characterized in terms of the off-channel-signal power necessary to degrade a system's SNR by a specified value.

Noise and Gain in Circuits/Systems, Cont'd.

1-dB Gain Compression Point:

Multiple signals need not be present for gain compression to occur. If only one signal is present, the ratio of gain with distortion to the network's idealized (linear) gain is given by

$$A_1' = \frac{k_1 + k_3\left(\frac{3}{4}A_1^2\right)}{k_1}$$

This is referred to as the *single-tone gain-compression factor*.

The point at which a network's power gain is down 1 dB from the ideal for a single signal is a figure of merit known as the *1-dB compression point* (P_{-1dB}).

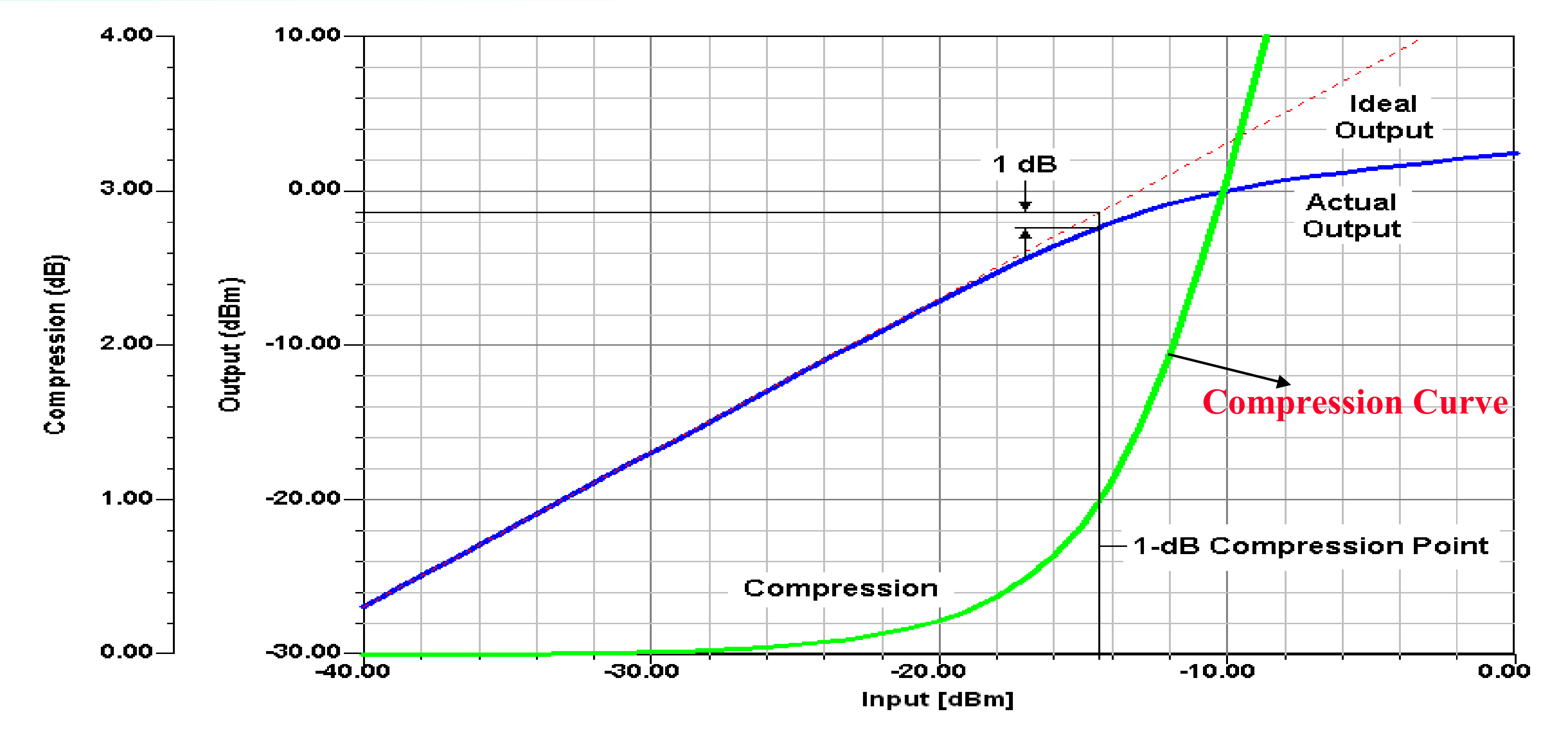
Many networks (including many receiving and low-level transmitting circuits, such as low-noise amplifiers, mixers and IF amplifiers) are usually operated under small-signal conditions--at levels sufficiently below P_{-1dB} to maintain high linearity.

However, some networks (including power amplifiers for wireless systems) may be operated under large-signal conditions--near or in compression--to achieve optimum efficiency at some specified level of linearity.

Noise and Gain in Circuits/Systems, Cont'd.

1-dB Gain Compression Point:

The 1-dB compression point can be expressed relative to input power ($P_{-1dB,in}$) or output power ($P_{-1dB,out}$). For the amplifier simulated here, $P_{-1dB,in} \approx -14.5$ dBm and $P_{-1dB,out} \approx -1.3$ dBm.

