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A Complete Analytical Approach for Designing Efficient Microwave FET and Bipolar Oscillators

For large-signal operation of oscillators, it is necessary to obtain the exact nonlinear device parameters of the active two-port network and calculate the external feedback elements for the circuit [1]-[13]. The circuit diagram for an oscillator below shows the intrinsic transistor, its parasitic and the external feedback circuit. The semiconductor depletion layer capacitances $(C_{gs}, C_{gd}\ and\ C_{gs})$ and the transconductance g_m are nonlinear, that is a function of voltage and current. The transistor in an oscillator is best described with the non-linear Materka model [2]-[6]. When using this model, the feedback element values are initially unknown. There is no good or efficient experimental solution for finding these feedback element values, only a small-signal approach [7], which does not handle power and noise.

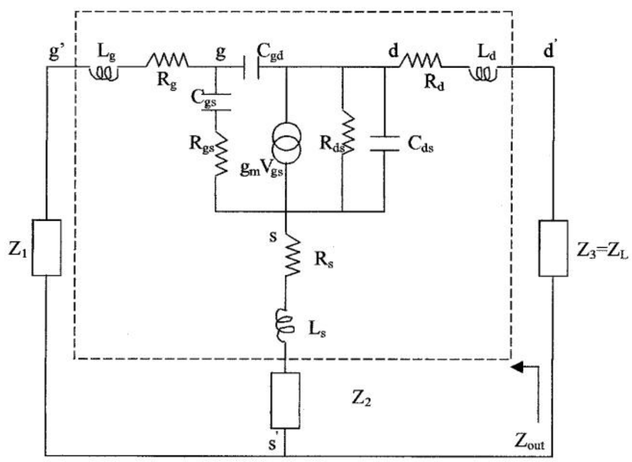

Figure 1 Series feedback topology.

Series Feedback (MESFET):

The best way to calculate series or parallel feedback oscillators with external elements is to use an analytical approach of designing a microwave oscillator that determines the explicit expression for the optimum feedback elements and the load impedance in terms of the transistor equivalent circuit parameters. These equations also provide a better understanding of the fundamental limitation in obtaining high output power for a given topology of the microwave oscillator [9]-[12].

Furthermore, maximizing the oscillator output power and the oscillator efficiency is the interest of many ongoing applications such as active phase-array antenna, etc.

Figure 1 shows the series feedback topology of the oscillator using a MESFET. External feedback elements $Z_{1,}$ Z_2, and Z_3 are shown outside the dotted line.

The optimum values of the feedback element $Z_{1,}$ Z_2, and Z_3 are given as

$$Z_1^{opt} = R_1^* + jX_1^* \tag{1}$$

$$Z_2^{opt} = R_2^* + jX_2^* \tag{2}$$

$$Z_3^{opt} = Z_L^{opt} = R_3^* + jX_3^* \tag{3}$$

$$Z_{out}^{opt} + Z_L^{opt} \Rightarrow 0 \tag{4}$$

$$Z_{out}^{opt} = R_{out}^* + jX_{out}^* \tag{5}$$

The general approach for designing an oscillator corresponding to the maximum output power at a given frequency is based on the optimum values of the feedback element and the load under steady-state large signal operation. The steady-state oscillation condition for a series feedback configuration can be expressed as

$$\left[Z_{out}(I_0, \omega_0) + Z_L(\omega_0)\right]_{w=w_0} = 0 \tag{6}$$

I_0 is amplitude of the load current and w_0 is the oscillator frequency. Assuming that the steady state current entering the active circuit is near sinusoidal, medium to high Q case, the output impedance $Z_{out}(I_0, \omega_0)$ and the load impedance $Z_L(\omega_0)$ can be expressed in terms of real and imaginary part as

$$Z_{out}(I_0, \omega_0) = R_{out}(I_0, \omega_0) + jX_{out}(I_0, \omega_0) \tag{7}$$

$$Z_L(\omega) = R_L(\omega) + jX_L(\omega) \tag{8}$$

$Z_{out}(I_0, \omega_0)$ is the current-amplitude and the frequency dependent-function, and $Z_L(w)$ is a function of the frequency.

The common source [Z] parameter of the MESFET is given as

$$[Z]_{cs} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}_{cs} \tag{9}$$

with

$$Z_{11} = R_{11} + jX_{11} \tag{10}$$

$$R_{11} = R_{gs}\left[\frac{a}{(a^2+b^2)} + \frac{b\omega R_{ds}C_{ds}(1+C_{gd}/C_{ds})}{(a^2+b^2)}\right] + \left[\frac{a\omega R_{ds}C_{ds}(1+C_{gd}/C_{ds})}{\omega C_{gs}(a^2+b^2)} - \frac{b}{\omega C_{gs}(a^2+b^2)}\right] \tag{11}$$

$$X_{11} = R_{gs}\left[\frac{a\omega R_{ds}C_{ds}(1+C_{gd}/C_{ds})}{(a^2+b^2)} - \frac{b}{(a^2+b^2)}\right] - \left[\frac{a}{\omega C_{gs}(a^2+b^2)} + \frac{b\omega R_{ds}C_{ds}(1+C_{gd}/C_{ds})}{\omega C_{gs}(a^2+b^2)}\right] \tag{12}$$

$$Z_{12} = R_{12} + jX_{12} \tag{13}$$

$$R_{12} = \frac{aR_{ds}C_{gd}}{C_{gs}(a^2+b^2)} + \frac{b\omega R_{ds}C_{gd}R_{gs}}{(a^2+b^2)} \tag{14}$$

$$X_{12} = \frac{a\omega R_{ds}C_{gd}R_{gs}}{(a^2+b^2)} - \frac{bR_{ds}C_{gd}}{C_{gs}(a^2+b^2)} \tag{15}$$

$$Z_{21} = R_{21} + jX_{21} \tag{16}$$

$$R_{21} = R_{ds}\left[\frac{C_{gd}}{C_{gs}}\frac{a}{(a^2+b^2)} + \frac{b\omega R_{gs}C_{gd}}{(a^2+b^2)} + \frac{g_m(b\cos\omega\tau + a\sin\omega\tau)}{\omega C_{gs}(a^2+b^2)}\right] \tag{17}$$

$$X_{21} = R_{ds}\left[\frac{a\omega R_{gs}C_{gd}}{(a^2+b^2)} - \frac{C_{gd}}{C_{gs}}\frac{b}{(a^2+b^2)} + \frac{g_m(a\cos\omega\tau - b\sin\omega\tau)}{\omega C_{gs}(a^2+b^2)}\right] \tag{18}$$

$$Z_{22} = R_{22} + jX_{22} \tag{19}$$

$$R_{22} = R_{ds}\left[\frac{a}{(a^2+b^2)} + \frac{C_{gd}}{C_{gs}}\frac{a}{(a^2+b^2)} + \frac{b}{(a^2+b^2)}\omega R_{gs}C_{gd}\right] \tag{20}$$

$$X_{22} = R_{ds}\left[\frac{a\omega R_{gs}C_{gd}}{(a^2+b^2)} - \frac{C_{gd}}{C_{gs}}\frac{b}{(a^2+b^2)} - \frac{b}{(a^2+b^2)}\right] \tag{21}$$

with

$$a = 1 + \frac{C_{gd}}{C_{gs}}(1 - \omega^2 R_{gs} C_{gs} R_{ds} C_{ds}) + \frac{g_m R_{ds} C_{gd}}{C_{gs}} \cos(\omega\tau) \tag{22}$$

$$b = \omega(R_{ds}C_{ds} + R_{ds}C_{gd}) + \omega \frac{C_{gd}}{C_{gs}}(R_{gs}C_{gs} + R_{ds}C_{ds}) - \frac{g_m R_{ds} C_{gd}}{C_{gs}} \sin(\omega\tau) \tag{23}$$

The expression of the output impedance, Z_{out} can be written as

$$Z_{out} = \left[[Z_{22} + Z_2] - \frac{[Z_{12} + Z_2][Z_{21} + Z_2]}{[Z_{11} + Z_1 + Z_2]} \right] \tag{24}$$

$$Z_{out} + Z_3 \Rightarrow Z_{out} + Z_L = 0 \tag{25}$$

where Z_{ij} (i,j=1,2) is the Z-parameters of the transistor model and can be expressed as

$$\lfloor Z_{i,j} \rfloor_{i,j=1,2} = [R_{ij} + jX_{ij}]_{i,j=1,2} \tag{26}$$

According to the criterion for the maximum output power at a given oscillator frequency, the negative real part of the output impedance Z_{out} has to be maximized, and the optimal values of the feedback reactance under which the negative value of R_{out} is maximized is given by the following condition as [2]:

$$\frac{\partial \operatorname{Re}}{\partial X_1}[Z_{out}(I,\omega)] = 0 \Rightarrow \frac{\partial}{\partial X_1}[R_{out}] = 0 \tag{27}$$

$$\frac{\partial \operatorname{Re}}{\partial X_2}[Z_{out}(I,\omega)] = 0 \Rightarrow \frac{\partial}{\partial X_2}[R_{out}] = 0 \tag{28}$$

The values of X_1 and X_2, which will satisfy the differential equations above are given as X_1^* and X_2^*. These can be expressed in terms of a two-port parameter of the active device (MESFET) as:

$$X_1^* = -X_{11} + \left[\frac{X_{12} + X_{21}}{2}\right] + \left[\frac{R_{21} - R_{12}}{X_{21} - X_{12}}\right]\left[\frac{R_{12} + R_{21}}{2} - R_{11} - R_1\right] \tag{29}$$

$$X_1^* = \frac{(1 - \omega\tau_g \tan\omega\tau)}{\omega C_{gs}(a - b\tan\omega\tau)} - \frac{(b + a\tan\omega\tau)(R_1 + R_g)}{(a - b\tan\omega\tau)} - \left[\frac{R_{ds}C_{ds}(\omega\tau_g + \tan\omega\tau)}{C_{gs}(a - b\tan\omega\tau)} - \frac{g_m R_{ds}}{2\omega C_{gs}\cos\omega\tau(a - b\tan\omega\tau)}\right] \tag{30}$$

where τ is the transit time in the MESFET-channel and $\tau_g = C_{gs}R_{gs}$ and $\tau_d = C_{ds}R_{ds}$.

$$X_2^* = -\left[\frac{X_{12}+X_{21}}{2}\right] - \left[\frac{(R_{21}-R_{12})(2R_2+R_{12}+R_{21})}{2(X_{21}-X_{12})}\right] \tag{31}$$

$$X_2^* = \frac{R_{ds}C_{gd}(\omega\tau_g + \tan\omega\tau)}{C_{gs}(a-b\tan\omega\tau)} - \frac{(b+a\tan\omega\tau)(R_2+R_s)}{(a-b\tan\omega\tau)} - \frac{g_m R_{ds}}{(a-b\tan\omega\tau)2\omega C_{gs}\cos\omega\tau} \tag{32}$$

$\left[R_{i,j}\right]_{i,j=1,2}$ *and* $=[X_{ij}]_{i,j=1,2}$ are the real and imaginary parts of the $[Z_{i,j}]_{i,j=1,2}$ of the transistor.