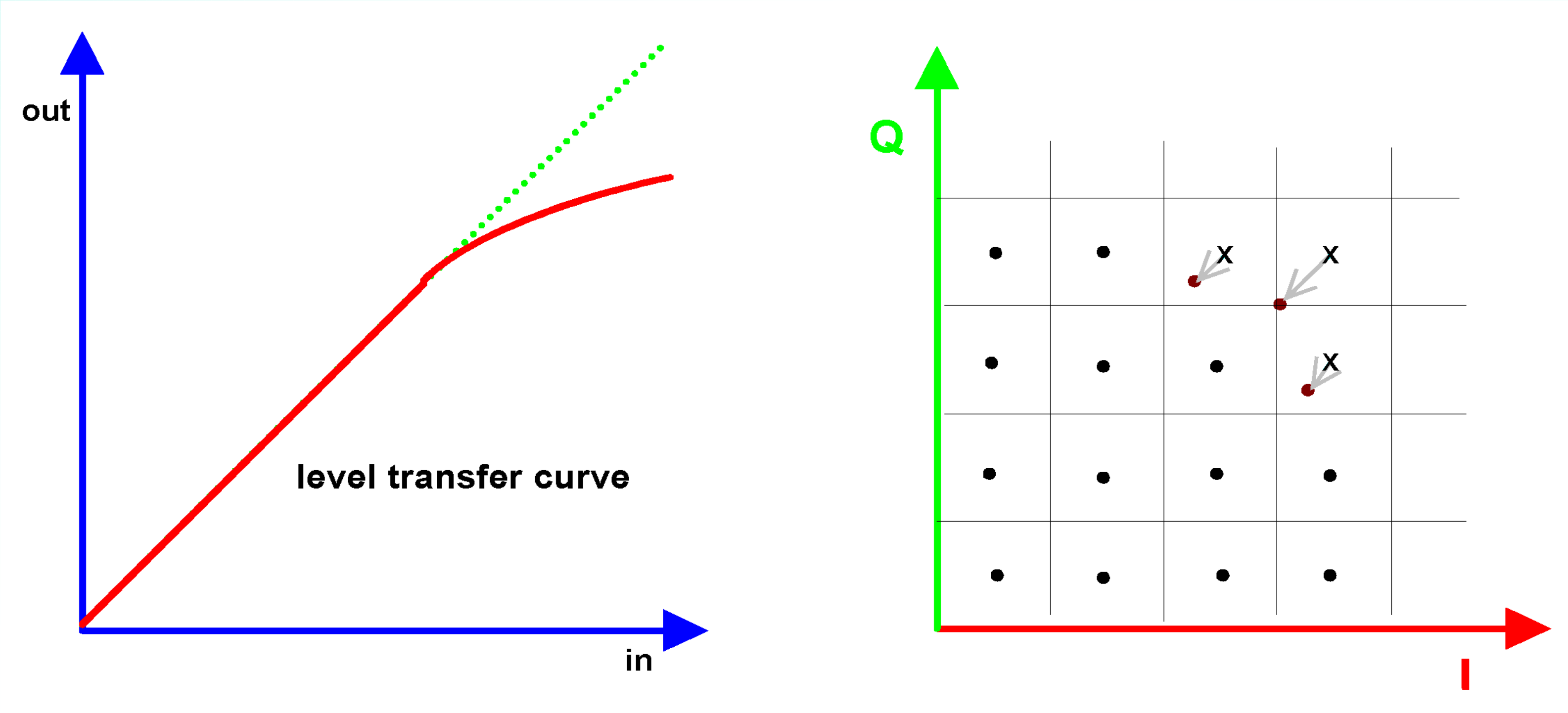


Noise and Gain in Circuits/Systems, Cont'd.

Amplitude Compression:

Figure below shows what happens when a digital emission that uses amplitude to convey information is subjected to amplitude compression.

Influence of differential amplitude error (compression) on a QAM constellation

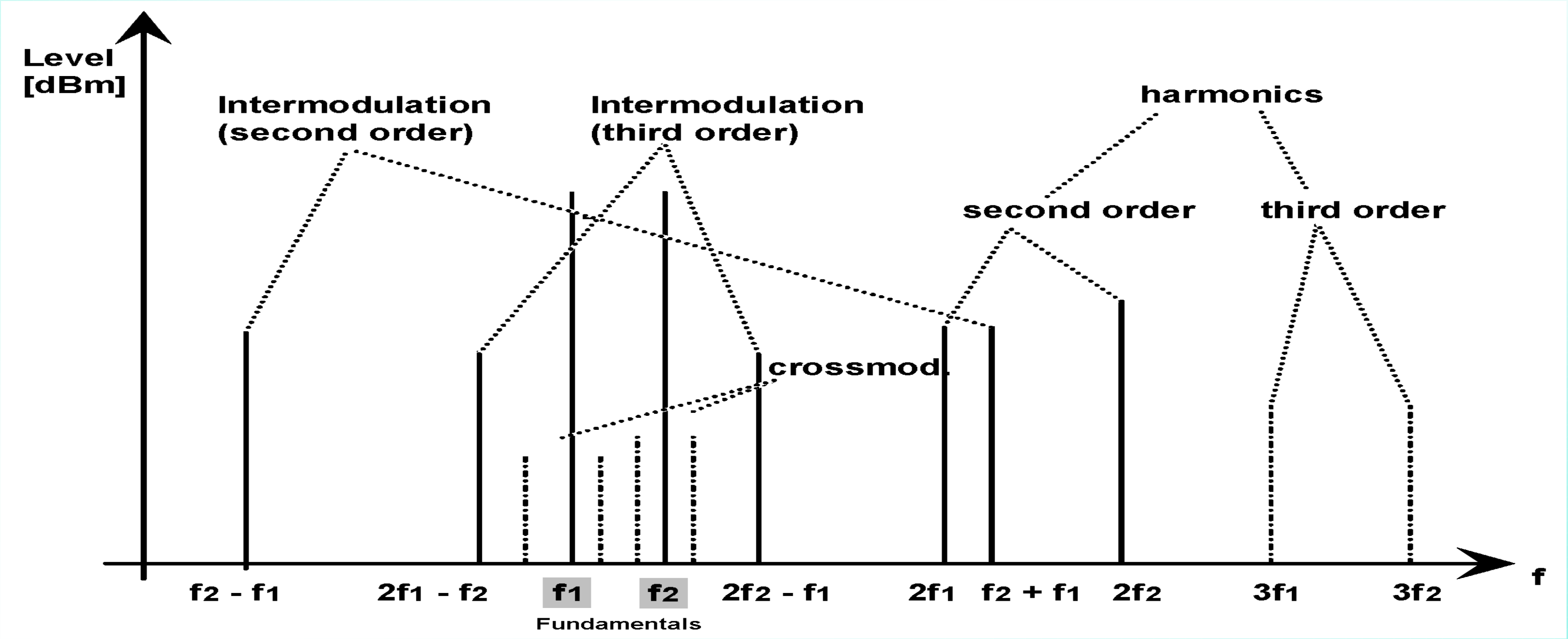


Noise and Gain in Circuits/Systems, Cont'd.

Intermodulation:

The new signals produced through intermodulation distortion (IMD) can profoundly affect the performance even of systems operated far below gain compression.

Figure below shows relationships between fundamental and spurious signals, including harmonics and products of intermodulation.



Noise and Gain in Circuits/Systems, Cont'd.

Intermodulation:

The intercept point for a given IM order *n* can be expressed, and should always be characterized, relative to input power ($IP_{n,\mathrm{in}}$) or output power ($IP_{n,\mathrm{out}}$); the IP_{in} and IP_{out} values differ by the network's linear gain. For equal-level test tones, $IP_{n,in}$ can be determined by:

$$IP_{n,in} = \frac{nP_A - P_{IM_n}}{n-1}$$

where *n* is the order, P_A is the input power (of one tone), P_{IMn} is the power of the IM product, and *IP* is the intercept point. The intercept point for cascaded networks can be determined from

$$IP_{2,in} = \frac{1}{\left(\frac{1}{\sqrt{IP1}} + \frac{G}{\sqrt{IP2}}\right)^2}$$

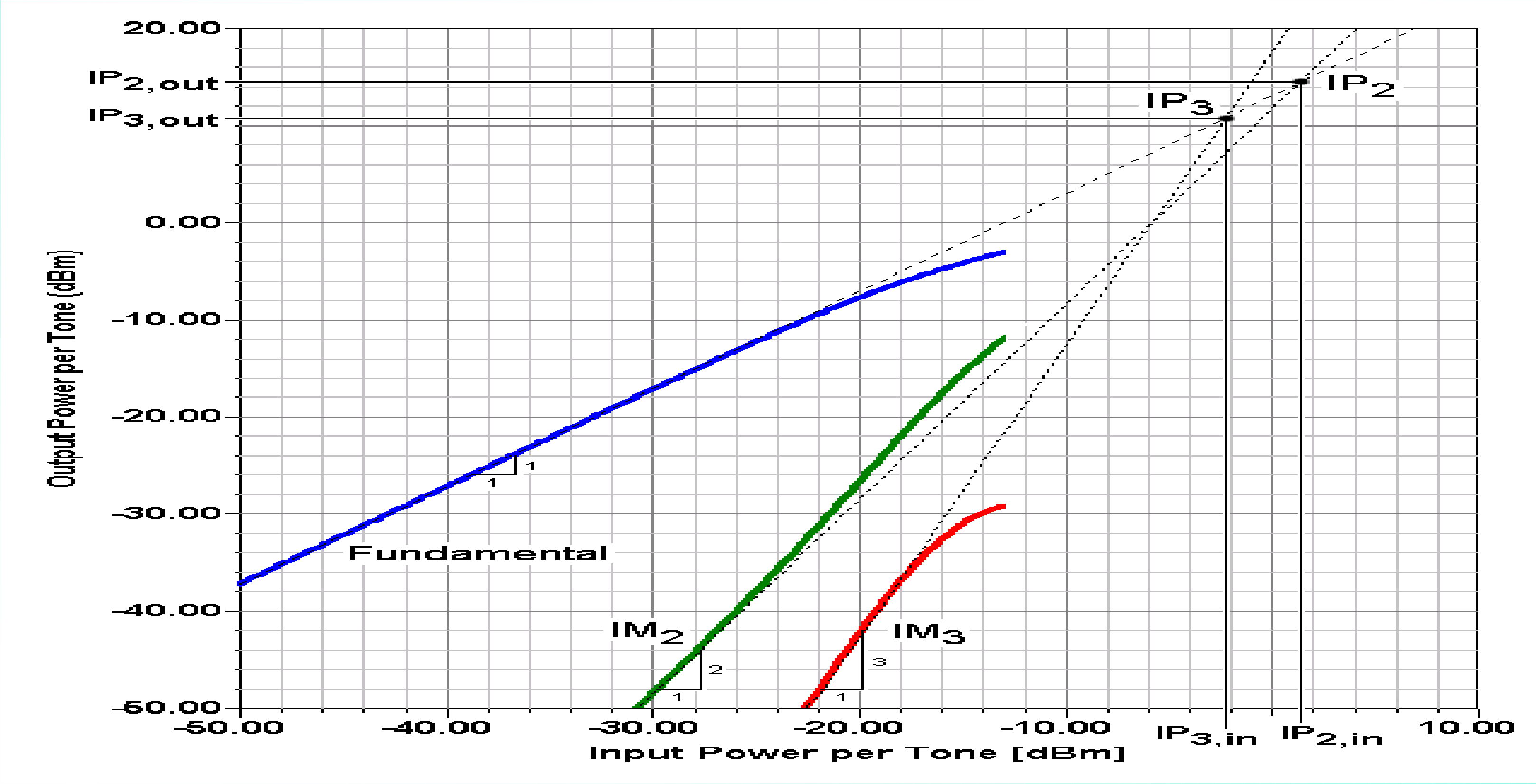
$$IP_{3,in} = \frac{1}{\frac{1}{IP1} + \frac{G}{IP2}}$$