The output impedance can be given as $Z_{out}(I,\omega) = R_{out}(I,\omega) + X_{out}(I,\omega)$ and the corresponding optimum output impedance for the given oscillator frequency can be derived analytically by substituting values of the optimum values of susceptance under which negative value of R_{out} is

$$\left[Z_{out}^*(I,\omega)\right]_{\omega-\omega_0} = \left[R_{out}^*(I,\omega) + X_{out}^*(I,\omega)\right]_{\omega=\omega_0} \tag{33}$$

$$\left[R_{out}^*(I,\omega_0)\right]_{X_1^*,X_2^*} = \left\{R_2 + R_{22} - \left[\frac{(2R_2+R_{21}+R_{12})^2 + (X_{21}-X_{12})^2}{4(R_{11}+R_2+R_1)}\right]\right\} \tag{34}$$

$$\left[X_{out}^*(I,\omega)\right] = \left\{\frac{X_{22}-X_{12}-X_{21}}{2} - \frac{(R_{21}-R_{12})(2R_2+R_{12}+R_{21})}{2(X_{21}-X_{12})} - \frac{(R_{21}-R_{12})(R_{out}^* - R_2 - R_{22})}{X_{21}-X_{12}}\right\} \tag{35}$$

$$\left[X_{out}^*(I,\omega)\right]_{X_1^*,X_2^*} = X_2^* + X_{22} - \left[\frac{R_{21}-R_{12}}{X_{21}-X_{12}}\right]\left[R_{out}^* - R_2 - R_{22}\right] \tag{36}$$

$$X_{out}^* = \frac{R_{ds}}{(a-b\tan\omega\tau)}\left[\tan\omega\tau - \frac{g_m}{2wC_{gs}\cos\omega\tau}\right] - \frac{(b+a\tan\omega\tau)R_{out}^*}{(a-b\tan\omega\tau)} \tag{37}$$

X_1^* and X_2^* in the equations above are the optimal values of the external feedback susceptance.

For easier analysis, the effects of the transit time and the gate-drain capacitance are neglected for preliminary calculation of an optimum value of the feedback element and the simplified expressions are given as

$$X_1^* = \frac{1}{\omega C_{gs}} + R_{ds}\left[-\omega C_{ds}(R_1 + R_g + R_{gs}) + \frac{g_m}{2\omega C_{gs}}\right] \tag{38}$$

$$X_2^* = -R_{ds}\left[\omega C_{ds}(R_2 + R_s) + \frac{g_m}{2\omega C_{gs}}\right] \tag{39}$$

$$X_{out}^* = -R_{ds}\left[\omega C_{ds} R_{out}^* + \frac{g_m}{2\omega C_{gs}}\right] \tag{40}$$

$$R_{out}^* = (R_2 + R_s) + \frac{R_{ds}}{1 + (\omega C_{ds} R_{ds})^2}\left[1 - \frac{R_{ds}}{R_g + R_s + R_1 + R_2 + R_{gs}}\left(\frac{g_m}{2\omega C_{gs}}\right)^2\right] \tag{41}$$

The simplified expressions above show accuracy with the harmonic balance-based simulated results for a gate length less than 1μm at an operating frequency range up to 20 GHz.

The differential drain resistance R_{ds} can be expressed in terms of the optimum output resistance as

$$R_{ds} = \frac{(1 + \sqrt{1 - 4(R_{out}^* - R_2 - R_s)G_{dso}})}{2G_{dso}} \tag{42}$$

where

$$G_{dso} = \frac{1}{R_g + R_s + R_2 + R_{gs}}\left(\frac{g_m}{2\omega C_{gs}}\right)^2 + (R_{out}^* - R_2 - R_s)(\omega C_{ds})^2 \tag{43}$$

Alternatively, a differential drain resistance can be obtained from a quasi-linear analysis. Under large signal operation, the transistor parameters vary with the drive level. If we restrict our interest to the fundamental signal frequency component, then V_{gs} and V_{ds} can be expressed as

$$V_{gs}(t) = V_{gso} + V_{gs} Sin(\omega t + \varphi) \tag{44}$$

$$V_{ds}(t) = V_{dso} + V_{ds} Sin(\omega t) \tag{45}$$

V_{gso} and V_{dso} are the DC operating bias voltages, V_{gs} and V_{ds} are the amplitude of the signal frequency components, and φ is the phase difference between the gate and drain voltages.

The drain current I_d can be expressed as

$$I_{ds} = I_{ds}(V_{gs}, V_{dso}) \tag{46}$$

Under the assumption of linear superposition of the DC and RF currents, an instantaneous drain current can be expressed as

$$I_{ds}(t) = I_{dso} + g_m v_{gs}\cos(\omega t + \varphi) + G_d v_{ds}\cos(\omega t) \quad (47)$$

where I_{dso} is the DC bias drain current.

The transconductance g_m and the drain conductance are defined as

$$g_m = \left[\frac{I_{ds}}{V_{gs}}\right]_{V_{ds=0}} \quad (48)$$

$$G_D = \left[\frac{I_{ds}}{V_{gs}}\right]_{V_{gs=0}} \quad (49)$$

Under large signal conditions, the transconductance and the drain conductance are given as

$$g_m = \frac{\omega}{\pi V_{gs}\sin\varphi}\int_0^{\frac{2\pi}{\omega}} I_{ds}\sin(\omega t)dt \quad (50)$$

$$G_d = \frac{\omega}{\pi V_{ds}\sin\varphi}\int_0^{\frac{2\pi}{\omega}} I_{ds}\sin(\omega t + \varphi)dt \quad (51)$$

The drain current can be expressed in terms of V_{gs}, V_p and V_{ds} as

$$I_d = I_{dss}\left[1 - \frac{V_g}{V_p}\right]^2 \tanh\left[\frac{\alpha V_d}{V_g - V_p}\right] \quad (52)$$

$$V_p = V_{p0} + \gamma V_d \quad (53)$$

I_{dss} is the saturation current and V_p is the gate pinch-off voltage, α, γ and V_{p0} are the model parameters of the MESFET.

Applying a Taylor-series expansion of the equation about the DC operating point and considering the fundamental frequency component terms, the large signal drain resistance, as function of the small signal drain voltage amplitude, can be given as

$$R_{DS}\big|_{Large-signal} = \frac{R_{ds}}{(1 + AV_d^2)} \tag{54}$$

where R_{DS} and R_{ds} are the large and signal differential resistances.

A is defined as

$$A = \left\{ \frac{3\tanh^2[\frac{\alpha V_{d0}}{V_{g0} - V_p}] - 1}{4[\frac{V_{g0} - V_p}{\alpha}]^2} \right\} \tag{55}$$

$$R_{ds} = \left\{ \frac{\cosh^2[\frac{\alpha V_{d0}}{V_{g0} - V_p}]}{I_{dss}[1 - \frac{V_{g0}}{V_p}]^2} \left[\frac{V_{g0} - V_p}{\alpha} \right] \right\} \tag{56}$$

From the expression above, $R_{DS}\big|_{Large-signal}$ has a maximum value in the absence of the RF drive signal and gets smaller as the amplitude of the RF signal increases. Consequently, the oscillator output impedance and the oscillator output power are a function of the change of the drain resistance under large-signal operation. To support the steady-state operation mode, the amplitude and the phase balance conditions can be written as

$$\left[R_{out}^*(I, \omega) + R_L(\omega) \right]_{\omega=\omega_0} = 0 \tag{57}$$

$$\left[X_{out}^*(I, \omega) + X_L^*(\omega) = 0 \right]_{\omega=\omega_0} = 0 \tag{58}$$

The output power of the oscillator can be expressed in terms of load current and load impedance as

$$P_{out} = \frac{1}{2} I_{out}^2 \,\mathrm{Re}[Z_L] \tag{59}$$

where I_{out} and V_{out} are the corresponding load current and drain voltage across the output.

$$I_{out} = \left[\frac{Z_{11} + Z_1 + Z_2}{Z_{22}(Z_{11} + Z_1 + Z_2) - Z_{21}(Z_{12} + Z_2)} \right] V_{out} \tag{60}$$

$$P_{out} = \frac{1}{2} I_{out}^2 \operatorname{Re}[Z_L] \Rightarrow \frac{1 + \left(\frac{R_{21} - R_{12}}{X_{21} - X_{12}} \right)^2}{(R_{22} + R)^2 + (X_{22} + X)^2} (R_{out} + R_d) \frac{V_d^2}{2} \tag{61}$$

where

$$R = \frac{X_{21}(X_{12} + X_2^*) - R_{21}(R_{12} + R_2 + R_s) - X_{22}(X_{11} + X_1^* + X_2^*)}{R_{11} + R_1 + R_2 + R_g + R_s} \tag{62}$$

$$X = \frac{R_{21}(X_{11} + X_1^* + X_2^*) - R_{21}(X_{12} + X_2^*) - X_{21}(R_{12} + R_2 + R_s)}{R_{11} + R_1 + R_2 + R_g + R_s} \tag{63}$$

Parallel Feedback (MESFET):

Figure 2 shows the parallel feedback topology of the oscillator using the MESFET, in which the external feedback elements $Y_{1,}$ Y_2, and Y_3 are shown outside the dotted line.