Where *IP*1 is the input intercept of Stage 1 in watts, *IP*2 is the input intercept of Stage 2 in watts, and *G* is the gain of Stage 1 (as a numerical ratio, *not* in decibels).

Noise and Gain in Circuits/Systems, Cont'd.

Intermodulation:

For the amplifier simulated here, $IP_{2,in} \approx 1.5$ dBm, $IP_{2,out} \approx 14.5$ dBm, $IP_{3,in} \approx -2.3$ dBm and $IP_{3,out} \approx 10.7$ dBm. Each curve depicts the power in one tone of the response evaluated.



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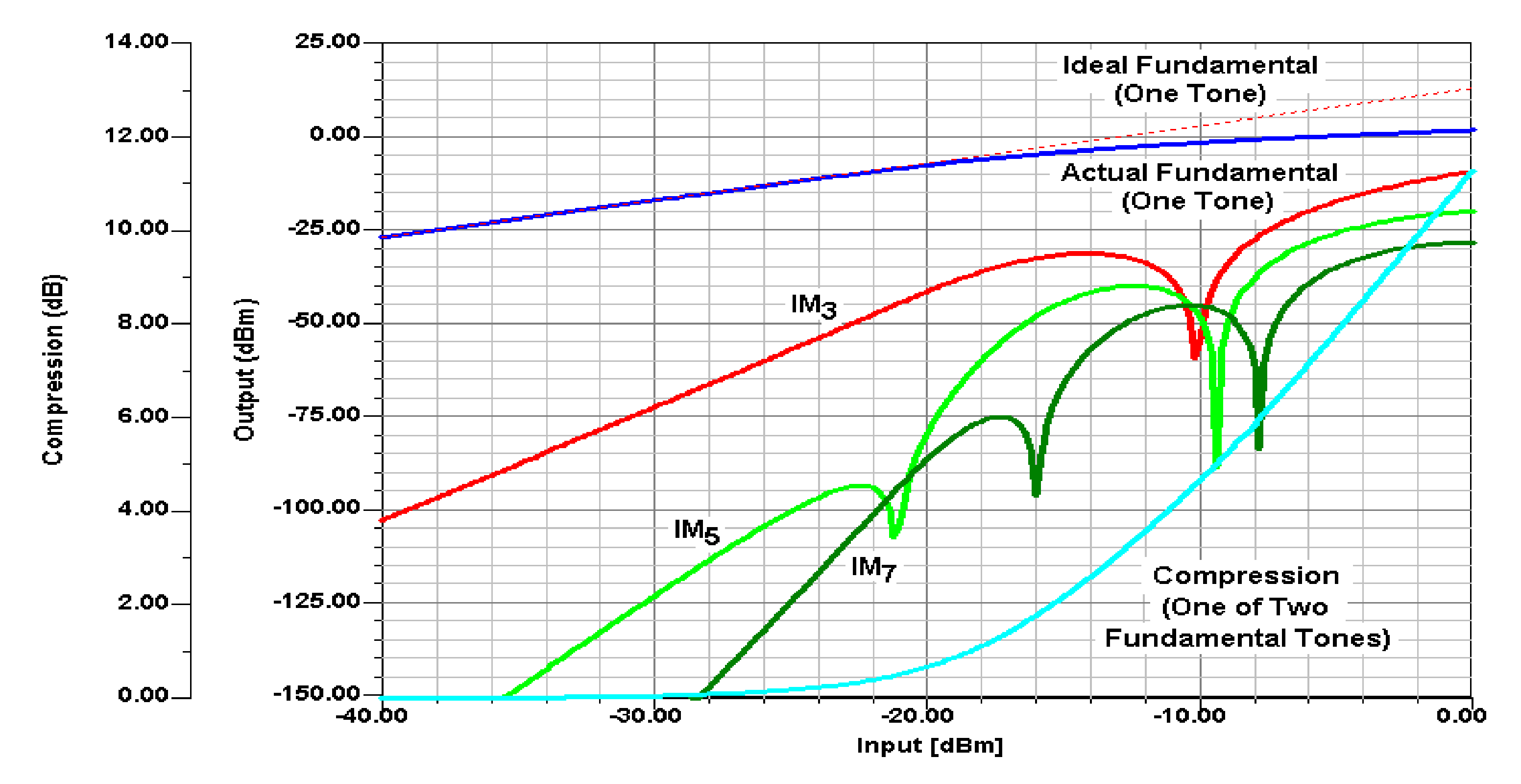
Noise and Gain in Circuits/Systems, Cont'd.