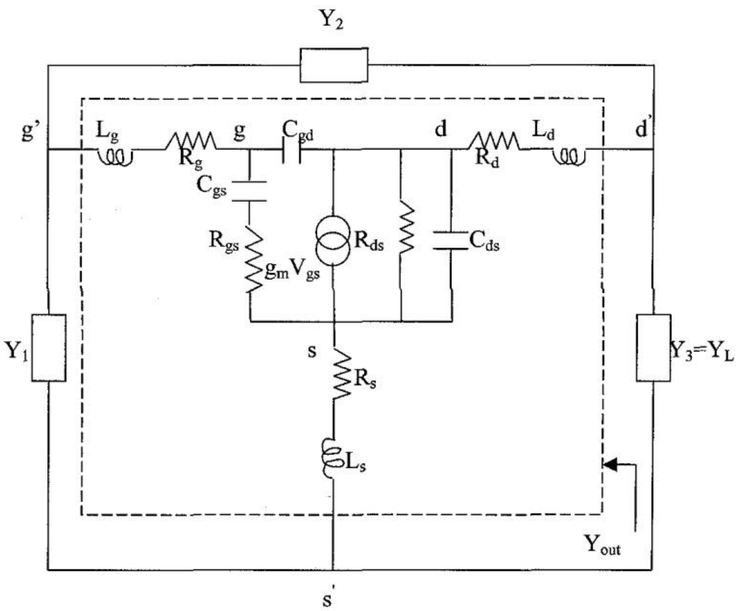

Figure 2 Parallel feedback topology.

The optimum values of the feedback element $Y_{1,}$ Y_2 and Y_3 are given as

$$Y_1^{opt} = G_1^* + jB_1^* \tag{64}$$

$$Y_2^{opt} = G_2^* + jB_2^* \tag{65}$$

$$Y_3^{opt} = Y_L^{opt} = G_3^* + jB_3^* \tag{66}$$

$$Y_{out}^{opt} + Y_L^{opt} \Rightarrow 0 \tag{67}$$

$$Y_{out}^{opt} = G_{out}^* + jB_{out}^* \tag{68}$$

The common source [Y] parameter of the MESFET is given as

$$[Y]_{cs} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \tag{69}$$

$$Y_{11} = \frac{j\omega C_{gs}}{1 + j\omega C_{gs} R_{gs}} + j\omega C_{gd} \Rightarrow G_{11} + jB_{11} \tag{70}$$

$$Y_{21} = \frac{g_m \exp(-j\omega\tau)}{1 + j\omega C_{gs} R_{gs}} - j\omega C_{gd} \Rightarrow G_{21} + jB_{21} \tag{71}$$

$$Y_{12} = -j\omega C_{gd} \Rightarrow G_{12} + jB_{12} \tag{72}$$

$$Y_{22} = \frac{1}{R_{ds}} + j\omega(C_{ds} + C_{gd}) \Rightarrow G_{22} + jB_{22} \tag{73}$$

The optimum values of the output admittance Y_{out}^* and the feedback susceptance B_1^* and B_2^*, which can be expressed in terms of the two-port Y-parameter of the active device given as

$$B_1^* = -\left\{ B_{11} + \left[\frac{B_{12} + B_{21}}{2}\right] + \left[\frac{G_{21} - G_{12}}{B_{21} - B_{12}}\right]\left[\frac{G_{12} + G_{21}}{2} + G_{11}\right] \right\} \tag{74}$$

$$B_1^* = \frac{g_m}{2\omega C_{gs} R_{gs}} \tag{75}$$

$$B_2^* = \left[\frac{B_{12} + B_{21}}{2}\right] + \left[\frac{(G_{12} + G_{21})(G_{21} - G_{12})}{2(B_{21} - B_{12})}\right] \tag{76}$$

$$B_2^* = -\omega C_{dg} - \frac{g_m}{2\omega C_{gs} R_{gs}} \tag{77}$$

The optimum values of the real and imaginary part of the output admittance as

$$Y_{out}^* = [G_{out}^* + jB_{out}^*] \tag{78}$$

where G_{out}^* and B_{out}^* is given as

$$G_{out}^* = G_{22} - \left[\frac{(G_{12}+G_{21})^2 (B_{21}-B_{12})^2}{4G_{11}}\right] \tag{79}$$

$$G_{out}^* = \frac{1}{R_{ds}} - \frac{1}{R_{gs}}\left[\frac{g_m}{2\omega C_{gs}}\right]^2 \tag{80}$$

$$B_{out}^* = B_{22} + \left[\frac{G_{21}-G_{12}}{B_{21}-B_{12}}\right] - \left[\frac{(G_{12}+G_{21})}{2} + G_{22} - G_{out}^*\right] + \left[\frac{B_{21}+B_{12}}{2}\right] \tag{81}$$

$$B_{out}^* = \omega C_{gd} - \frac{1}{R_{gs}}\left[\frac{g_m}{2\omega C_{gs}}\right]\left[1 - \frac{1}{R_{gs}}\frac{1}{\omega C_{gs}}\frac{g_m}{2\omega C_{gs}}\right] \tag{82}$$

The value of the output susceptance B_{out}^* may be positive or negative, depending on the values of the transistor transconductance and $\tau_{gs} = R_{gs}C_{gs}$.

The voltage feedback factor-n and phase-φ_n can be expressed in terms of transistor *Y*-parameters as

$$n(V_{ds}/V_{gs}) = \frac{\sqrt{(G_{12}+G_{21}-2G_2)^2 + (B_{21}-B_{12})^2}}{2(G_{12}+G_{21}-G_2)} \Rightarrow \frac{1}{2}\sqrt{1+(\omega R_s C_{gs})^2} \tag{83}$$

$$\Phi_n(phase) = \tan^{-1}\frac{B_{21}-B_{12}}{G_{12}+G_{21}-2G_2} \Rightarrow -\tan^{-1}(\omega R_s C_{gs}) \tag{84}$$

The output power of the oscillator can be expressed in terms of the load current and the load impedance as

$$P_{out} = \frac{1}{2} I_{out}^2 \operatorname{Re}[Z_L] \tag{85}$$

where I_{out} and V_{out} is the corresponding load current and drain voltage across the output.

$$I_{out} = \left[\frac{Z_{11} + Z_1 + Z_2}{Z_{22}(Z_{11} + Z_1 + Z_2) - Z_{21}(Z_{12} + Z_2)} \right] V_{out} \tag{86}$$

Series Feedback (Bipolar):

Figure 3 shows the series feedback oscillator topology for deriving explicit analytical expressions for the optimum values of the external feedback elements and the load impedance for maximum power output at a given oscillator frequency through [*Z*]-parameters of a bipolar transistor. The modified physics based Gummel-Poon [8, 9] models nicely describes the physical behavior of the bipolar transistor.

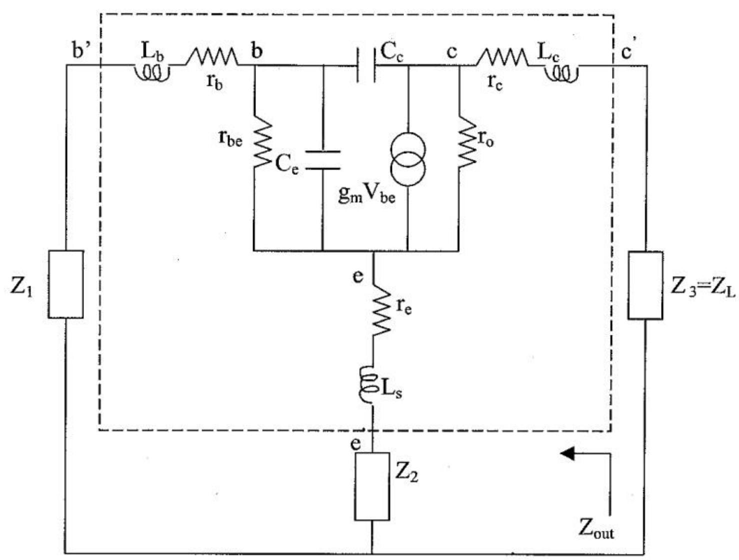

Figure 3 Series feedback topology of the oscillator using bipolar transistor.

Figure 3 shows the series feedback topology of the oscillator using a bipolar transistor, in which external feedback elements $Z_{1,}$ Z_2 and Z_3 are shown outside the dotted line.

The optimum values of the feedback element $Z_{1,}$ Z_2 and Z_3 are given as

$$Z_1^{opt} = R_1^* + jX_1^* \tag{87}$$

$$Z_2^{opt} = R_2^* + jX_2^* \tag{88}$$

$$Z_3^{opt} = Z_L^{opt} = R_3^* + jX_3^* \tag{89}$$

$$Z_{out}^{opt} + Z_L^{opt} \Rightarrow 0 \tag{90}$$

$$Z_{out}^{opt} = R_{out}^* + jX_{out}^* \tag{91}$$

The $[Z]$ parameters of the internal bipolar transistor in a common-emitter, small signal condition are given as [2, 10, 11]

$$[Z]_{ce} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \tag{92}$$

$$Z_{11} = R_{11} + jX_{11} \Rightarrow a\left[\frac{1}{g_m} + r_b\left(\frac{\omega}{\omega_T}\right)^2\right] - ja\frac{\omega}{\omega_T}\left[\frac{1}{g_m} - r_b\right] \tag{93}$$

$$Z_{12} = R_{12} + jX_{12} \Rightarrow a\left[\frac{1}{g_m} + r_b\left(\frac{\omega}{\omega_T}\right)^2\right] - ja\frac{\omega}{\omega_T}\left[\frac{1}{g_m} - r_b\right] \tag{94}$$

$$Z_{21} = R_{21} + jX_{21} \Rightarrow a\left[\frac{1}{\omega_T C_c} + \frac{1}{g_m} + r_b\left(\frac{\omega}{\omega_T}\right)^2\right] - ja\frac{\omega}{\omega_T}\left[\frac{1}{g_m} - \frac{1}{\omega_T C_c} - r_b\right] \tag{95}$$

$$Z_{22} = R_{22} + jX_{22} \Rightarrow a\left[\frac{1}{\omega_T C_c} + \frac{1}{g_m} + r_b\left(\frac{\omega}{\omega_T}\right)^2\right] - ja\frac{\omega}{\omega_T}\left[\frac{1}{g_m} + \frac{1}{\omega_T C_c} - r_b\right] \tag{96}$$

where

$$a = \left\{ \frac{1}{1 + \left[\frac{\omega}{\omega_T} \right]^2} \right\} \tag{97}$$

$$\omega_T = 2\pi f_T \tag{98}$$

$$f_T = \frac{g_m}{2\pi C_e} \tag{99}$$

According to the optimum criterion for the maximum power output at a given oscillator frequency, the negative real part of the output impedance Z_{out} has to be maximized and the possible optimal values of the feedback reactance, under which the negative value of R_{out} is maximized, is given by the following condition as [10]

$$\frac{\partial \operatorname{Re}}{\partial X_1}\left[Z_{out}(I,\omega) \right] = 0 \Rightarrow \frac{\partial}{\partial X_1}[R_{out}] = 0 \tag{100}$$

$$\frac{\partial \operatorname{Re}}{\partial X_2}\left[Z_{out}(I,\omega) \right] = 0 \Rightarrow \frac{\partial}{\partial X_2}[R_{out}] = 0 \tag{101}$$

The values of X_1 and X_2, which will satisfy the differential equations above, are given as X_1^* and X_2^*. This can be expressed in terms of a two-port parameter of the active device (bipolar) as [10]

$$X_1^* = -X_{11} + \left[\frac{X_{12} + X_{21}}{2} \right] + \left[\frac{R_{21} - R_{12}}{X_{21} - X_{12}} \right] \left[\frac{R_{12} + R_{21}}{2} - R_{11} - R_1 \right] \tag{102}$$

$$X_1^* = \frac{1}{2\omega C_c} - r_b \frac{\omega}{\omega_T} \tag{103}$$

$$X_2^* = -\left[\frac{X_{12} + X_{21}}{2} \right] - \left[\frac{(R_{21} - R_{12})(2R_2 + R_{12} + R_{21})}{2(X_{21} - X_{12})} \right] \tag{104}$$

$$X_2^* = -\frac{1}{2\omega C_c} - (r_{be} + r_e)\frac{\omega}{\omega_T} \tag{105}$$

By substituting the values of X_1^* and X_2^* into the equation above, the optimal real and imaginary parts of the output impedance Z_{out}^* can be expressed as

$$Z_{out}^* = R_{out}^* + X_{out}^* \tag{106}$$

$$R_{out}^* = R_2 + R_{22} - \left[\frac{(2R_2 + R_{21} + R_{12})^2 + (X_{21} - X_{12})^2}{4(R_{11} + R_2 + R_1)}\right] \tag{107}$$

$$R_{out}^* = r_c + \frac{r_b}{r_b + r_e + R_{11}}\left[r_e + R_{11} + \frac{a}{\omega_T C_e}\right] - \frac{a}{r_b + r_e + R_{11}}\left[\frac{1}{2\omega C_e}\right] \tag{108}$$

$$X_{out}^* = X_2^* + X_{22} - \left[\frac{R_{21} - R_{12}}{X_{21} - X_{12}}\right]\left[R_{out}^* - R_2 - R_{22}\right] \tag{109}$$

$$X_{out}^* = \frac{1}{2\omega C_e} - (R_{out}^* - r_c)\frac{\omega}{\omega_T} \tag{110}$$

thus, in the steady-state operation mode of the oscillator, the amplitude, and the phase balance conditions can be written as

$$R_{out}^* + R_L = 0 \tag{111}$$

$$X_{out}^* + X_L^* = 0 \tag{112}$$

The output power of the oscillator can be expressed in terms of load current and load impedance as

$$P_{out} = \frac{1}{2} I_{out}^2 \mathrm{Re}[Z_L] \tag{113}$$

I_{out} and V_{out} are the corresponding load current and drain-voltage across the output.

$$I_{out} = \left[\frac{Z_{11} + Z_1 + Z_2}{Z_{11}Z_2 - Z_{12}(Z_1 + Z_2)} \right] V_{be} \tag{114}$$

$$V_{out} = V_c = \left[\frac{Z_{22}(Z_{11} + Z_1 + Z_2) - Z_{21}(Z_2 + Z_{12})}{Z_{12}(Z_1 + Z_2) - Z_{11}Z_2} \right] V_{be} \tag{115}$$

$$P_{out} = \frac{1}{2} I_{out}^2 \mathrm{Re}[Z_L] \tag{116}$$

$$P_{out} = aG_m^2(x)R_{out}^* \frac{(r_b + r_e + R_{11})}{(r_b + r_c - R_{out}^*)} \frac{V_1^2}{2} \tag{117}$$

V_1 is the signal voltage and x is the drive level across base-emitter junction of bipolar transistor. The large signal transconductance $G_m(x)$ is given as

$$G_m(x) = \frac{qI_{dc}}{kTx} \left[\frac{2I_1(x)}{I_0(x)} \right]_{n=1} = \frac{g_m}{x} \left[\frac{2I_1(x)}{I_0(x)} \right]_{n=1} \tag{118}$$

$$V_1 \big|_{peak} = \frac{kT}{q} x \tag{119}$$

$$g_m = \frac{I_{dc}}{kT/q} \tag{120}$$

where g_m is the small signal transconductance.

Parallel Feedback (Bipolar):

Figure 4 shows the parallel feedback topology of the oscillator using a bipolar transistor in which the external feedback elements Y_1, Y_2, and Y_3 are shown outside the dotted line.

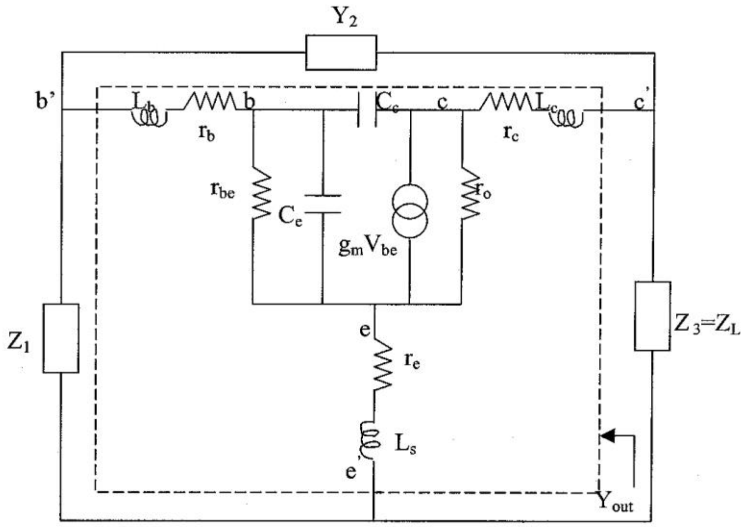

Figure 4 A parallel feedback topology of the oscillator using a bipolar transistor.

The optimum values of the feedback elements $Y_{1,}$ Y_2 and Y_3 are given as

$$Y_1^{opt} = G_1^* + jB_1^* \tag{121}$$

$$Y_2^{opt} = G_2^* + jB_2^* \tag{122}$$

$$Y_3^{opt} = G_L^{opt} = G_3^* + jB_3^* \tag{123}$$

$$Y_{out}^{opt} + Y_L^{opt} \Rightarrow 0 \tag{124}$$

$$Y_{out}^{opt} = G_{out}^* + jB_{out}^* \tag{125}$$

The common source [Y] parameter of the bipolar transistor is given as

$$[Y]_{cs} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \tag{126}$$

$$Y_{11} = G_{11} + jB_{11} \tag{127}$$

$$Y_{21} = G_{21} + jB_{21} \tag{128}$$

$$Y_{12} = G_{12} + jB_{12} \tag{129}$$

$$Y_{22} = G_{22} + jB_{22} \tag{130}$$

The optimum values of the output admittance Y_{out}^{*} and feedback susceptance B_1^{*} and B_2^{*}, which can be expressed in terms of the two-port Y-parameter of the active device, is given as

$$B_1^{*} = -\left\{B_{11} + \left[\frac{B_{12}+B_{21}}{2}\right] + \left[\frac{G_{21}-G_{12}}{B_{21}-B_{12}}\right]\left[\frac{G_{12}+G_{21}}{2} + G_{11}\right]\right\} \tag{131}$$

$$B_2^{*} = \left[\frac{B_{12}+B_{21}}{2}\right] + \left[\frac{(G_{12}+G_{21})(G_{21}-G_{12})}{2(B_{21}-B_{12})}\right] \tag{132}$$

The optimum values of the real and imaginary part of the output admittance as

$$Y_{out}^{*} = [G_{out}^{*} + jB_{out}^{*}] \tag{133}$$

where G_{out}^{*} and B_{out}^{*} is given as

$$G_{out}^{*} = G_{22} - \left[\frac{(G_{12}+G_{21})^2(B_{21}-B_{12})^2}{4G_{11}}\right] \tag{134}$$

$$B_{out}^{*} = B_{22} + \left[\frac{G_{21}-G_{12}}{B_{21}-B_{12}}\right] - \left[\frac{(G_{12}+G_{21})}{2} + G_{22} - G_{out}^{*}\right] + \left[\frac{B_{21}+B_{12}}{2}\right] \tag{135}$$

The output power of the oscillator can be expressed in terms of the load current and the load impedance as

$$P_{out} = \frac{1}{2} I_{out}^2 \,\mathrm{Re}[Z_L] \tag{136}$$

where I_{out} and V_{out} is the corresponding load current and drain voltage across the output.

$$I_{out} = \left[\frac{Z_{11}+Z_1+Z_2}{Z_{22}(Z_{11}+Z_1+Z_2) - Z_{21}(Z_{12}+Z_2)}\right] V_{out} \tag{137}$$

An FET Example

Figure 5 shows a 950MHz MESFET oscillator circuit configuration [1] and the analytical approach for optimum operating conditions for maximum oscillator output power. The analysis is based on a quasi-linear approach and is experimentally supported with the conversion efficiency of 54%, which is the maximum conversion efficiency published for this topology. However, the publication does not give any emphasis on the optimum phase noise, which is the key parameter for the oscillator design.