Intermodulation:

This graph shows the simulated performance of a single-BJT broadband amplifier driven by two equal-amplitude tones at 10 and 11 MHz.



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Noise and Gain in Circuits/Systems, Cont'd.

Intermodulation: Comments/Discussions

Discussions of IMD have traditionally downplayed the importance of IM_2 because the incidental distributed filtering contributed by the tuned circuitry once common in radiocommunication systems was usually enough to render out-of-passband IM_2 products caused by in-passband signals, and in-passband IM_2 products caused by out-of-passband signals, vanishingly weak compared to fundamental and IM_3 signals.

In broadband systems that operate at bandwidths of an octave or more, however, in-passband signals may produce significantly strong in-passband IM_2 and second-harmonic products.

In such applications, balanced circuit structures (such as push-pull amplifiers and balanced mixers) can be used to minimize IM_2 and other even-order nonlinear products.

Noise and Gain in Circuits/Systems, Cont'd.

Distortion Ratio :

The ratio of the signal power to the IM-product power, the *distortion ratio*, can be expressed as

$$R_{dn} = (n-1)\left[IP_{n(in)} - P_{(in)}\right]$$

where n is the order, R_{dn} is the distortion ratio, $IP_{n(in)}$ is the input intercept point, and $P_{(in)}$ is the input power of one tone.

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Noise and Gain in Circuits/Systems, Cont'd.

Dynamic Range :

The ratio of the noise-floor power to the upper-limit signal power is referred to as the network's *dynamic range (DR)*, often more carefully characterized as *two-tone IMD dynamic range*, which, when evaluated with equal-power test tones, is a figure of merit commonly used to characterize receivers. DR can be given as

$$DR_n = \frac{(n-1)[IP_{n(in)} - MDS_{in}]}{n}$$

where *DR* is the dynamic range in decibels, *n* is the order, $IP_{(in)}$ is the input intercept power in dBm, and MDS is the minimum detectable signal power in dBm.

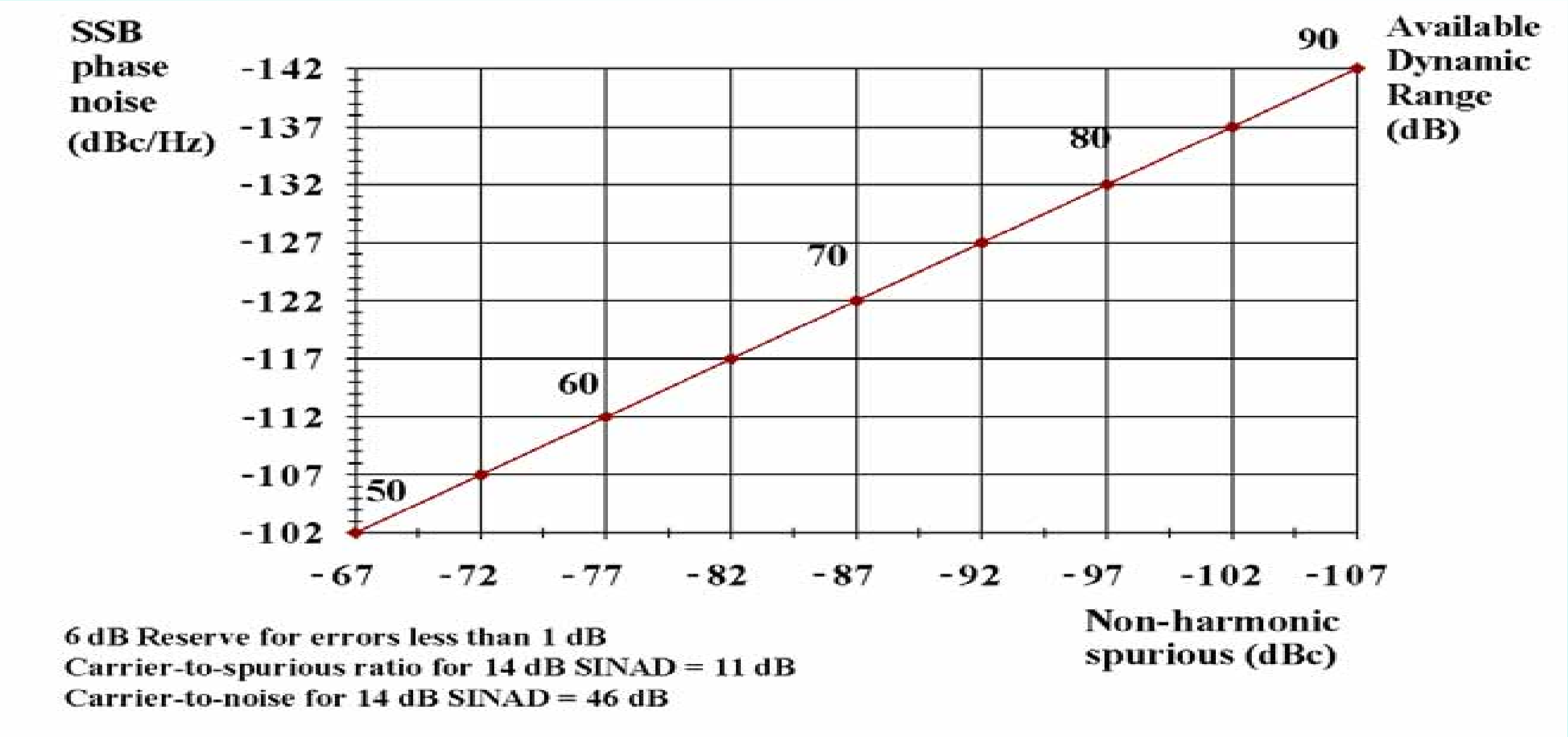
The *spurious-free dynamic range* (*SFDR* or DR_{SF}) is given by