Power optimization of a GaAs-950MHz-MESFET oscillator [1]:

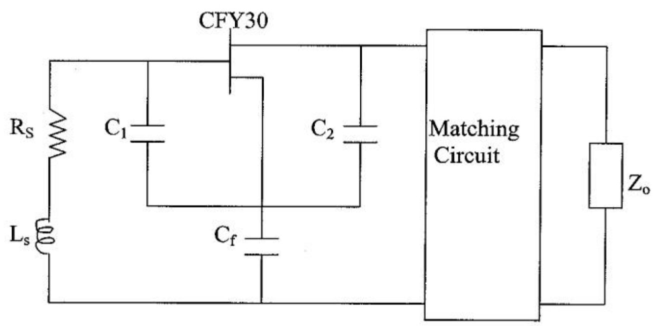

Figure 5 A 950MHz MESFET oscillator circuit configuration.

The derivations of the analytical expressions are based on the open loop model of the oscillator. Figure 6 shows an equivalent circuit of the oscillator.

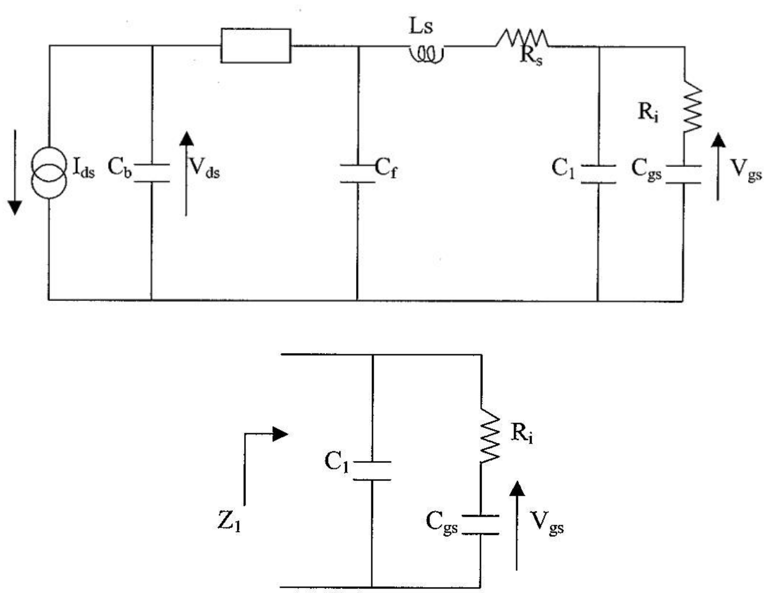

Figure 6 An equivalent circuit of the open model MESFET oscillator.

Z_1 can be expressed as

$$Z_1 = \frac{\left[R_i + \frac{1}{j\omega C_{gs}}\right]\frac{1}{j\omega C_1}}{\left[R_i + \frac{1}{j\omega C_{gs}} + \frac{1}{j\omega C_1}\right]} = \frac{-\left[\frac{jR_i}{\omega C_1} + \frac{1}{\omega^2 C_{gs} C_1}\right]}{\left[R_i - j\left(\frac{1}{\omega C_{gs}} + \frac{1}{\omega C_1}\right)\right]} \tag{138}$$

Multiplying the numerator and the denominator by the conjugate yields:

$$Z_1 = \frac{\left[\frac{-jR_i^2}{\omega C_1} - \frac{R_i}{\omega^2 C_{gs} C_1}\right] + \frac{R_i}{\omega^2 C_1}\left[\frac{1}{C_1} + \frac{1}{C_{gs}}\right] - \frac{j}{\omega^3 C_{gs} C_1}\left[\frac{1}{C_1} + \frac{1}{C_{gs}}\right]}{\left[R_i^2 + \left(\frac{1}{\omega C_{gs}} + \frac{1}{\omega C_1}\right)^2\right]} \tag{139}$$

The following assumptions are made for simplification purposes.

$$\frac{R_i}{\omega^2 C_{gs} C_1} << \frac{R_i}{\omega^2 C_1}\left[\frac{1}{C_1} + \frac{1}{C_{gs}}\right] \tag{140}$$

$$\frac{jR_i^2}{\omega C_1} << \frac{j}{\omega^3 C_{gs} C_1}\left[\frac{1}{C_1} + \frac{1}{C_{gs}}\right] \tag{141}$$

$$R_i << \left[\frac{1}{\omega C_{gs}} + \frac{1}{\omega C_1}\right] \tag{142}$$

then modified Z_1 can be represented as

$$Z_1 = \left[\frac{\frac{R_i}{\varpi^2 C_1}\left(\frac{1}{C_1} + \frac{1}{C_{gs}}\right) - \frac{j}{\omega^3 C_{gs} C_1}\left(\frac{1}{C_1} + \frac{1}{C_{gs}}\right)}{\left(\frac{1}{\omega C_{gs}} + \frac{1}{\omega C_1}\right)^2}\right] \tag{143}$$

$$Z_1 = \left[\frac{\left(\frac{R_i}{C_1}\right)}{\left(\frac{1}{C_1} + \frac{1}{C_{gs}}\right)} - \left(\frac{j}{\omega[C_1 + C_{gs}]}\right) \right] \tag{144}$$

defining the three new variables as

$$C_a = C_1 + C_{gs} \tag{145}$$

$$C_b = C_2 + C_{ds} \tag{146}$$

$$R_a = R_s + \frac{C_{gs}^2}{C_a^2} R_i \tag{147}$$

$$X_a = \omega L_s - \frac{1}{\omega C_a} \tag{148}$$

$$\omega(X_a = 0) = \frac{1}{\sqrt{L_s C_a}} \tag{149}$$

Figure 7 shows a simplified open loop model of the oscillator for easy analysis. In this open loop model, the parasitic elements of the device are absorbed into the corresponding embedding impedances.

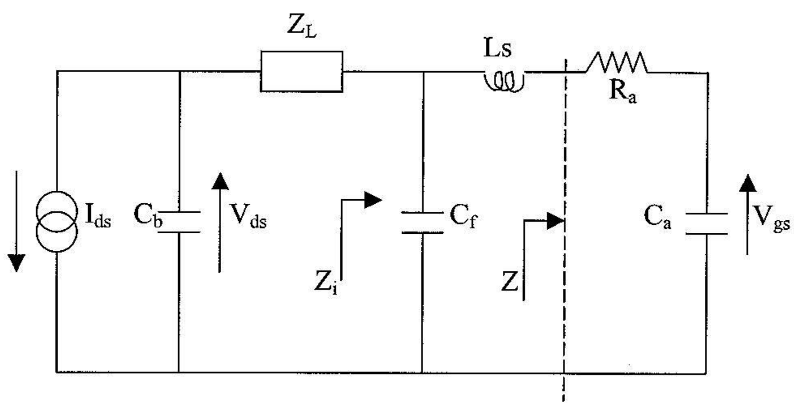

Figure 7 A simplified loop model of the oscillator.

$$Z = R_a + \frac{1}{j\omega C_a} \Rightarrow R_s + \frac{C_{gs}^2}{C_a^2} R_i - \frac{j}{\omega C_a} \tag{150}$$

$$Z + j\omega L_s = R_s + \frac{C_{gs}^2}{C_a^2} R_i - \frac{j}{\omega C_a} + j\omega L_s \Rightarrow \left[R_s + \frac{C_{gs}^2}{C_a^2} R_i \right] + j\left[\omega L_s - \frac{1}{\omega C_a} \right] \quad (151)$$

$$Z_a = Z + j\omega L_s \Rightarrow R_a + jX_a \quad (152)$$

$$R_a = R_s + \frac{C_{gs}^2}{C_a^2} R_i \quad (153)$$

$$X_a = \omega L_s - \frac{1}{\omega C_a} \quad (154)$$

$$Z_i = Z_a \parallel C_f \Rightarrow [Z + j\omega L_s] \parallel C_f \quad (155)$$

$$Z_i = \frac{-j\left[\dfrac{R_a + jX_a}{\omega C_f}\right]}{\left[R_a + jX_a - \dfrac{j}{\omega C_f}\right]} \Rightarrow \frac{[R_a + jX_a]}{[1 + jR_a\omega C_f - \omega C_f X_a]} = \frac{[R_a + jX_a]}{1 + j\omega C_f [R_a + jX_a]} \quad (156)$$

The circuit model of the oscillator is shown in Figure 8, in which the output current through Z_L is given as

$$I = \frac{I_{ds}}{1 + j\omega C_b [Z_i + Z_L]} \quad (157)$$

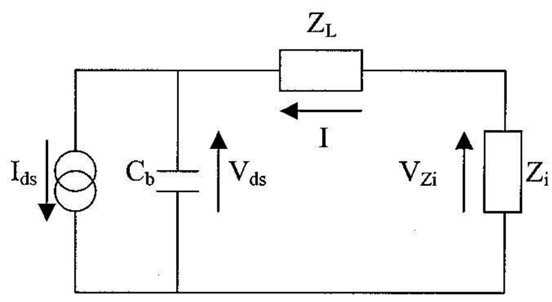

Figure 8 A circuit model of an oscillator.

The voltage across Z_i is given as:

$$V_{zi} = IZ_i = -I_{ds}\left[\frac{[R_a + jX_a]}{1 + j\omega C_f[R_a + jX_a]}\right]\left[\frac{1}{1 + j\omega C_b[Z_i + Z_L]}\right] \qquad (158)$$

Applying the voltage divider in Figure 8, V_{gs} can be expressed as

$$V_{gs} = -I_{ds}\left[\frac{1}{j\omega C_b[R_a + jX_a]}\right]\left[\frac{Z_i}{1 + j\omega C_b[Z_i + Z_L]}\right] \qquad (159)$$

Steady-state oscillation occurs when $I_{ds}(t)=I_I$ and $V_{gs}=V_p$. Consequently, the equation above can be written as

$$1 + j\omega C_b\left[Z_i + Z_L\right] = -\frac{I_{ds}}{V_{gs}}\frac{Z_i}{j\omega C_a(R_a + jX_a)} \qquad (160)$$

$$1 + j\omega C_b\left[Z_i + Z_L\right] = \frac{-g_{mc}Z_i}{j\omega C_a(R_a + jX_a)} \Rightarrow \frac{-g_{mc}[R_a + jX_a]}{j\omega C_a(R_a + jX_a)[1 + j\omega C_f(R_a + jX_a)]} \qquad (161)$$

$$1 + j\omega \mathrm{C_b}\left[\mathrm{Z_i} + \mathrm{Z_L}\right] = \frac{-\mathrm{g_{mc}}}{j\omega \mathrm{C_a}[1 + \mathrm{jwC_f}(\mathrm{R_a} + \mathrm{jX_a})]} = \frac{\mathrm{g_{mc}}}{\omega^2\mathrm{C_f C_a} - \mathrm{j}[\omega\mathrm{C_a} - \omega^2\mathrm{C_f C_a X_a}]} \qquad (162)$$

$$\mathrm{Z_L} = \mathrm{Z_i}\frac{\mathrm{g_{mc}}}{\omega^2\mathrm{C_b C_a}\left[\mathrm{R_a} + \mathrm{jX_a}\right]} - \mathrm{Z_i} - \frac{1}{j\omega\mathrm{C_b}} \qquad (163)$$

$$Z_L = \frac{g_{mc}(R_a + jX_a)}{[1 + j\omega C_f(R_a + jX_a)][\omega^2 C_b C_a(R_a + jX_a)]} - \frac{(R_a + jX_a)}{[1 + j\omega C_f(R_a + jX_a)]} - \frac{1}{j\omega C_b} \qquad (164)$$

$$Z_L = \frac{g_{mc}}{\omega^2 C_b C_a[1 + j\omega C_f(R_a + jX_a)]} - \frac{(R_a + jX_a)}{[1 + j\omega C_f(R_a + jX_a)]} - \frac{1}{j\omega C_b} \qquad (165)$$

where

$$g_{mc} = \frac{I_1}{V_p} = \frac{I_{\max}}{2V_p} \qquad (166)$$

In addition, V_{ds} can be determined by calculating $I_{cb,}$ the current through C_b with the help of Figure 8.

$$I_{cb} = I_{ds}\frac{[Z_L + Z_i]}{Z_L + Z_i + 1/j\omega C_b} \qquad (167)$$

Based on the last result we can conclude that

$$V_{ds} = I_{cb}\frac{j}{\omega C_b} = \frac{[Z_L + Z_i]}{Z_L + Z_i + 1/j\omega C_b} I_1 \tag{168}$$

or in squared magnitude form

$$V_{ds}^2 = \frac{[Z_L + Z_i]^2}{[1 + j\omega C_b (Z_L + Z_i)]^2} I_1^2 \tag{169}$$

Also, Re[Z_L] can be defined as follows

$$\mathrm{Re}[Z_L] = \frac{g_{mc}[1 - \omega C_f X_a - j\omega C_f R_a]}{\omega^2 C_b C_a [(1 - \omega C_f X_a)^2 + \omega^2 C_f^2 R_a^2]} - \frac{(R_a + jX_a)[1 - \omega C_f X_a - j\omega C_f R_a]}{[(1 - \omega C_f X_a)^2 + \omega^2 C_f^2 R_a^2]} \tag{170}$$

$$\mathrm{Re}[Z_L] = \frac{g_{mc}[1 - \omega C_f X_a - j\omega C_f R_a] - \omega^2 C_b C_a (R_a + jX_a)[1 - \omega C_f X_a - j\omega C_f R_a]}{\omega^2 C_b C_a [(1 - \omega C_f X_a)^2 + \omega^2 C_f^2 R_a^2]} \tag{171}$$

$$\mathrm{Re}[Z_L] = \frac{g_{mc}[1 - \omega C_f X_a] - \omega^2 C_b C_a R_a + \omega^3 C_b C_a C_f X_a - \omega^3 C_b C_a C_f R_a)}{\omega^2 C_b C_a [(1 - \omega C_f X_a)^2 + \omega^2 C_f^2 R_a^2]} \tag{172}$$

The power delivered to the load Z_L and the magnitude of V_{ds} can be determined by

$$P_{out} = \frac{1}{2} I^2 \,\mathrm{Re}[Z_L] \tag{173}$$

$$P_{out} = \frac{1}{2} I_1^2 \frac{\mathrm{Re}[Z_L]}{[1 + j\omega C_b (Z_L + Z_i)]^2} \tag{174}$$

$$V_{ds}^2 = \frac{[Z_L + Z_i]^2}{[1 + j\omega C_b (Z_L + Z_i)]^2} I_1^2 \tag{175}$$

$$[1 + j\omega C_b (Z_i + Z_L)]^2 = \frac{g_{mc}^2}{[\omega^2 C_f C_a R_a - j(\omega C_a - \omega^2 C_f C_a X_a)][\omega^2 C_f C_a R_a + j(\omega C_a - \omega^2 C_f C_a X_a)]} \tag{176}$$

$$1+j\omega C_b(Z_i+Z_L)]^2=\frac{g_{mc}^2}{(\omega^2 C_f C_a R_a)^2+(\omega C_a-\omega^2 C_f C_a X_a)^2}=\frac{g_{mc}^2}{\omega^2 C_a^2[(1-\omega^2 C_f C_a X_a)^2+(\omega^2 C_f C_a R_a)^2} \tag{177}$$

Based on the equations above, the output power can be estimated as

$$P_{out}=\frac{1}{2}I^2\,\mathrm{Re}[Z_L] \tag{178}$$

$$P_{out}=\frac{1}{2}I_1^2\frac{\mathrm{Re}[Z_L]}{[1+j\omega C_b(Z_L+Z_i)]^2} \tag{179}$$

$$P_{out}=\frac{1}{2}I_1^2\frac{\left\{\dfrac{g_{mc}[1-\omega C_f X_a]-\omega^2 C_b C_a R_a+\omega^3 C_b C_a C_f X_a-\omega^3 C_b C_a C_f R_a)}{\omega^2 C_b C_a[(1-\omega C_f X_a)^2+\omega^2 C_f^2 R_a^2]}\right\}}{\left\{\dfrac{g_{mc}^2}{\omega^2 C_a^2[(1-\omega^2 C_f C_a X_a)^2+(\omega^2 C_f C_a R_a)^2}\right\}} \tag{180}$$

$$P_{out}=\frac{1}{2}I_1^2 C_a\frac{[1-\omega C_f X_a]-\omega^2 C_b C_a R_a+\omega^3 C_b C_a C_f X_a-\omega^3 C_b C_a C_f R_a)}{g_{mc}C_b} \tag{181}$$

Below 5 GHz, it is valid to ignore some of the terms by assuming that

$$\omega^2 C_b C_a R_a >> \omega^3 C_b C_a C_f X_a \tag{182}$$

$$\omega^2 C_b C_a R_a >> \omega^3 C_b C_a C_f R_a \tag{183}$$

The power output is now expressed as

$$P_{out}=\frac{1}{2}I_1^2 C_a\frac{g_{mc}[1-\omega C_f X_a]-\omega^2 C_b C_a C_f R_a)}{g_{mc}^2 C_b} \tag{184}$$

$$P_{out}=\frac{1}{2}I_1^2\left[C_a w\frac{[1-\omega C_f X_a]}{\omega C_b}-\frac{\omega^2 C_a^2 R_a}{g_{mc}^2}\right] \tag{185}$$

$$P_{out}=\frac{1}{2}I_1^2\left[\alpha\frac{[1-\omega C_f X_a]}{\omega C_b}-\alpha^2 R_a\right] \tag{186}$$

$$\alpha = \frac{\omega C_a}{g_{mc}} \tag{187}$$

In a similar manner, V_{ds} is given by

$$V_{ds}^2 = I_1^2 \left[\frac{\alpha^2 [1 - \omega C_f X_a]^2 + [1 - \omega C_f R_a]^2}{\omega^2 C_b^2} \right] \tag{188}$$

Both the output power and V_{ds} depend on C_b if the other parameters are fixed.