$$DR_{SF} = \frac{2}{3}\left(IP_3 - 174\,\text{dBm} + NF + 3\,\text{dB}\right)$$

The equation allows us to determine the spurious-free dynamic range by applying the two-tone signals (in the case of IP_3) and increasing the two signals to the point where the signal-to-noise ratio deteriorates by 3 dB or, if the measurement is done relative to MDS, where the noise floor rises by 3 dB. The factor 2/3 is derived from the fact that the levels of IM_3 outputs increase 3 dB for 1 dB of input increase. This definition of dynamic range now is referenced to a noise figure rather than a minimum level in dBm, and is therefore independent of bandwidth.

Noise and Gain in Circuits/Systems, Cont'd.

Dynamic Range : This graph shows the available dynamic range, which is determined either by the masking of the unwanted signal by phase noise or by discrete spurii. As far as the culprit synthesizer is concerned, it can be either the local oscillator or the effect of a strong adjacent-channel signal that takes over the function of the local oscillator .



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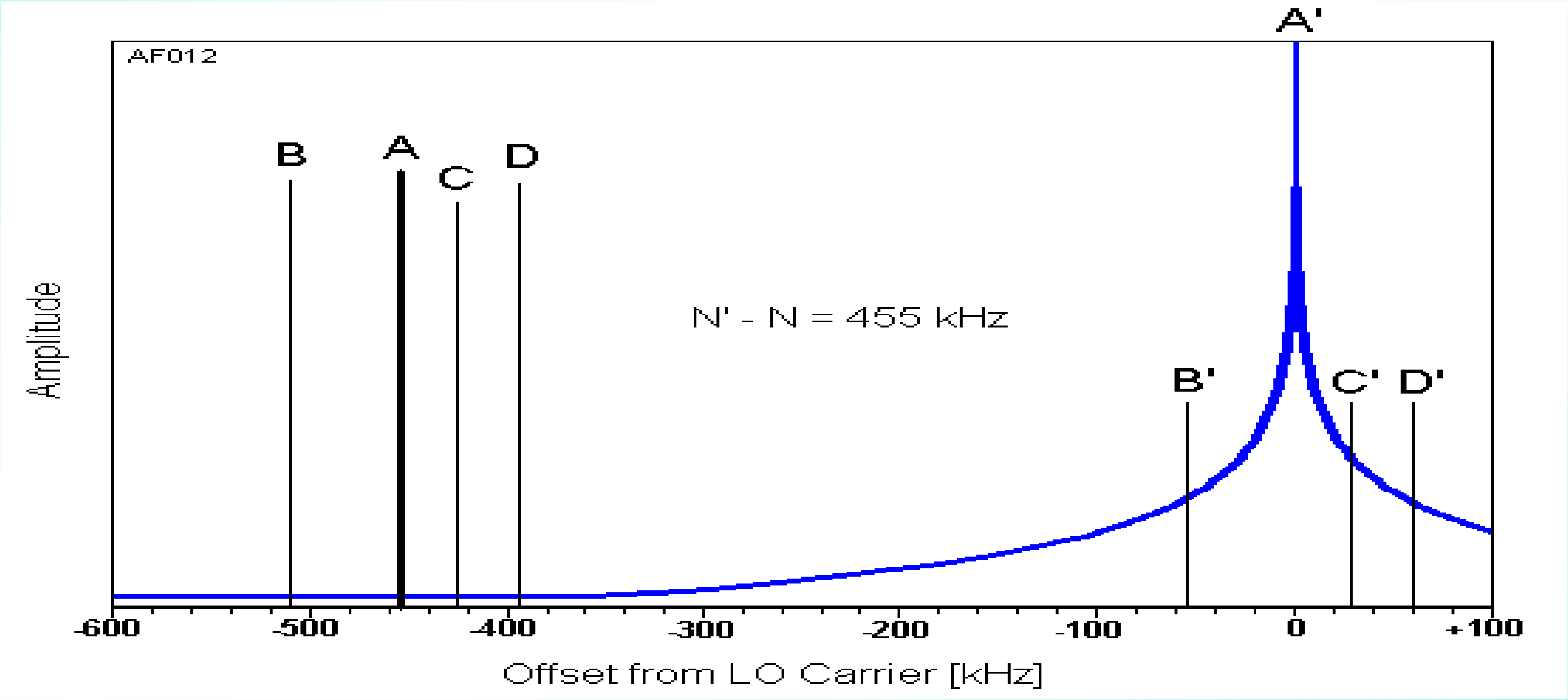
Noise and Gain in Circuits/Systems, Cont'd.