This is a limitation for the maximum value. However, a maximum value of the current and the voltage a transistor can take before burn-out. Therefore, by setting $|V_{ds}| = V_{dsm}$ an optimal condition can be given by [1]:

$$\frac{|V_{ds}|^2}{I_1^2} = \frac{|V_{dsm}|^2}{I_1^2} = \frac{\alpha^2 [1 - \omega C_f X_a]^2 + [1 - \omega C_f R_a]^2}{\omega^2 C_b^2} \tag{189}$$

The optimum load impedance that the device needs to see to deliver the highest power is defined as

$$\frac{|V_{ds}|}{I_1} = \frac{2|V_{dsm}|}{I_{\max}} = R_{opt} \tag{190}$$

leading to the following definition

$$\omega C_b R_{opt} = \sqrt{\alpha^2 [1 - \omega C_f X_a]^2 + [1 - \omega C_f R_a]^2} \tag{191}$$

Using the result above, the optimum P_{out} is, therefore, given by

$$P_{out} = \frac{V_{dsm} I_{dsm}}{4} \alpha \frac{[1 - \omega C_f X_a]}{\sqrt{\alpha^2 [1 - \omega C_f X_a]^2 + [1 - \omega C_f R_a]^2}} - (\omega C_a V_p)^2 \frac{R_a}{2} \tag{192}$$

The first term is the power available from the current source and the second term is the power absorbed by R_a. This also indicates that a high Q inductor minimizes the absorbed power, increasing the power available from the current source. P_{out} simplifies further at the oscillation frequency since $X_a \approx 0$.

$$P_{out} = \frac{V_{dsm} I_{dsm}}{4} \alpha \frac{1}{\sqrt{\alpha^2 + [1 - \alpha\omega C_f R_a]^2}} - (\omega C_a V_p)^2 \frac{R_a}{2} \tag{193}$$

The above analytical analysis gives the following important results:

1) Maximum output power is attained if we set

$$C_f = \frac{1}{\alpha\omega R_a} \tag{194}$$

and

$$P_{out}(\max) = \frac{V_{dsm} I_{\max}}{4} \left[1 - \frac{1}{G}\right] \tag{195}$$

$$\frac{1}{G} = \frac{P_f}{P_{av}} = \omega^2 C_a^2 R_a \frac{2V_p^2}{V_{dsm} I_{\max}} \tag{196}$$

Accordingly, the DC/RF conversion efficiency is calculated by

$$P_{dc} = \frac{V_{DS} I_{\max}}{\pi} \tag{197}$$

$$\eta_{\max} = \frac{P_{out}(\max)}{P_{dc}} \tag{198}$$

$$\eta_{\max} = \left[1 - \frac{1}{G}\right] \frac{V_{dsm}}{V_{DS}} \tag{199}$$

In order to maximize the oscillator output power and efficiency, the loss resistance R_a of the input circuit has to be reduced (increasing G), and an optimal biasing condition V_{DS} has to be selected.

2)

$$C_b = \frac{[1 - \omega C_f R_a] C_a}{g_{mc} R_{opt}} \tag{200}$$

$$C_b(C_f = 0) = \frac{C_a}{g_{mc} R_{opt}} \tag{201}$$

3) Combining the above equation leads to expressions for Z_L in terms of

$$Z_L = \frac{1 + j\alpha}{1 + \alpha^2} R_{opt} \tag{202}$$

From these analytical calculations, the following results were achieved. The circuit simulation of the oscillator done was using a nonlinear Materka model.

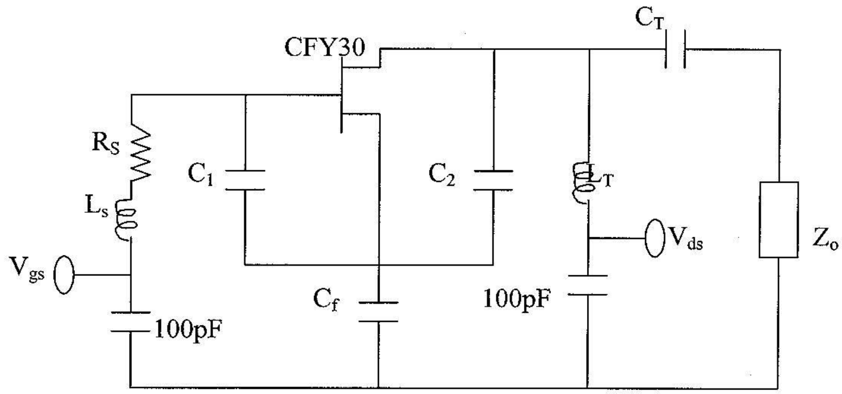

Figure 9 Schematic diagram of the oscillator operating at 950 MHz as published in [1].

Figure 9 above shows the schematic diagram of a practical oscillator operating at 950MHz. A simple high-pass filter consisting of L_T and C_T is used to transfer the Z_0/50Ω load to the required Z_L value.

From above expression all the effective components of the oscillator can be given as:

1. Bias condition:

$$V_{DS} = 5V$$

$$I_{DS} = 18mA$$

2. Device Parameters:

$$I_{\max} = 45mA$$

$$V_P = 1.25V$$

$$V_K(knee - voltage) = 0.5V$$

3. Device Parasitic:

$$C_{gs} = 0.5pF$$

$$C_{ds} = 0.2pF$$

$$C_{gd} = 0.0089pF$$

4. Oscillator Parameters:

$$\omega = \frac{1}{\sqrt{L_s C_a}} \Rightarrow f = 950MHz$$

$$C_1 = 6pf$$

$$C_2 = 1.5pf$$

$$C_f = 20pF$$

$$L_s = 3.9nH$$

$$C_a = C_1 + C_{gs} = 6.5pF$$

$$C_b = C_2 + C_{ds} = 1.7pF$$

$$L_f^{'} = 18nH$$

$$C_f^{'} = 15pF$$

$$R_a = R_s + \frac{C_{gs}^2}{C_a^2} R_i = 4\Omega$$

5. Output matching circuit:

$$L_d(package) = 0.7nH$$

$$L_T = 8.9nH$$

$$L_T^{'} = 8.9nH - L_d = 8.7nH$$

$$C_T = 1.91pF$$

6. Calculation of R_{opt}:

$$I_{dc} = \frac{I_{\max}}{\pi}$$

$$I_1 = \frac{I_{\max}}{2} = 22.5mA$$

$$R_{opt} = \frac{V_{dsm}}{I_1} = \frac{V_{DS} - V_K}{I_1} = \frac{5V - 0.5V}{22.5mA} = 200\Omega$$

7. Calculation of Z_L:

$$Z_L = \frac{1 + j\alpha}{1 + \alpha^2} R_{opt}$$

$$g_{mc} = \frac{I_1}{V_p} = \frac{I_{\max}}{2V_p} = \frac{45mA}{2 * 1.25} = 18.8mS$$

$$\alpha = \frac{w_0 C_a}{g_{mc}} = \frac{2 * \pi * 950E + 6 * 6.5E - 12}{0.0188} = 2.0$$

$$Z_L = \frac{1 + j\alpha}{1 + \alpha^2} R_{opt} = \frac{1 + j2}{1 + 4} * 200 = 40 + j80\Omega$$

8. Output power:

$$P_{out}(\max) = \frac{V_{dsm} I_{\max}}{4} \left[1 - \frac{1}{G} \right] = 16.6dBm$$

$$\frac{1}{G} = \frac{P_f}{P_{av}} = \omega^2 C_a^2 R_a \frac{2V_p^2}{V_{dsm} I_{\max}}$$

9. DC-RF conversion efficiency:

$$P_{dc} = \frac{V_{DS} I_{\max}}{\pi} = \frac{5 * 45mA}{\pi} = 71.62mW$$

$$\eta_{\max} = \frac{P_{out}(\max)}{P_{dc}} = \frac{45.7mW}{71.62mW} = 0.64$$

$$\eta_{\max} = 64\%$$

Simulated Results

Figures 10, 11, 12, 13, 14, and 15 show the oscillator test circuit and its simulated results. After the oscillator circuit is analyzed in the harmonic-balance program, the oscillator frequency is found to be 1.08 GHz, and some tuning is required to bring the oscillator frequency back to the required value by changing L_s from 3.9nH to 4.45nH. The slight shift in the oscillator frequency may be due to the device parasitic. The simulated power output is 17.04 dBm, which is about the same as the measured value by [1]. The DC to RF conversion efficiency at the fundamental frequency is 55%. The calculation in [1], as well as the calculation here, assumes an ideal transistor. By finding a better value between C_1 and C_2, the efficiency was increased to 64%, compared to the published result of 55%. This means that the circuit in [1] has not been fully optimized.

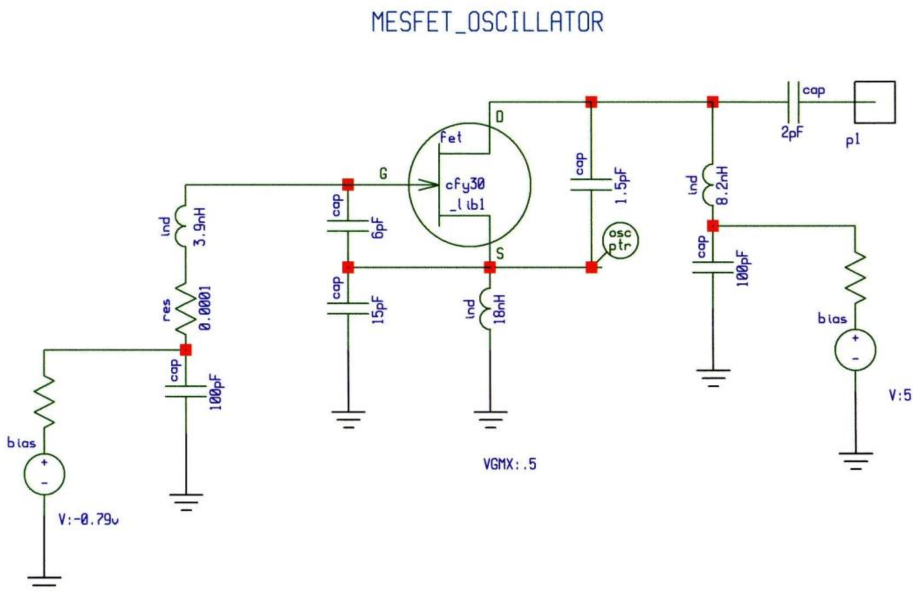

Figure 10 Schematic of the test oscillator based on [1].

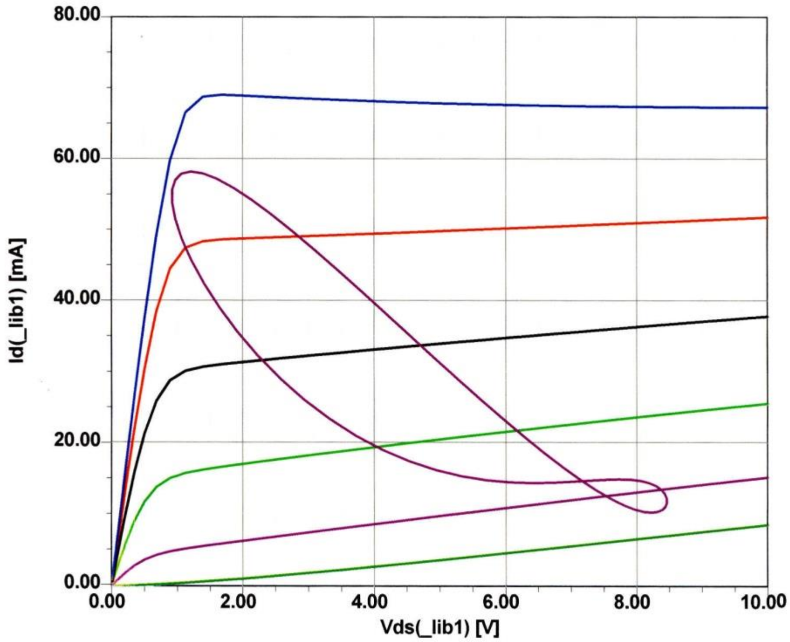

Figure 11 Load line of the oscillator shown in Figure 10. Because the load is a tuned circuit, the "load line" is a curve and not a straight line.

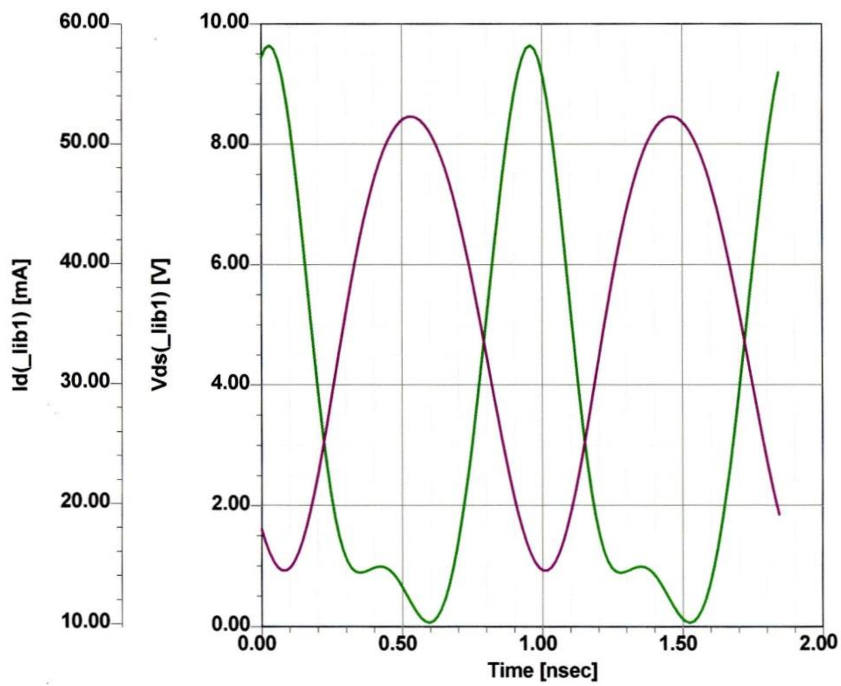

Figure 12 Plot of drain current and drain source voltage as a function of time.

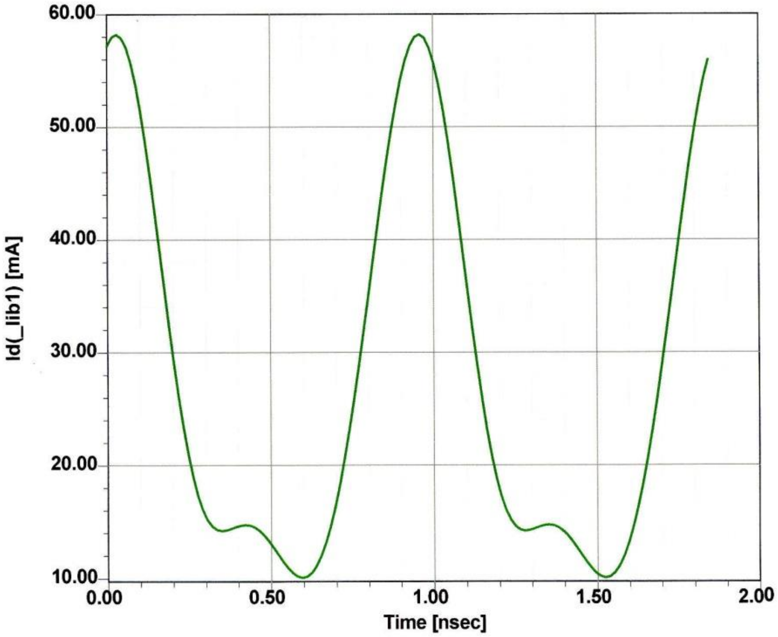

Figure 13 AC drain current simulated for Figure 10.

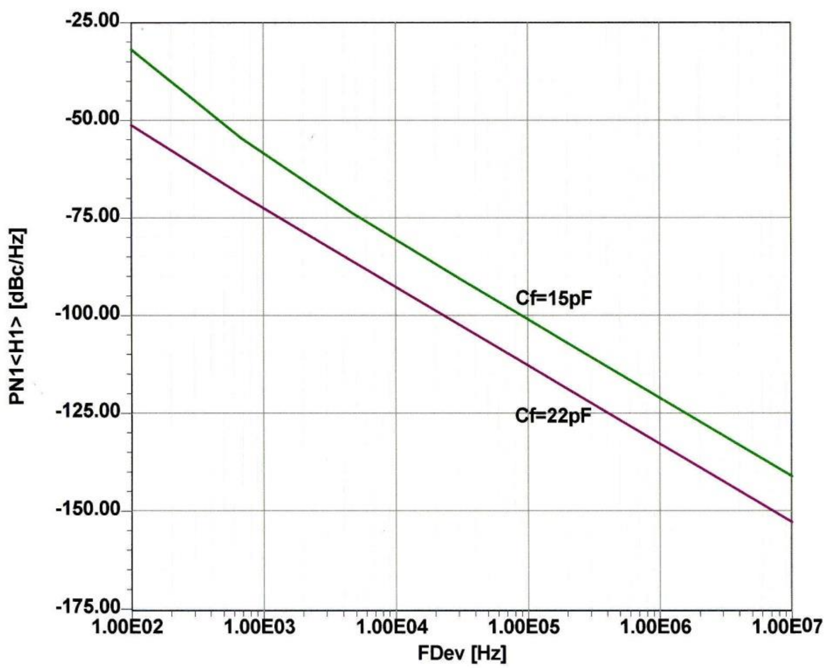

Figure 14 Simulated noise figure of the circuit shown in Figure 10. An increase of the feedback capacitor from 15 to 22pF improves the phase noise.

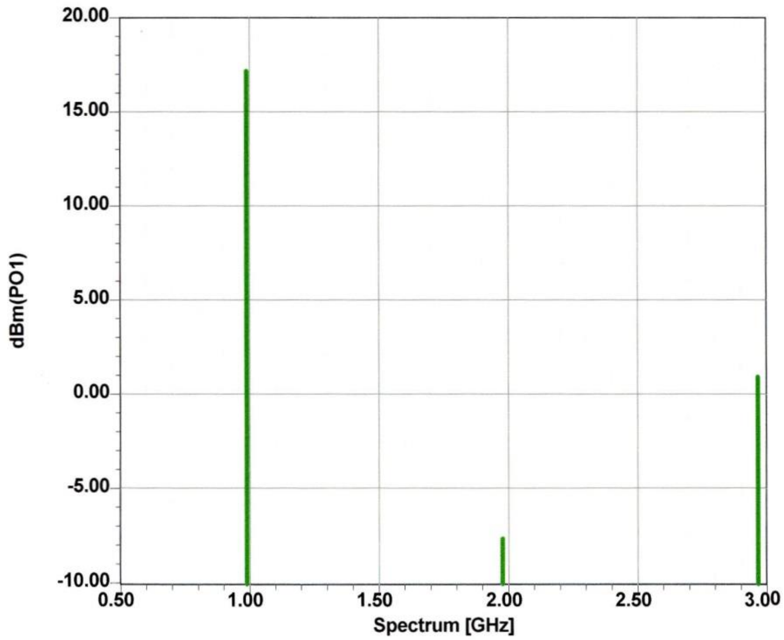

Figure 15 Simulated output power of the oscillator shown in Figure 10.

Taking the published experimental results [1] into consideration, the analytical expression gives excellent insight into the performance of the oscillator circuit.

The maximum achievable output power and efficiency for a given active device can be predicted through the closed-form expressions without the need of a large-signal device characterization and an HB-simulation. The publication [1] has not addressed the power optimization and best phase noise, which is a very important requirement for the oscillator. By proper selection of the feedback ratio at the optimum drive level, the noise is improved by 8dB, keeping the output power approximately the same. In [2] we discuss fixing the optimum feedback ratio and the absolute values of the feedback capacitor, with consideration for the best possible phase noise.

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