Reciprocal mixing:

In reciprocal mixing, incoming signals mix with LO-sideband energy to produce IF output. In this example, the oscillator is tuned so that its carrier, at A', heterodynes the desired signal, A, to the 455 kHz as intended; at the same time, the undesired signals B, C and D mix the oscillator noise-sideband energy at B', C' and D', respectively, to the IF.



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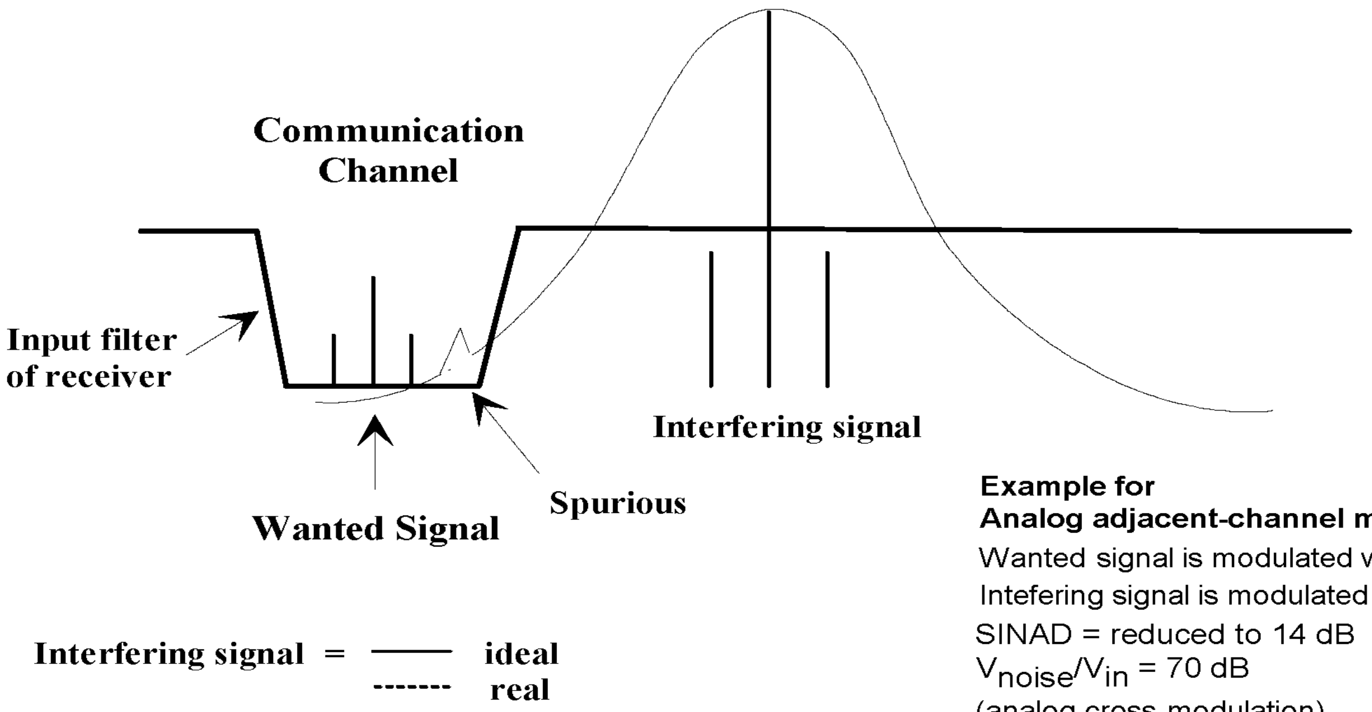
Noise and Gain in Circuits/Systems, Cont'd.

Selectivity:

Figure shows a typical arrangement of principle of selectivity measurement for analog receivers.

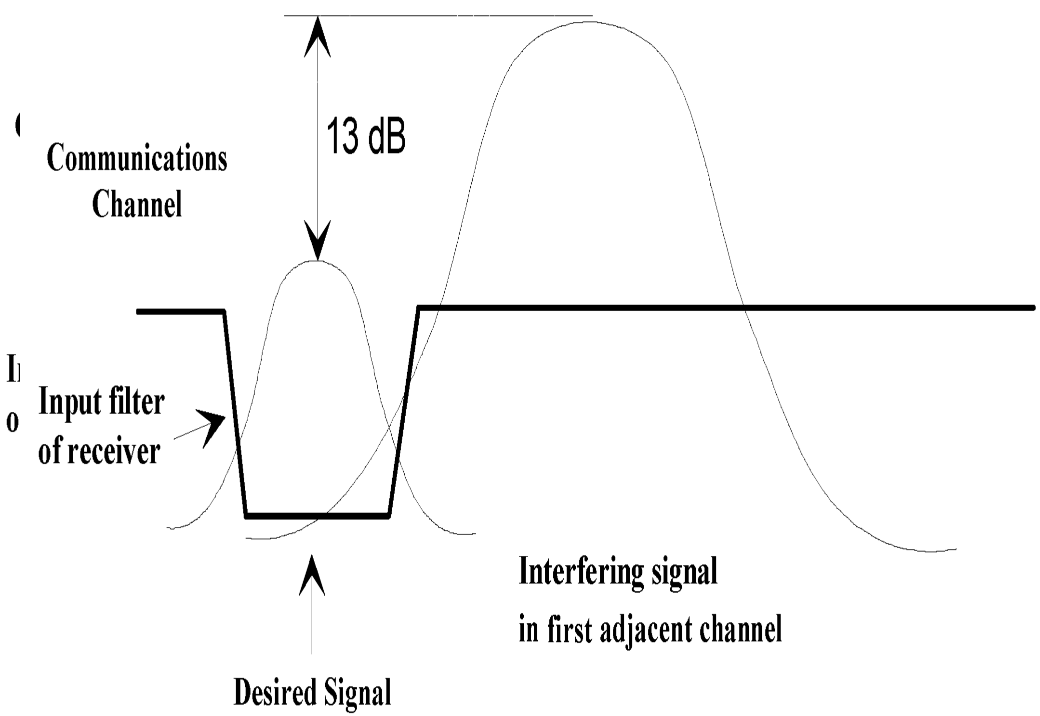


Example for
Analog adjacent-channel measurement:
Wanted signal is modulated with 1 kHz
Intefering signal is modulated with 400 Hz
SINAD = reduced to 14 dB
V_{noise}/V_{in} = 70 dB
(analog cross-modulation)

Noise and Gain in Circuits/Systems, Cont’d.

Selectivity:

Figure shows a typical arrangement of principle of selectivity measurement for digital receivers.



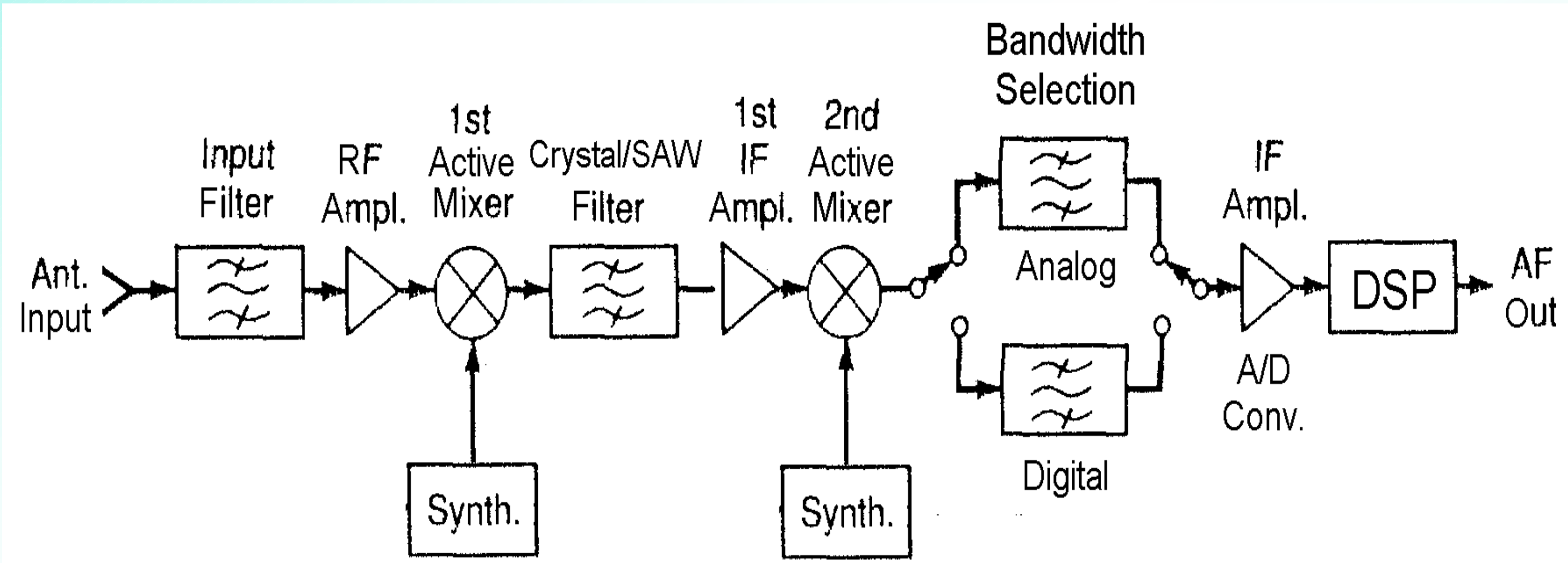
ACP suppression:

	Requirement
NADC	
30kHz from carrier	<-26 dBc
60kHz "	<-45 dBc
90kHz "	<-49 dBc
PDC	
50kHz from carrier	<-45 dBc
100kHz "	<-60 dBc

Noise and Gain in Circuits/Systems, Cont'd.

Example: Dual Conversion Receiver

Figure shows a typical arrangement of a dual-conversion receiver with local oscillators. The signal coming from the antenna is filtered by an arrangement of tuned circuits referred to providing as *input selectivity*.



Block diagram of an analog/digital receiver showing the signal path from antenna to audio output. No AGC or other auxiliary circuits are shown. This receiver principle can be used for all types of modulation, since the demodulation is done in the DSP block.

References

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[6] U. L. Rohde and A. K. Poddar, "Noise analysis of Systems of Coupled Oscillators," *INMMIC* workshop, Italy, Nov. 2004.

[7] U. L. Rohde and A. K. Poddar, "Ultra Low Noise Low Cost Multi Octave Band VCO", *IEEE Sarnoff Symposium*, Princeton, NJ, USA, April 18-19, 2005.

[8] U. L. Rohde and A. K. Poddar, " Configurable Ultra Low Ultra Wideband Power Efficient VCOs", 11th European Wireless, Conference, Cyprus, 10-13 April 2005.

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